# 1

# Introduction

## 1.1 Motivation and Structure of the Book

A variety of on-going research is focused on the development and analysis of methods to decrease the time required to progress from the computer modeling of the design surface to the machining while maintaining or improving the quality of the surface. One of the most important areas is tool path planning for numerical control (NC) machining. The main goal is obtaining the cutter location and orientation data that allow for an efficient surface milling within an allowed machining error.

Five-axis NC machines are becoming increasingly popular due to their ability to handle geometrically complex workpieces composed of raw material such as wood, wax, rubber, metal, stone, plastic, etc. Moreover, up-to-date five-axis NC machines are characterized by a high material removal rate and an efficient surface finish up.

Typically, manufacturing of the design surface by an NC machine comprises two stages, a rough cutting and a finish machining. During the rough cutting, the raw material is removed as fast as possible while ensuring no excessive cutting or gouging. During the finish machining, the tool is placed at the maximum contact with the surface to remove the remaining excess and create a well-finished and accurate surface. After finishing, the remaining scallops which are inevitably generated on the machined surface must be removed by manual surface grinding and polishing. The finish machining and manual polishing stages require as much as 75% of the total machining time. Besides, manual polishing is prone to error and undesirable irregularities.

Five-axis machining offers an improvement in efficiency of both the rough and finish machining stages over the three-axis counterpart. In five-axis machining, the tool orientation relative to the workpiece can be controlled by two additional degrees of freedom so as to achieve higher machining efficiency. With these advantages, a large number of tool path planning methods for five-axis machining has been developed and presented in the literature.

In chapter 1 we present the most popular CAD/CAM data formats and give a short literature survey on mathematical methods for optimization of five-axis machining. The survey has been focused on tool path interpolators, adaptable geometric patterns and methods for tool posture and gouging avoidance.

Chapter 2 exposes the readers to basic knowledge required to perform five-axis cutting. The G-code programming, examples of five-axis machining of simple shapes and verification of the cut using solid modeling software is presented and discussed. The chapter can be used as a short introduction into five-axis machining in the framework of an undergraduate course in Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM).

Chapter 3 introduces theories required to embrace the concepts of the tool path optimization for five-axis machining. The chapter presents such fundamental issues as kinematics of the five-axis machines, part surface representation, machining strip, tool orientation and gouging avoidance as well as the forward step error. A variety of configurations of the five-axis machines is also discussed and analyzed. This chapter can also be used at an undergraduate or a postgraduate level for CAD/CAM related studies.

Chapters 4 and 5 present advanced optimization schemes based on the adaptable geometric patterns, namely, the space-filling curves (SFC) and adaptive curvilinear grids. The SFC tool path has a number of attractive features such as the possibility to locally adapt the curve in such a way that the cutting device travels along the optimal direction. In addition, the entire surface is cut in one path eliminating the need of tool retractions. The use of the concept of curvilinear grids allows to simultaneously adapt the points on the tool paths to create more efficient zigzag, spiral or even SFC structures. The combination of the SFC and grid generation allows for tool paths on surfaces with complex irregular boundaries, cuts off, pockets, islands, etc.

Three-axis machines are often thought of as three dimensional plotters. However the five-axis machine is more like a big bore machine. That is why changing from three-axis to five-axis programming is not an easy task. In particular, the idea of optimizing rotations may seem totally foreign from the viewpoint of the three-axis machining. Therefore, chapter 6 presents the theory and practice of optimization of rotations for five-axis machining. Several optimization algorithms based on the shortest path techniques are presented and discussed.

Chapter 7 presents a theoretical background developed to construct numerical algorithms to minimize kinematics error introduced by the initial setup of five-axis milling machines. The initial setup consists of the position and orientation of the workpiece with respect to the mounting table and, optionally, the machine's initial configuration. Given a set of cutter contact points and tool orientations, a least-squares optimization procedure finds the optimal setup parameters.

## 1.2 CAD/CAM Formats

Sculptured or free-form surfaces are widely used in today's manufacturing industries for a variety of applications such as the production of dies, molds, aerospace and automotive parts, etc. The surfaces are usually characterized by complex geometries and variable curvatures. A single surface is usually composed of patches represented mathematically by parametric forms such as the Bezier surfaces, B-splines and NURBS. A design and manufacture of the sculptured surface parts is an expensive and time-consuming process. First, a design surface is transformed into a computer model (possibly with the help of CAD programs). The computer model is then used by the CAM programs to generate commands to move the cutting tool of the machine. The resulting set of tool positions and orientations constitutes a tool path to machine the desired surface.

