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Negative Refraction of Electromagnetic and Electronic Waves in Uniform Media

Y. Zhang and A. Mascarenhas

Summary. We discuss various schemes that have been used to realize negative refraction and zero reflection, and the underlying physics that dictates each scheme. The requirements for achieving both negative refraction and zero reflection are explicitly given for different arrangements of the material interface and different structures of the electric permittivity tensor $\boldsymbol{\varepsilon}$. We point out that having a left-handed medium is neither necessary nor sufficient for achieving negative refraction. The fundamental limitations are discussed for using these schemes to construct a perfect lens or "superlens," which is the primary context of the current interest in this field. The ability of an ideal "superlens" beyond diffraction-limit "focusing" is contrasted with that of a conventional lens or an immersion lens.

1.1 Introduction

1.1.1 Negative Refraction

Recently, negative refraction has attracted a great deal of attention, largely due to the realization that this phenomenon could lead to the development of a perfect lens (or superlens) [1]. A perfect lens is supposed to be able to focus all Fourier components (i.e., the propagating and evanescent modes) of a two-dimensional (2D) image without missing any details or losing any energy. Although such a lens has yet to be shown possible either physically and practically, the interest has generated considerable research in electromagnetism and various interdisciplinary areas in terms of fundamental physics and material sciences [2–4]. Negative refraction, as a physical phenomenon, may have much broader implications than making a perfect lens. Negative refraction achieved using different approaches may involve very different physics and may find unique applications in different technology areas. This chapter intends to offer some general discussion that distinguishes the underlying physics of various approaches, bridges the physics of different disciplines (e.g., electromagnetism and electronic properties of the material), and provides some detailed discussions for one particular approach, that is, negative refraction involving uniform media with conventional dielectric properties. By uniform medium we mean that other than the microscopic variation on the atomic or molecular scale the material is spatially homogeneous.

The concept of negative refraction was discussed as far back as 1904 by Schuster in his book *An Introduction to the Theory of Optics* [5]. He indicated that negative dispersion of the refractive index, n, with respect to the wavelength of light, λ, i.e., $\mathrm{d}n/\mathrm{d}\lambda < 0$, could lead to negative refraction when light enters such a material (from vacuum), and the group velocity, v_{g}, is in the opposite direction to the wave (or phase) velocity, v_{p}. Although materials with $\mathrm{d}n/\mathrm{d}\lambda < 0$ were known to exist even then (e.g., sodium vapor), Schuster stated that "in all optical media where the direction of the dispersion is reversed, there is a very powerful absorption, so that only thicknesses of the absorbing medium can be used which are smaller than a wavelength of light. Under these circumstances it is doubtful how far the above results have any application." With the advances in material sciences, researchers are now much more optimistic 100 years later. Much of the intense effort in demonstrating a "poor man's" superlens is directed toward trying to overcome Schuster's pessimistic view by using the spectral region normally having strong absorption and/or thin-film materials with film thicknesses in the order of (or even a fraction of) the wavelength of light [2]. However, with regard to the physics of refraction, for a "lens" of such thickness, one may not be well justified in viewing the transmission as refraction, because of various complications (e.g., the ambiguity in defining the layer parameters [6] and the optical tunnel effect [7]).

The group velocity of a wave, $\mathbf{v}_{\mathrm{g}}(\omega, \mathbf{k}) = \mathrm{d}\omega/\mathrm{d}\mathbf{k}$, is often used to describe the direction and the speed of its energy propagation. For an electromagnetic wave, strictly speaking, the energy propagation is determined by the Poynting vector $\mathbf{S}$. In certain extreme situations, the directions of $\mathbf{v}_{\mathrm{g}}$ and $\mathbf{S}$ could even be reversed [8]. However, for a quasimonochromatic wave packet in a medium without external sources and with minimal distortion and absorption, the direction of $\mathbf{S}$ does coincide with that of $\mathbf{v}_{\mathrm{g}}$ [9]. For simplicity, we will focus on the simpler case, where the angle between $\mathbf{v}_{\mathrm{g}}$ and wave vector $\mathbf{k}$ is of significance in distinguishing two types of media: when the angle is acute or $\mathbf{k} \cdot \mathbf{v}_{\mathrm{g}} > 0$, it is said to be a right-handed medium (RHM); when the angle is obtuse or $\mathbf{k} \cdot \mathbf{v}_{\mathrm{g}} < 0$, it is said to be a left-handed medium (LHM) [10]. If one prefers to define the direction of the energy flow to be positive, an LHM can be referred to as a material with a negative wave velocity, as Schuster did in his book. A wave with $\mathbf{k} \cdot \mathbf{v}_{\mathrm{g}} < 0$ is also referred to as a backward wave (with negative group velocity), in that the direction of the energy flow is opposite to that of the wave determined by $\mathbf{k}$ [11, 12]. Lamb was perhaps the first to suggest a one-dimensional mechanical device that could support a wave with a negative wave velocity [13], as mentioned in Schuster book [5]. Examples of experimental demonstrations of backward waves can be found in other review papers [4, 14]. Unusual physical phenomena are expected to emerge either in an individual LHM (e.g., a reversal of the group velocity and a reversal of Doppler shift) or jointly with an RHM (e.g., negative refraction that occurs at the interface of an LHM and RHM) [10]. The effect that has received most attention lately is the negative refraction at the interface of an RHM and LHM, which relies on the property $\mathbf{k} \cdot \mathbf{v}_{\mathrm{g}} < 0$ in the LHM.