Every CAD or CAD/CAM software uses an internal format to represent and control the required part. When the geometrical data is transferred from a CAD system to a CAD or CAM system, a neutral format for the data transfer is used. One of the most popular is the IGES (Initial Graphics Exchange Specification) format (see the history of the IGES format in [40]). The IGES format supports the use of surfaces defined by NURBS (Non Uniform Rational B-Splines) or derivatives of these representation. A good book for a beginner in NURBS is [31]. An advanced reader could use [98] and [24]. There are also several free libraries designed to control and manipulate the NURBS, such as NURBS++ package [1] and NURBS Toolbox [2]. Some free IGES-file processing tools are collected at the NIST/IGES web page [3].

The STL files, originally employed by the so-called layered manufacturing technologies such as the rapid prototyping, are now becoming more and more popular due to the simplicity of describing the part surfaces. As opposed to the complex description of surfaces employed by the IGES format, the STL format defines the surface as a collection of triangles each described by the coordinates of its three corners and a normal-vector. This technology provides an important platform for CAD/CAM applications due to the existence of many robust triangulation algorithms. Besides, the surface models are often composed of many patches. Therefore, by tessellating the patches and creating groups of triangles one can use many well established methods for treating intersections, trimming, shading, hidden surface removal and gouge protection [36]. The machining is usually performed by creating contours obtained by slicing the STL-surface [85, 115]. Of course, slicing of the NURBS surfaces is also possible, but it requires much more sophisticated techniques such as [81]. The contours are then saved using the SLC format (see [4] for instance).

Other popular CAD/CAM formats include STEP (Standard for the Exchange of Product Data), DXF (The Drawing Exchange Format from AutoDesk) and many others. As a matter of fact, the difference in data formats has created a large software industry specializing in transferring, adapting and processing the CAD/CAM files. A table of compatibility of the CAD/CAM formats can be found, for instance, at [5] or [6].

## 1.3 Short Literature Survey

Optimization of tool paths for five-axis machining may include many features and multiple criteria such as the accuracy, the length of the tool path, the machining time, the size of the remaining scallops, etc. It may also include gouging avoidance, satisfying the machine axis limits, maximizing the volume of the removed material, reducing the tool wear. The optimization may also take into account the thermal characteristics of the cutting process, the tool bending, the vibrations and jacks, the workpiece positioning and many other parameters. The criteria could also include the configuration of the machine or specific parts of the machine as well as the design of the clamping device. Readers interested in citations before 1997 could use a fairly comprehensive survey by Dragomatz and Mann [25]. The survey presents a classification of research papers on three-axis and five-axis machining related to geometries of the tool paths and tool positioning. These categories include: 1) systems, 2) isoparametric paths, 3) non-isoparametric paths, 4) planar pocketing paths, 5) sculptured surface pocketing paths, 6) roughing paths, 7) tool positioning, 8) offset surface methods, 9) five-axis machining, 10) mesh models, 11) pixel and point models, 12) simulation and verification. Of course, the above groups overlap. Techniques involved in one group could be also involved in another group. For example, systems for tool path generation may include all of the above mentioned techniques. Roughing paths may be generated by the isoparametric or non-isoparametric schemes and so on.

Our survey is focused on five-axis machining. It also includes the three-axis methods but as long as they can be extended to the five-axis case. Besides, we confine ourselves by techniques designed for cutting the part surface by bottom-edge of the tool, e.g., flat-end milling and fillet milling. Many interesting methods designed for five-axis grinding (flank milling) and plunge milling have not been included.

The survey has been focused on the following categories: 1) *tool path interpolators*, 2) *adaptable geometric patterns* and 3) *methods for tool posture and gouging avoidance.* We believe that the above procedures are the most important part for efficient design of the numerical methods for five-axis machining.

### Tool path interpolators

In the CNC machines, the tool motion is controlled by a sequence of reference points that are fed to the servo control system. The NC controllers employ linear interpolation techniques [66] or a circular interpolation which may result in discontinuities of the velocity at the junctions of the segments. They may also result in high accelerations, subsequent surface inaccuracies and long machining time required to eliminate them. Furthermore, the modern high speed machining requires feedrates up to 40 m/min with accelerations up to 2 g. At such high speeds, small discontinuities in the reference tool path can result in undesirable high frequency harmonics in the reference trajectory, which may end up exciting the natural modes of the mechanical structure and the servo control system.