There are a number of ways to realize negative refraction [4]. Most ways rely on the above-mentioned LH behavior, i.e., $\mathbf{k} \cdot \mathbf{v}_{\mathrm{g}} < 0$, although LH behavior is by no means necessary or even sufficient to have negative refraction. Actually, LH behavior can be readily found for various types of wave phenomena in crystals. Examples may include the negative dispersion of frequency $\omega(\mathbf{k})$ for phonons and of energy $E(\mathbf{k})$ for electrons; however, they are inappropriate to be considered as uniform media and thus to discuss refraction in the genuine sense, because the wave propagation in such media is diffractive in nature. For a simple electromagnetic wave, it is not trivial to find a crystal that exhibits LH behavior. By "simple electromagnetic wave," we refer to the electromagnetic wave in the transparent spectral region away from the resonant frequency of any elementary excitation in the crystal. In this case, the light–matter interaction is mainly manifested as a simple dielectric function $\varepsilon(\omega)$, as in the situation often discussed in crystal optics [15], where $\varepsilon(\omega)$ is independent of $\mathbf{k}$.

1.1.2 Negative Refraction with Spatial Dispersion

The first scheme to be discussed for achieving negative refraction relies on the $\mathbf{k}$ dependence of ε to produce the LH behavior. The dependence of $\varepsilon(\mathbf{k})$ or $n(\lambda)$ is generally referred to as spatial dispersion [16, 17], meaning that the dielectric parameter varies spatially. Thus, this scheme may be called the *spatial-dispersion scheme.* The negative refraction originally discussed by Schuster in 1904 could be considered belonging to this scheme, although the concept of spatial dispersion was only introduced later [17] and discussed in greater detail in a book by Agranovich and Ginzburg, *Spatial Dispersion in Crystal Optics and the Theory of Excitons* [9]. If one defines $v_{\mathrm{p}} = \omega/k = c/n$, and assumes $n > 0$, then according to Schuster, v_{g} is related to v_{p} by [5]

$$v_{\mathrm{g}} = v_{\mathrm{p}} - \lambda \frac{\mathrm{d}v_{\mathrm{p}}}{\mathrm{d}\lambda}, \tag{1.1}$$

and the condition for having a negative wave velocity is given as $\lambda\, \mathrm{d}v_{\mathrm{p}}/\mathrm{d}\lambda > v_{\mathrm{p}}$, which is equivalent to $\mathrm{d}n/\mathrm{d}\lambda < -n/\lambda < 0$. Negative group velocity and negative refraction were specifically associated with spatial dispersion by Ginzburg and Agranovich [9, 17]. Recently, a generalized version of this condition has been given by Agranovich et al. [18]. In their three-fields $(\mathbf{E}, \mathbf{D}, \mathbf{B})$ approach, with a generalized permittivity tensor $\tilde{\varepsilon}(\omega, \mathbf{k})$ (see the chapter of Agranovich and Gartstein for more details), the time-averaged Poynting vector in an isotropic medium is given as

$$\mathbf{S} = \frac{c}{8\pi} \mathrm{Re}(\mathbf{E}^* x \mathbf{B}) - \frac{\omega}{16\pi} \nabla_k \tilde{\varepsilon}(\omega, \mathbf{k}) \mathbf{E}^* \mathbf{E}, \tag{1.2}$$

where the direction of the first term coincides with that of $\mathbf{k}$, and that of the second term depends on the sign of $\nabla_{\mathrm{k}}\tilde{\varepsilon}(\omega, \mathbf{k})$, which could lead to the reversal of the direction of $\mathbf{S}$ with respect to $\mathbf{k}$ under certain conditions. If permeability $\mu = 1$ is assumed, the condition can be simplified to $\mathrm{d}\varepsilon/\mathrm{d}k > 2\varepsilon/k > 0$ (here, ε is the conventional permittivity or dielectric constant), which is essentially the same as that derived from (1.1). Spatial dispersion is normally very weak in a crystal, because it is characterized by a parameter a/λ, where a is the lattice constant of the crystal and λ is the wavelength in the medium. However, when the photon energy is near that of an elementary excitation (e.g., exciton, phonon, or plasmon) of the medium, the light–matter interaction can be so strong that the wave is neither pure electromagnetic nor electronic, but generally termed as a polariton [19, 20]. Thus, the spatial dispersion is strongly enhanced, as a result of coupling of two types of waves that normally belong to two very different physical scales. With the help of the polariton effect and the negative exciton dispersion $\mathrm{d}E(\mathbf{k})/\mathrm{d}k < 0$, one could, in principle, realize negative refraction for the polariton wave inside a crystal if the damping is not too strong [21]. Because damping or dissipation is inevitable near the resonance, similar to the case of sodium vapor noted by Schuster [5], a perfect lens is practically impossible with this *spatial-dispersion scheme*.

It is worth mentioning that the damping could actually provide another possibility to induce $\mathbf{k} \cdot \mathbf{v}_{\mathrm{g}} < 0$ for the polariton wave in a crystal, even though in such a case the direction of $\mathbf{v}_{\mathrm{g}}$ may not be exactly the same as that of $\mathbf{S}$. In the *spatial-dispersion scheme*, the need to have $\mathrm{d}E(\mathbf{k})/\mathrm{d}k < 0$ is based on the assumption of the ideal polariton model, i.e., with vanishing damping. However, with finite damping, even with the electronic dispersion $\mathrm{d}E(\mathbf{k})/\mathrm{d}k > 0$, one may still have one polariton branch exhibiting $\mathrm{d}\omega/\mathrm{d}\mathbf{k} < 0$ near the frequency window Δ_{LT}, splitting the longitudinal and transverse mode, and thus, causing the exhibiting of LH behavior [7].

1.1.3 Negative Refraction with Double Negativity

Mathematically, the simplest way to produce LH behavior in a medium is to have both $\varepsilon < 0$ and $\mu < 0$, as pointed out by Pafomov [22]. Double negativity, by requiring energy to flow away from the interface and into the medium, also naturally leads to a negative refractive index $n = -\sqrt{\varepsilon\mu}$, thus facilitating negative refraction at the interface with an RHM, as discussed by Veselago [10]. At first glance, this *double-negativity scheme* would seem to be more straightforward than the *spatial-dispersion scheme*. However, $\varepsilon < 0$ is only known to occur near the resonant frequency of a polariton (e.g., plasmon, optical phonon, exciton). Without damping and spatial dispersion, the spectral region of $\varepsilon < 0$ is totally reflective for materials with $\mu > 0$. $\mu < 0$ is also known to exist near magnetic resonances, but is not known to occur in the same material and the same frequency region where $\varepsilon < 0$ is found. Indeed, if in the same material and spectral region one could simultaneously have $\varepsilon < 0$ and $\mu < 0$ yet without any dissipation, the material would then turn transparent. In recent years, metamaterials have been developed to extend material response and thus allow effective ε and μ to be negative in an overlapped frequency region [3]. The hybridization of the metamaterials with, respectively, $\varepsilon_{\mathrm{eff}} < 0$ and $\mu_{\mathrm{eff}} < 0$ has made it possible to realize double negativity or $n_{\mathrm{eff}} < 0$ in a small microwave-frequency window, and to demonstrate negative refraction successfully [23]. However, damping or dissipation near the resonant frequency still remains a major obstacle for practical applications of metamaterials. There is a fundamental challenge to find any natural material with nonunity μ at optical frequencies or higher, because of the ambiguity in defying μ at such frequencies [18, 24]. Although there have been a few demonstrations of metamaterials composed of "artificial atoms" exhibiting nonunity or even negative effective μ and negative effective refractive index at optical frequencies [25–29], no explicit demonstration of negative refraction or imaging has been reported, presumably because of the relative large loss existed in such materials. Thus, the *double-negativity scheme* essentially faces the same challenge that the *spatial-dispersion scheme* does in realizing the dream of making a perfect lens.