Although the NC program cannot change the way the controller moves the machine parts, the cutter location points and the rotation angles required to cut the prescribed curve can be changed in such a way that these errors are minimized or at least decreased. The task of generating such a sequence of points is called interpolation. Early interpolation schemes solved the problem of discontinuities by smoothing the tool path at the corners [17] and using low pass filters [126]. However, the problem is due to the difference between the chord and the arc lengths. That is why, generating the tool positions by incrementing the chord length leads to the feedrate instabilities. Therefore, if the fit curve is parameterized with respect to the arc length, these type of the inaccuracies will be eliminated. Therefore, many modern interpolation schemes are focused on finding a suitable polynomial interpolation (such as the B-splines) parameterized by means of the arc length.

Unfortunately, such parameterization is not analytically possible for general spline curves. Therefore, a number of approximate solutions were proposed. For instance, Wang and Yang [122] generate the trajectory by means of cubic and quintic splines using the chord length and a *nearly arc length parameterization*, Zhang and Greenway [133] implemented a similar B-spline based interpolation. Coordinate transformations between the workpiece and machine coordinate systems for five-axis milling were incorporated in the interpolator by Lo [77, 79] and Bohez et al. [14]. An extra jerk continuity condition has been included into the solution in [121].

Furthermore, the limitations of the machine tool drivers may cause failure in maintaining the commanded feedrate which in turn may lead to the tool chatter or breakage. Therefore, Weck et al. [125] have implemented cubic spline interpolation where adaptation of the feedrate was based on the physical limitations of the drives. The smooth transitions were obtained using fourth order acceleration profiles. Erkorkmaz and Altintas [30] presented a quintic spline trajectory generation algorithm that produced continuous position, velocity, and acceleration profiles. Smooth accelerations and decelerations were provided by imposing constrains on the first and second time derivatives of the feedrate.

In 1994, Farouki and Sakkalis [33] introduced the Pythagorean-Hodograph (PH) curves to solve the problem of feedrate control for three-axis machines. The curves provide a mathematically elegant solution to the above mentioned problems occurring in NC machining. In particular, the arc length was represented by a polynomial function of the curve parameter. In [32], a 2D Hermite interpolation combined with the PH was proposed and analyzed. The ideas were further developed in [34, 35]. It was shown that since the arc length of the PH curves can be represented by a polynomial function of the curve parameter, they can be successfully used for the interpolation. Consequently, a variety of planar PH curves matching given Hermite type boundary data were developed (see, for instance, [61, 88, 119]).

Müller et al. [90] presented an algorithm for simultaneous five-axis spline interpolation which merges the PH interpolation and the analytic solution of the inverse kinematics problem using the template equation method. The result is a time-dependent spline which represents the given tool path with a high accuracy. Langeron et al. [69] suggested a polynomial B-spline interpolation which took into account the kinematics of the five-axis machine. The B-spline interpolation of the tool path in the part coordinate system includes the accuracy requirements and describes a five-axis tool path in a format adapted to the communication between the CAM software and the NC unit. The CAM output is directly expressed through the B-spline curves. Lo [80] introduced spline interpolators for *isoparametric, iso-scallop* and *iso-planar* machining methods (see the forthcoming section). Šír et al. [118] presented biarc interpolation techniques based on spline curves composed of circular arcs and compared them with the PH curves.

Finally, a number of papers introduce interpolators designed for high speed milling. During the high speed machining the actual average feedrate could be significantly lower than the programmed feedrate due to the physical restrictions of the machine tool and the block processing time of the CNC controller. In many cases the machine tool hardly reaches the maximum feedrates offered by the manufacturer. This happens when the block processing time is longer than the block execution time and the machine reaches the end point of the segment before information required for the next movement is available. In this case modern CNCs automatically reduce the programmed feedrate which results in a lower real feedrate and, consequently, a longer machining time. This relatively new issue has been discussed in [55, 64, 87, 109].

### Adaptable geometric patterns

This section surveys research aimed to construct geometric patterns adaptable to a criteria which represents a certain estimate of the quality of the tool path such as the kinematics error, scallop heights, undercuts, overcuts, etc. It also includes methods for complex pocket milling since they often require special geometric patterns. Finally, the construction of the geometric patterns might or might not take into account the actual machine kinematics. In many cases, patterns employed for three-axis machining are also applicable, with certain modifications, to the five-axis machining.

The simplest tool path planning algorithms employ structured zigzag or spiral patterns due to their simplicity and the ease of computation (see [25, 108]). The zigzag and spiral motions employ uniform steps along a coordinate which parametrizes the desired curve extracted from the part surface.