1.1.4 Negative Refraction Without Left-Handed Behavior

It is perhaps understandable that the general public might have the impression that negative refraction never occurs in nature [23, 30]. One could only make such a claim if one insists on using isotropic media [4, 31, 32]. The simplest example of negative refraction is perhaps refraction of light at the interface of air and an anisotropic crystal without any negative components of ε and μ, as illustrated in Fig. 1.1 [32–36]. A standard application of such an optical component is a beam displacer. Thus, negative refraction is a readily observable phenomenon, if one simply allows the use of an anisotropic medium. This *anisotropy scheme* has enabled the demonstration of negative refraction in the most genuine sense – that is, the classic refraction phenomenon in uniform media or optical crystals in a broad spectral range and involving neither electronic nor magnetic resonances [31, 34, 35]. As in the case of the double-negativity scheme, to eliminate the reflection at the medium interface, the *anisotropy scheme* also needs to satisfy certain conditions for matching the dielectric properties of the two media, as illustrated by a special case of a bicrystal structure [31]. In general, eliminating the reflection loss requires material parameters to automatically ensure the continuity of the energy flux

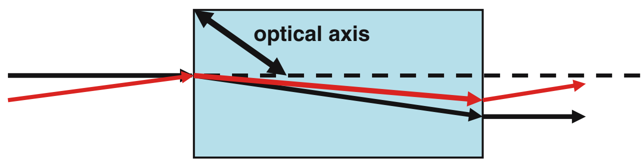

Fig. 1.1. Refraction of light at the interface of air and a (positive) uniaxial crystal

along the interface normal [32]. Generalization has been discussed for the interface of two arbitrary uniaxially anisotropic media [33, 37, 38]. Note that negative refraction facilitated by the anisotropy scheme does not involve any LH behavior and thus cannot be used to make a flat lens, in contrast to that suggested by Veselago, using a double-negativity medium [10], which is an important distinction from the other schemes based on negative group velocity. However, one could certainly envision various important applications other than the flat lens.

1.1.5 Negative Refraction Using Photonic Crystals

The last scheme we would like to mention is the *photonic crystal scheme*. Although it is diffractive in nature, one may often consider the electromagnetic waves in a photonic crystal as waves with new dispersion relations, $\omega_n(\mathbf{k})$, where n is the band index, and $\mathbf{k}$ is the wave vector in the first Brillouin zone. For a three-dimensional (3D) or 2D photonic crystal [39, 40], the direction of the energy flux, averaged over the unit cell, is determined by the group velocity $\mathrm{d}\omega_n(\mathbf{k})/\mathrm{d}\mathbf{k}$, although that might not be generally true for a 1D photonic crystal [40]. If the dispersion is isotropic, the condition $\mathbf{q}\cdot \mathrm{d}\omega_n(\mathbf{q})/\mathrm{d}\mathbf{q} < 0$, where $\mathbf{q}$ is the wave vector measured from a local extremum, must be satisfied to have LH behavior. Similar to the situations for the spatial-dispersion and double-negativity schemes, $\mathbf{q}\cdot \mathrm{d}\omega_n(\mathbf{q})/\mathrm{d}\mathbf{q} < 0$ also allows the occurrence of negative refraction at the interface of air and photonic crystal as well as the imaging effect with a flat photonic slab [4, 41–44]. However, similar to the situation for the anisotropy scheme, one may also achieve negative refraction with positive, but anisotropic, dispersions [40]. Because of the diffractive nature, the phase matching at the interface of the photonic crystal often leads to complications, such as the excitation of multiple beams [40, 45].

1.1.6 From Negative Refraction to Perfect Lens

Although the possibility of making a flat lens with the double-negativity material was first discussed by Veselago [10], the noted unusual feature alone, i.e., a lens without an optical axis, would not have caused it to receive such broad interest. It was Pendry who suggested perhaps the most unique aspect of the double-negativity material – the potential for realizing a perfect lens beyond negative refraction [1] – compared to other schemes that can also achieve negative refraction. Apparently, not all negative refractions are equal. To make Pendry's perfect lens, in addition to negative refraction, one also needs (1) zero dissipation, (2) amplification of evanescent waves, and (3) matching of the dielectric parameters between the lens and air. Exactly zero dissipation is physically impossible for any real material. For an insulator with an optical bandgap, one normally considers that there is no absorption for light with energy below the bandgap, if the crystal is perfect (e.g., free of defects). However, with nonlinear optical effects taken into account, there will always be some finite absorption below the bandgap due to harmonic transitions [46]. Although it is typically many orders of magnitude weaker than the above-bandgap linear absorption, it will certainly make the lens imperfect. Therefore, a perfect lens may simply be a physically unreachable singularity point. For the schemes working near the resonant frequencies of one kind or the other, the dissipation is usually strong, and thus more problematic to allow such a lens to be practically usable.

Mathematically, double-negativity material is the only one, among all the schemes mentioned above, that automatically provides a correct amount of amplification for each evanescent wave [1]. Unfortunately, this scheme becomes problematic at high frequencies because of the ambiguity in defining nonunity μ at high frequencies [18, 24]. The other schemes – spatial dispersion and photonic crystals – may also amplify the evanescent components when the effective refractive index $n_{\mathrm{eff}} < 0$, but typically with some complications (e.g., the amplification magnitude might not be exactly correct or the resolution is limited by the periodicity of the photonic crystal) [47, 48].

One important requirement of negative refraction for making a perfect lens is matching the dielectric parameters ("impedances") of the two media to eliminate reflection, as well as aberration [49], for instance, $n_1 = -n_2$ for the double-negativity scheme. In addition to the limitation caused by finite damping, another limitation faced by both the spatial-dispersion and double-negativity schemes is frequency dispersion, which prohibits the matching condition of the dielectric parameters to remain valid in a broad frequency range. For the spatial-dispersion scheme, the frequency dispersion $\varepsilon(\omega)$ is apparent [9]. It is less trivial for the double-negativity scheme, but it was pointed out by Veselago that "the simultaneous negative values of ε and μ can be realized only when there is frequency dispersion," in order to avoid the energy becoming negative [10]. For the photonic crystal, the effective index is also found to depend on frequency. Therefore, even for the ideal case of vanishing damping, the matching condition can be found at best for discrete frequencies, using any one of the three schemes discussed above.

However, even with the practical limitations on the three aspects – damping, incorrect magnitude of amplification, and dielectric mismatch – one can still be hopeful of achieving a finite improvement in "focusing" light beyond the usual diffraction limit [50], in addition to the benefits of having a flat lens. A widely used technique, an immersion lens [51], relies on turning as many evanescent waves as possible into propagating waves inside the lens, and it requires either the source or image to be in the near-field region. Compared to this immersion lens, the primary advantage of the "superlens" seems to be the ability to achieve subwavelength focusing with both the source and image at far field. An immersion lens can readily achieve $\sim \lambda/4$ resolution at $\sim$200 nm in semiconductor photolithography [52]. With a solid immersion lens, even better resolution has been achieved (e.g., $\sim 0.23\lambda$ at $\lambda = 436$ nm [53], $\sim\lambda/8$ at $\lambda = 515$ nm [54]). Thus far, using negative refraction, there have only been a few experimental demonstrations of non-near-field imaging with improved resolution (e.g., 0.4λ image size at 1.4λ away from the lens [55], using a 2D quasicrystal with $\lambda = 25$ mm; 0.36λ image size at 0.6λ away from the lens [56], using a 3D photonic crystal with $\lambda = 18.3$ mm). In addition, plasmonic systems (e.g., ultra thin metal film) have also be used for achieving subwavelength imaging in near field [57, 58], although not necessarily related to negative refraction.

Some further discussion is useful on the meaning of "focusing" as used by Pendry for describing the perfect lens [1]. The focusing power of a lens usually refers to the ability to provide an image smaller than the object. What the hypothetical flat lens can do is exactly reproduce the source at the image site, or equivalently, spatially translate the source by a distance of $2d$, where d is the thickness of the slab. Thus, mathematically, a δ-function source will give rise to a δ-function image, without being subjected to the diffraction limit of a regular lens, i.e., $\lambda/2$ [59]. And such a "superlens" can, in principle, resolve two objects with any nonzero separation, overcoming the Rayleigh criterion of 0.61λ for the resolving power of a regular lens [59]. However, what this "superlens" cannot do is focus an object greater than λ to an image smaller than λ; thus, it cannot bring a broad beam to focus for applications such as photolithography, whereas a regular lens or an immersion lens can, in principle, focus an object down to the diffraction limit $\lambda/2$ or $\lambda/(2n)$ (n is the refractive index of the lens material). Therefore, it might not be appropriate to call such optical device of no magnification a "lens," though it is indeed very unique. One could envision using the "superlens" to map or effectively translate a light source, while retaining its size that is already below the diffraction limit to begin with.