The early adaptable methods replace the uniform spacing in favor of distributing the cutter location points by analogy with interpolation characterized by a variable step. First, the trajectories were assumed to be linear. Next, the desired curve was approximated using a certain technique, for example, employing arcs. Next, the forward step was selected by considering the deviation between the approximation and the straight line (see, for instance, [73]). The choice of the forward step can be performed by bisection or another inexpensive method. Some of the recent developments of these ideas are presented in [22, 73].

As mentioned before the most popular geometric solutions are the *zigzag* and the *spiral isoparametric* patterns constructed for single-patch or multi-patch parametric surfaces $S(u, v)$. In this context, the term *isoparametric* means that the zigzag tool path is generated in the parametric space $u - v$ along one of the coordinates, say, $u$. The $v$ coordinate is then used to generate the forward steps.

Another approach is the *contour based* or *iso-planar* machining. In this case the cutter path follows intersection curves of the parametric surface and a series of vertical planes. One of the first papers reporting such techniques is [19], see also [104].

The both methods calculate the maximum allowable distance between the consecutive tracks using a scallop height limitation. However, if the maximum allowable distance is calculated globally, that is, the minimal allowable distance is taken from all the maximum allowable (pointwise) distances, then the method does not produce a constant scallop height. As the result, the machining efficiency is limited. Methods to maintain the constant scallop height called the *iso-scallop* machining methods were first proposed in [76, 112].

Lo [78] developed these approach and adapted it to five-axis machining. His algorithm starts with an initial curve in the parametric domain and calculates offset curves so that the scallop height remains approximately constant. The algorithm is designed for flat-end cuter and includes adaptive inclination which maximizes the machining strip. The algorithm also includes *a local gouging avoidance*. The local gouging refers to the removal of an excess material in the vicinity of the cutter contact point (CC point) due to the mismatch in curvatures between the tool as it is carried along the tool path and the desired surface (see the forthcoming section for details). Rao and Sarma [103] introduced similar *local gouging* avoidance algorithms applicable to surfaces characterized by low curvature and cut by the flat-end cutter. Finally, Lo [80] presents an iso-scallop tool path for *ball nose* cutters.

Evaluation of the machining strip versus the inclination and the direction could lead to complicated tool path topologies. For each tool position on the surface there exists at least one direction which maximizes the machining strip. The corresponding set of vectors mapped onto the parametric space $(u, v)$ constitutes a 2D vector field which could be further analyzed. A continuous tool path which visits every point and follows the optimal direction at every point constitutes the optimal tool path which will maximize the machining strip globally. However, such a path can rarely be found in practice due to the complexity of the resulting vector field. An algorithm to find a suboptimal solution of this problem is presented in [20]. The authors introduce an "initial" tool path which has the largest average machining strip. Next, the entire tool path is constructed by propagating the initial path inside the region until it substantially deviates from the streamlines.

A few papers explore other "iso" methods such as the iso-distance and the iso-curvature methods (see, for instance, [47]). In [63] an additional tool path segments are appended to the basic tool path in order to achieve constant cutting forces and to avoid chatter vibrations in the entire machining area. Furthermore, the necessity to create geometric patterns suitable for the so-called pocket milling leaded to a series of methods designed for parts with one or more complex shaped "islands" inside. The methods are also needed to machined the so-called trimmed surfaces when the boundaries of the surface are defined by intersections with other surfaces.

In 1998, Choi and Jerard [21] introduced a term *regional milling* referring to situations when the machining operation, occurs in a region specified by boundary curves. According to [96] the regional milling can be performed using the same types of tool path topologies, namely, the *contour-based offset type* and the *direction-parallel type*. The *direction-parallel type* has been analyzed in [45, 46, 49], whereas the *direction-parallel type* in [21, 48, 51, 94].

One of the most important problems in the contour-based machining is linking the contours in such a way that the number of tool retractions is minimized. Held et al. [51] presents an algorithm designed for this type of machining based on the proximity maps and the Voronoi diagrams. The author suggests a linking procedure requiring a spanning tree of the planar graph of the monotonic pouches. Park and Chung [95] propose a contour linking algorithm accommodating minimization of slotting, tool-retractions and drilling holes.