1.2 Conditions for Realizing Negative Refraction and Zero Reflection

Let us consider a fairly general case of refraction of light at the interface of two uniform media, as shown in Fig. 1.2. The media are assumed to have anisotropic permittivity tensors ε_L and ε_R, both with uniaxial symmetry,

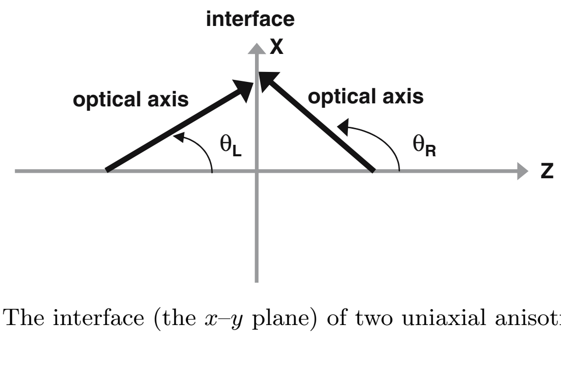

Fig. 1.2. The interface (the x–y plane) of two uniaxial anisotropic media

but isotropic permeabilities μ_{L} and μ_{R}, where L and R denote the left-hand and right-hand side, respectively. Their symmetry axes are assumed to lie in the same plane as the plane of incidence, which is also perpendicular to the interface, but nevertheless may incline at any angles with respect to the interface normal. In the principal coordinate system (x', y', z'), the relative permittivity tensors are given by

$$\varepsilon_{\mathrm{L,R}} = \begin{pmatrix} \varepsilon_1^{\mathrm{L,R}} & 0 & 0 \\ 0 & \varepsilon_1^{\mathrm{L,R}} & 0 \\ 0 & 0 & \varepsilon_3^{\mathrm{L,R}} \end{pmatrix}, \tag{1.3}$$

where ε_1 and ε_3 denote the dielectric components for electric field $\mathbf{E}$ polarized perpendicular and parallel to the symmetric axis. In the (x,y,z) coordinate system shown in Fig. 1.2, the tensor becomes

$$\varepsilon_{\mathrm{L,R}} = \begin{pmatrix} \varepsilon_1^{\mathrm{L,R}} \cos^2(\theta_{\mathrm{L,R}}) + \varepsilon_3^{\mathrm{L,R}} \sin^2(\theta_{\mathrm{L,R}}) & 0 & (\varepsilon_3^{\mathrm{L,R}} - \varepsilon_1^{\mathrm{L,R}}) \sin(\theta_{\mathrm{L,R}}) \cos(\theta_{\mathrm{L,R}}) \\ 0 & \varepsilon_1^{\mathrm{L,R}} & 0 \\ (\varepsilon_3^{\mathrm{L,R}} - \varepsilon_1^{\mathrm{L,R}}) \sin(\theta_{\mathrm{L,R}}) \cos(\theta_{\mathrm{L,R}}) & 0 & \varepsilon_3^{\mathrm{L,R}} \cos^2(\theta_{\mathrm{L,R}}) + \varepsilon_1^{\mathrm{L,R}} \sin^2(\theta_{\mathrm{L,R}}) \end{pmatrix}. \tag{1.4}$$

Rather generalized discussions for the reflection–refraction problem associated with the interface defined in Fig. 1.2 have been given in the literature for the situation of ε_1 and ε_3 both being positive [37, 38]. For an ordinary wave with electric field $\mathbf{E}$ polarized in the y-direction, i.e., perpendicular to the plane of incidence (a TE wave), the problem is equivalent to an isotropic case with different dielectric constants $\varepsilon_1^{\mathrm{L}}$ and $\varepsilon_1^{\mathrm{R}}$ for the left-hand and right-hand side. It is the reflection and refraction of the extraordinary or **H**-polarized wave, i.e., with **E** polarized in the x–z plane (a TM wave), that has generally been found to be more interesting. For the E- and H-polarized waves, the dispersion relations are given below for the two coordinate systems:

$$k_x'^2 + k_z'^2 = \frac{\omega^2}{c^2} \mu \varepsilon_1, \tag{1.5E}$$

$$\frac{k_x'^2}{\varepsilon_3} + \frac{k_z'^2}{\varepsilon_1} = \frac{\omega^2}{c^2} \mu, \tag{1.5H}$$

and

$$k_x^2 + k_z^2 = \frac{\omega^2}{c^2} \mu \varepsilon_1, \tag{1.6E}$$

$$\frac{(k_x \cos\theta_0 - k_z \sin\theta_0)^2}{\varepsilon_3} + \frac{(k_x \sin\theta_0 + k_z \cos\theta_0)^2}{\varepsilon_1} = \frac{\omega^2}{c^2} \mu, \tag{1.6H}$$

where θ_0 is the inclined angle of the uniaxis of the medium with respect to the z-axis. The lateral component k_x is required to be conserved across the interface and the two solutions for k_z (of $\pm$) are found to be (with k in the unit of ω/c) the following:

$$k_z^{\pm} = \pm\sqrt{\mu\varepsilon_1 - k_x^2}, \tag{1.7E}$$

$$k_z^{\pm} = \frac{k_x\delta \pm 2\sqrt{\gamma(\beta\mu - k_x^2)}}{2\beta}, \tag{1.7H}$$

where $\gamma = \varepsilon_1\varepsilon_3$, $\beta = \varepsilon_1 \sin^2\theta_0 + \varepsilon_3 \cos^2\theta_0$, $\delta = \sin(2\theta_0)(\varepsilon_1 - \varepsilon_3)$. The Poynting vector $\mathbf{S} = \mathbf{E}^* \times \mathbf{H}$, corresponding to $k_z^{\pm}$, can be given as

$$S_x^{\pm} = |E_y|^2 \frac{k_x}{c\mu\mu_0}, \tag{1.8E}$$

$$S_x^{\pm} = |H_y|^2 \frac{2\gamma k_x \mp \delta\sqrt{\gamma(\beta\mu - k_x^2)}}{2c\varepsilon_0\beta\gamma}, \tag{1.8H}$$

and

$$S_z^{\pm} = \pm|E_y|^2 \frac{\sqrt{\mu\varepsilon_1 - k_x^2}}{c\mu\mu_0}, \tag{1.9E}$$

$$S_z^{\pm} = \pm|H_y|^2 \frac{\sqrt{\gamma(\beta\mu - k_x^2)}}{c\varepsilon_0\gamma}, \tag{1.9H}$$

where E_y and H_y are the y components of $\mathbf{E}$ and $\mathbf{H}$, respectively. If the incident beam is assumed to arrive from the left upon the interface (i.e., energy flows along the $+z$ direction), one should choose from (1.7) the solution that can give rise to a positive S_z. Note that (1.8) and (1.9) are valid for either side of the interface, and positive as well as negative ε_1, ε_3, and μ. With these equations, we can conveniently discuss the conditions for realizing negative refraction and zero reflection. Note that for the E-polarized wave, the sign of $\mathbf{k}\cdot\mathbf{S}$ is only determined by that of ε_1, since $\mathbf{k}\cdot\mathbf{S} = |E_y|^2\omega\varepsilon_0\varepsilon_1$; for the H-polarized wave, it is only determined by μ, since $\mathbf{k}\cdot\mathbf{S} = |H_y|^2\omega\mu_0\mu$.

Since S_z is always required to be positive, the condition to realize negative refraction is simply to request a sign change of S_x across the interface. For realizing zero reflection, if one can assure the positive component of S_z to be continuous across the interface, the reflection will automatically be eliminated. Therefore, one does not need to consider explicitly the reflection [32].

If both media are isotropic, i.e., $\varepsilon_1 = \varepsilon_3 = \varepsilon$, we have $S_x \propto k_x/\mu$ and $S_z^{\pm} \propto \pm\sqrt{\mu\varepsilon - k_x^2}/\mu$ for the E-polarized wave, $S_x \propto k_x/\varepsilon$ and $S_z^{\pm} \propto \pm\sqrt{\varepsilon^2(\varepsilon\mu - k_x^2)}/\varepsilon^2$ for the H-polarized wave. To have negative refraction for both of the polarizations, the only possibility is to have ε and μ changing sign simultaneously. To have zero reflection for any k_x, the conditions become $|\varepsilon^{\mathrm{L}}| = |\varepsilon^{\mathrm{R}}|$ and $|\mu^{\mathrm{L}}| = |\mu^{\mathrm{R}}|$, and $(\varepsilon\mu)^{\mathrm{L}} = (\varepsilon\mu)^{\mathrm{R}}$. Since $\varepsilon\mu > 0$ is necessary for the propagating wave, the conditions become $\varepsilon^{\mathrm{L}} = -\varepsilon^{\mathrm{R}}$ and $\mu^{\mathrm{L}} = -\mu^{\mathrm{R}}$, as derived by Veselago [10]. It is interesting to note that if one of the media is replaced with a photonic crystal with a negative effective refractive index, the "impedance" matching conditions become much more restrictive. In has been found that to minimize the reflection the surrounding medium has to have a pair of specific ε and μ for a given photonic crystal [60] and the values could even depend on the surface termination of the photonic crystal [61].