Park et al. [96] presented a tool path linking algorithm, which guarantees a "zero" number of tool-retractions. The algorithm employs the concept of tool path element net providing information on the parent/child relationships. Jeong and Kim [57] present an algorithm designed to offset the boundary curves in the complex shaped region using the Voronoi diagram. Each curve segment is offset within the corresponding Voronoi polygon to avoid the degeneracy problem. Jeong and Kim [58] introduce a distance map algorithm which effectively finds the characteristic points and self intersection points of the offset curve segments and as the result eliminates such topological problems as loops, ridges and cusps. A forward locus tracing method is introduced in [68]. The algorithm searches for all intervals split by intersections of the planar curves and maps the 2D transversal intersections onto a 1D interval.

Suppose that the part is partitioned into a grid of cells each of them being a curvilinear triangle or a curvilinear rectangle. The optimization can then be considered as constructing a path which visits each cell, does not have intersections, requires minimal number of tool retractions and satisfies some error related criteria. Pocket machining using staircase or window frame patterns were proposed in [97]. Hansen and Arbab [46] developed a scan line algorithm for generating NC tool paths for arbitrarily shaped flat bottom pockets with islands. Flat pocket machining based on grids was suggested by Bao and Yim [11].

Treating the tool path generation as a navigation problem on grids leaded to approaches exploiting the shortest path optimization and related techniques. Suh and Shin [110] developed a neural network model to obtain the tool path in rough pocket machining as a solution to the traveling salesman problem. A good mathematical analysis of such strategies is given in [9]. The problem is formulated as follows: given a region in the parametric plane and the shape of a cutter find a shortest tour/path for the cutter such that every point within the region is covered by the cutter at some position along the tour (tool path). Additionally the cutter could be constrained to stay within a certain region. Narayanaswami and Choi [91] present a grid-based 3D navigation approach for generating NC tool path data for both linear interpolation and a combination of linear and circular interpolation for three-axis milling. The approach can be extended to the five-axis case.

The space-filling curves (SFC), having been applied in computer graphics, image processing, information systems, can be also seen as a suitable navigation pattern for generation of five-axis paths. The first application of the SFC to NC tool path generation was reported in [23, 44]. Griffiths [44] proposed the use of the Hilbert's curve as a tool path, while Cox et al. [23] used various forms of space-filling curves such as Moore's curve. Fractal based techniques were suggested by Chen et al. [18].

However, neither SFCs nor fractals have never been very popular in the five-axis machining community due to a large number of sharp turns produced by the conventional SFCs. A concept of an adaptive space-filling curve for tool path planning for five-axis NC machining was proposed in [8]. The space-filling curves, adapted to the local optimal cutting direction, produce shorter tool paths. Besides, the tool path correction stage suggested in [8] makes it possible to eliminate the effect of sharp angular turns which characterize the standard SFC patterns. These techniques will be presented in Chap. 4 of this book.

Finally, the entire tool path can be considered in the framework of the grid generation technologies. The concept was first introduced by Makhanov [82] and developed in [15, 83]. The grid generation techniques are surprisingly well-adapted to tool path optimizations. As a matter of fact, the concept of a grid refinement contains almost all the main ingredients for tool path planning, such as grid adaptation to the regions of large milling errors, possibility to easily construct curvilinear versions of the conventional zigzag and spiral patterns and adaptation to constraints related to the tool diameter and the scallop height. Moreover, in contrast to the standard techniques characterized by a local error estimate, grid generation deals with a global spatial error and consequently adapts all the CL points simultaneously. These ideas were developed further in [84], specifically for five-axis machining whereas Bieterman and Sandstrom [12] suggested a similar approach, independently. Finally, Sun et al. [111] presented a spiral version of the grid generation algorithm applied to tool path generation. The advanced grid generation techniques for five-axis machining will be considered in Chap. 5 of this book.

### Tool posture and gouging avoidance

This section deals with techniques providing an optimal position and orientation of the tool in a particular neighborhood of the machined surface. The emphasis is on the orientations prescribed independently with regard to a certain criteria such as the local gouging avoidance, machining strip, scallop height, avoiding global gouging constraints, etc. The most important application of these techniques is cutting the part surface by the flat-end or fillet mill.

In 1987, Marciniak [86] showed that in five-axis machining the maximum width of machined strip on the surface could be obtained if the tool moved on the surface approximately along the minimum curvature line. The maximum width of the strip depends on the difference of the surface main curvatures at the contact point. Some of the early research papers exploiting this idea are [37, 67, 74]. Furthermore, Kruth and Klewais [67] introduced an optimal milling direction parallel to the principal direction of the surface with the minimum curvature.

Gani et al. [37] notice that "One of the critical problems in five-axis milling is the positioning of the cutter in relation to the surfaces in order to machine without having overcut (gouging) or undercut. Because of this problem, ball-end cutters are preferred. Undercutting does not cause a big problem when using ball-end cutters. The calculation of the NC tool path for ball-end cutters is mainly a problem of surface offset. An important drawback of ball-end cutters is the varying cutting speed along the tool radius. The maximal cutting speed is reached on the tool diameter, and at the tool tip it is zero. This leads to cutting edge chipping as well as poor surface roughness".

Recall that as long as the five-axis machines are considered, the tool has five degrees of freedom relative to the surface. The three spatial degrees are used to locate the tool at the cutter location points point. The extra two rotational degrees are used to establish the orientation of the tool represented by the *inclination angle* and the *tilt angle* (see, for instance, [103]) or the *tilt angle* and the *yaw angle* [60]. The angles are evaluated in a local coordinate system usually defined by the feed direction, the surface normal and the corresponding cross product vector. In the case of the flat-end mill the boundary of the base of the tool, which is the part of the tool cylinder is called the *cutting circle* of the tool. The *effective cutting shape* (also referred to as the *tool swept section*) is defined as a projection of the base of the tool onto the plane normal to the feed direction. Actually, in the case of flat-end mill the projected bottom edge becomes an ellipse called the *effective cutting ellipse.*

The parameters of the ellipse depend on the tool orientation. The *local gouging* (or the *curvature interference*) is usually defined as the excess material removal in the vicinity of the cutter contact point due to the mismatch in curvatures between the tool cutting edge and the desired surface. Detecting and avoidance of the local gouging includes comparing the *curvature of the effective cutting shape* (also referred to as *effective cutting curvature*) and the normal curvature of the surface evaluated in the same plane where the effective cutting shape is defined.

If the effective cutting curvature is greater than the normal curvature of the surface then the local gouging will not occur. The mathematical description of the effective cutting shape for a flat-end cutter is given in [129] and for a fillet end mill in [71]. The gouging is then avoided by determining the smallest inclination angle that ensures the largest material removal, that is, the largest machining strip. Of course, gouging is still possible because the curvatures are compared only in one section. In order to eliminate this source of errors, Lee and Ji [72] suggested to compare the curvature of the effective cutting shape evaluated in two planes: along the tool path and normal to the tool path. These effective cutting curvatures are compared to the normal curvatures of the surface in the respective planes and the inclination angle is computed as the maximum from the two minimal inclinations. Unfortunately, the method is not applicable to the non-convex surfaces when the radius of the curvature of the part surface is negative in the both directions but the maximum principle curvature is positive. In these cases, the method produces a zero inclination. This “bug” often leads to local gouging. Lo [78] solves this problem by continuously checking for gouging in all directions. Some improvements and modifications of these techniques are given in [8].

Li and Chen [75] write “Not only the parameters of the part of cutter body that pierces into the stock, but also the parameters of the area on the designed surface that may have relations to the cutter is yet to be studied. But the cutter location point, just as its name, is only the common point both on the cutter and the designed surface, any methods only based on the geometric properties of it will not obtain the best cutter positions.” In other words, the accuracy of the above single point gouging model may be insufficient. In this case multipoint strategies [123, 124, 130] could be applied to further enhance the accuracy of the tool positioning. Furthermore, Rao and Sarma [103] present a closed form, coordinate free method for the detection and elimination of local gouging, at a CC point, in five-axis machining of sculptured surfaces using flat-end tools. The method is based on finding the curvatures of the tool swept surface at CC points along the tool path. Local gouging can then be detected and eliminated by sampling a finite set of points on the tool path, while comparing curvatures of the tool swept surface and the designed surface. Pottmann et al. [100] proposed a local millability criterion that guarantees global millability (i.e., rear-gouge and collision free milling) for three-axis machining using ball-end tools. The local millability criterion is based on curvature matching, using Dupin indicatrices in the tangent plane at the CC point, between the designed surface and the tool swept surface. A five-axis version of these method is presented in [131].

The above curvature matching methods require iterative gouge checking and correction strategies. To eliminate the need for iterative gouge checking and correction algorithms, some five-axis tool positioning strategies attempt to match the tool's cutting geometry to the surface geometry such as the principal axis method [104] developed for three-axis ball nose cutter and the five-axis arc intersection methods [42] based on the fact that the widest machined strip width is cut when the tool is tilted along the feed direction. Then for a given feed direction, a tool position is computed for each CC point along the tool path. The tool vector is restricted to lie in the tilting plane. The tilt angles are measured around the cross vector. The idea is to find the minimum tilt angle of the tool axis about the cross vector at which the tool contacts another point on the surface and maintain its contact with the CC point without gouging the surface.

Gray et al. [43] propose a modification of the five-axis arc intersection method for the so-called $3^1/_2{}^1/_2$-axis machining. This type of machining is characterized by three linear axes and two temporary locked rotary axes. The rotary axis are locked during the entire cut (resulting in a fixed tool orientation) or during a certain fraction of the cut. The rotary axis, represented by a high precision indexing device, constitute an interesting inexpensive alternative to the five-axis machining.

It should be noted that the above methods are based on the properties of a single or several contact points and therefore the errors still are unavoidable during, for instance, the wide strip precision machining. Besides, there is always a possibility of the so-called rear gouging when the back side of the tool gouges the surface with an attempt to obtain a wider machining strip. When the gouge is detected the tool must be inclined further and checked for gouging again until it clears the part. This secondary check and gouge elimination can be performed using the rolling ball method suggested in [41]. The basic idea is to roll a varying radius ball along the tool path and position the tool inside the ball.

An interesting approach has been developed by Li and Chen [75]. An envelop surface created by the cutter movement is discretized into infinite characteristic curves. Each of these curves will exactly copy themselves on to the stock. Then an analysis of the characteristic curves is performed to solve the problem of cutter positioning. The authors use the concept of the instantaneous cutter position error employing the virtual cutting edge of the tool. The effective bandwidth of cutting strip is calculated and used in the optimization algorithm.

Finally, regardless of the tool orientation there always exists the possibility of a global interference of the workpiece with the tool holder, fixture or other parts of the machine. There also exists a possibility that given any orientation, the tool still flank-mills an unwanted part of the surface. It means that this portion of the surface is not accessible. Elber and Cohen [28] write "The problem of accessibly, or the ability to verify and possibly correct gouging into the machined surface or even into other surfaces, is apparently the most fundamental hindering factor in the broad use of five-axis machining".

Solid modeling systems offer the possibility of doing both simulation and verification of tool paths off-line. However, the solid modeling approach is computationally expensive. The cost of simulation using the so-called constructive solid geometry is proportional to the fourth power of the number of the tool movements $O(N^4)$ [16]. On the other hand, the solid model can detect both the local and the global interference, including collisions with the clamping device and the machine parts. A typical program for surface machining could contain more than 10,000 tool movements, therefore, current solid modeling research focuses on efficient and fast algorithms to compute the swept volume of the tool and perform Boolean operations to subtract the intersection from the stock. Bohez et al. [16] presents a short introduction to solid modeling schemes such as the extended Z-buffer algorithm [52], line graphic simulation approaches [59] and others.

The partition into elements and the corresponding data structures are the most important components of these procedures. The Z-buffer structure [7, 113], ray representation [56], Octree method [93, 106], K-D trees [50], BSP-trees [92], Brep-indices [65, 117], tetrahedral meshes [93] and regular grids [38] are examples of such spatial decomposition techniques. Each solid modeling algorithm has advantages and disadvantages in terms of accuracy, robustness, data structure and computation time. However, it seems that the simplicity of the data structure required for the Z-buffer scheme and the possibility to generate and update the part model very fast made many commercial CAM program to use the Z-buffer algorithm or its ramifications for the NC code verification and optimization [107].

The recent research papers include many improvements of the Z-buffer techniques such as the enhanced Z-buffer model [107], the stencil buffer [16], the adaptive depth buffer [105], the undo facilities for the Z-buffer scheme [13], etc. However, the above methods are not designed specifically for global interference detection. As a matter of fact, the solid model visualizes a general cut which may or may not include the global interferences. It may take hours of simulation and possibly an operator to visually detect possible collisions.

Therefore, methods based on the closed form mathematical solutions or their approximation are still valuable. The problem of the global gouging is treated mathematically using the concept of *accessibility.* The *accessibility of a point in a given direction* is defined as follows: a point belonging to a geometric entity is *accessible* in a given direction if a ray can be drawn from it in the given direction without intersecting with interior of the geometric entity.

The problem of accessibility in three-axis machining can be solved by a method of hidden surface removal of the same scene from a direction collinear with the tool axis [26, 53]. The fact that the tool has a finite thickness can be compensated for, by offsetting all the check surfaces by the radius of the tool.

The use of Z-buffer based hidden surface removal techniques to verify and correct the three-axis tool path, is a common practice in many contemporary computer-aided manufacturing schemes. For three-axis machining under certain conditions the absence of the local gouging implies the complete absence of collisions [99].

For five-axis machining it has been also shown that if all axis positions pass through a fixed point and if all points of the workpiece surface can be seen from this point then the local millability implies global millability [99]. Wallner and Pottmann [120] presented a global millability theorem for general workpieces. They analyzed several possible configuration manifolds of tool positions relative to a workpiece under different aspects; the degree of freedom of the motion of the tool, the correspondence between the contact point and the tool position, and the presence or absence of unwanted collisions between tool and workpiece.

Takeuchi et al. [114] proposed a method for computing the collision-free CL data using a trial and error approach. Morishige et al. [89] used the so-called C-space techniques to generate a smooth, continuously varying tool path. The C-space is a general concept of robotics where the configuration of a mechanism is specified by a sequence of values. A rigid body, for example, can be located in space by specifying six parameters related to all six of its degrees of freedom. The configuration space (C-space) of a mechanism is the space of these parameters, and a point in the C-space specifies a particular configuration. Obstacles can be mapped to the C-space as well, and the required collision-free access can theoretically be inferred by navigating the point in the C-space around the obstacles. Unfortunately, though intuitive and intellectually appealing, the C-space approach could lead to computationally intractable tasks.

The problem of accessibility can be approximated by a simpler requirement called visibility. A point on an object is visible from a point at infinity if there exists a straight line segment connecting the two points which does not intersect with the object. Visibility is a useful precursor for the accessibility computation because, for a certain class of tools and probes, visibility is a necessary condition for accessibility. Seminal theoretical results in the area were obtained by Elber and Cohen [27], Elber and Zussman [29], Woo [127]. Later research was focused on generating the so-called product visibility cones (see [62, 116]).

Lauwers et al. [70] describe a multi-axis tool path generation software where the tool orientation is optimized to avoid machine collisions and at the same time to maximize the material removal rate along the tool track. To perform efficient collision avoidance, the tool path generation module, the post processing and machine simulation has been integrated into one system. Xu et al. [128] combine the machine limits, collisions, and gouging to generate feasible gouge free tool orientations. Gian et al. [39] developed open regions and vector fields techniques to find rapidly the cutter paths and tool orientations for parts with cavity areas. In 2003, Balasubramaniam et al. [10] developed methods for five-axis tool positioning that account for accessibility of the tool using visibility maps of the triangulated data. Using this visibility data for finish machining the authors show how it can be used to generate globally collision-free five-axis finishing tool paths while considering machine limits, tool tilt, cusp height limits and the tool pitch limits.

Young et al. [132] presented a new parametric method with an approximate constant cutting depth for the rough machining of an impeller. The initial tool spindle axis is considered as the initial orientation to determine the cutting tool posture for which the variation of rotational axes of the five-axis machine tool will be reduced. Hsueh et al. [54] propose to prevent the collisions using the two stages: the first stage is to obtain the tilting and collision-free angle range in the plane that is normal to the tool path obtained. Next, a checking cone generated from this collision-free tool axis range is used for the second collision check. The collision region is formed by the intersection of the neighboring surfaces.

Analyzing a proper sculptured surface orientation on the worktable of multi-axis CNC machine, Radzevich and Goodman [102] proposed the so-called spherical indicatrix of the sculptured surface machinability. This characteristic curve indicates whether the sculptured surface is machinable under a known scenario. The theory is developed in connection with a sculptured surface orientation on the worktable of a multi-axis CNC machine.

Radzevich [101] presents an approach that enables us to detect regions of a sculptured surface which are not accessible for a cutting tool of a given design, Furthermore, if any not-machinable regions exist, the developed approach enables us to subdivide the sculptured surface into the cutter-accessible and the cutter-not-accessible regions.

We have presented a survey on three selected topics in five-axis machining, namely, tool path interpolators, adaptable geometric patterns and methods for tool posture and gouging avoidance. We believe that the above procedures are the most important for constructing efficient numerical methods for five-axis machining

It should be noted that complete software systems designed for tool path generation and verification are now very important. The commercial CAD/CAM systems such as Unigraphics, PowerMill, Catia, MasterCam, each of them to a certain degree, include five-axis capabilities. In Chap. 2 we will demonstrate how the solid modeling features of Unigraphics can be used for the tool path simulation and verification.

Finally, we apologize to those authors whose works have not been cited. The exclusion of any such papers is due to our not being aware of their work.

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