If the media are allowed to be anisotropic, several ways exist to achieve negative refraction, even if we limit ourselves to μ being isotropic. For the E-polarized wave, since $S_x \propto k_x/\mu$, negative refraction requires $\mu < 0$ on one side, assuming $\mu^{\mathrm{R}} < 0$ (the left-hand side is assumed to have everything positive). In the meantime, because $S_z \propto \pm\sqrt{\mu\varepsilon_1 - k_x^2}/\mu$, one also needs to have $\varepsilon_1^{\mathrm{R}} < 0$ to make the wave propagative. Thus, with $\varepsilon_1^{\mathrm{R}} < 0$ and $\mu^{\mathrm{R}} < 0$ while keeping $\varepsilon_3^{\mathrm{R}} > 0$, one can have negative refraction, and zero reflection for the E-polarized wave occurring for any k_x when $\mu^{\mathrm{R}} = -\mu^{\mathrm{L}}$ and $(\varepsilon_1\mu)^{\mathrm{L}} = (\varepsilon_1\mu)^{\mathrm{R}}$. This situation is similar to the isotropic case with $\varepsilon = \varepsilon_1$, although there will be no negative refraction for the H-polarized wave.

For the H-polarized wave, if both media are allowed to be anisotropic but the symmetry axes are required to be normal to the interface (i.e., $\theta_{\mathrm{L}} = \theta_{\mathrm{R}} = 0°$), we have $S_x \propto k_x/\varepsilon_3$ and $S_z \propto \pm\sqrt{\varepsilon_1\varepsilon_3(\varepsilon_3\mu - k_x^2)}/(\varepsilon_1\varepsilon_3) > 0$. Negative refraction requires $\varepsilon_3 < 0$ on one side, again assumed to be the right-hand side (the left-hand side is assumed to have everything positive). If $\varepsilon_1 < 0$, then $\mu^{\mathrm{R}} < 0$ is also needed to have a propagating wave; we have $S_z^R \propto \sqrt{\varepsilon_1^{\mathrm{R}}\varepsilon_3^{\mathrm{R}}(\varepsilon_3^{\mathrm{R}}\mu^{\mathrm{R}} - k_x^2)}/(\varepsilon_1^{\mathrm{R}}\varepsilon_3^{\mathrm{R}})$, and the conditions for zero reflection are $(\varepsilon_1\varepsilon_3)^{\mathrm{L}} = (\varepsilon_1\varepsilon_3)^{\mathrm{R}}$ and $(\varepsilon_3\mu)^{\mathrm{L}} = (\varepsilon_3\mu)^{\mathrm{R}}$. If $\varepsilon_1^{\mathrm{R}} > 0$, then $\mu^{\mathrm{R}} > 0$ is necessary to have a propagating wave; we have $S_z^{\mathrm{R}} \propto -\sqrt{\varepsilon_1^{\mathrm{R}}\varepsilon_3^{\mathrm{R}}(\varepsilon_3^{\mathrm{R}}\mu^{\mathrm{R}} - k_x^2)}/(\varepsilon_1^{\mathrm{R}}\varepsilon_3^{\mathrm{R}})$, but zero reflection is not possible except for $k_x = 0$ and when $(\varepsilon_1\varepsilon_3)^{\mathrm{L}} = |\varepsilon_1\varepsilon_3|^{\mathrm{R}}$ and $|\varepsilon_3\mu|^{\mathrm{L}} = |\varepsilon_3\mu|^{\mathrm{R}}$. The results for $\theta_{\mathrm{L}} = \theta_{\mathrm{R}} = 90°$ can be obtained by simply replacing ε_3 with ε_1 in the results for $\theta_{\mathrm{L}} = \theta_{\mathrm{R}} = 0°$. Similar or somewhat different cases have been discussed in the literature for either $\theta_{\mathrm{L}} = \theta_{\mathrm{R}} = 0°$ or $\theta_{\mathrm{L}} = \theta_{\mathrm{R}} = 90°$, leading to the conclusion that at least one component of either ε or $\boldsymbol{\mu}$ tensor needs to be negative to realize negative refraction [62–66].

However, the relaxation on the restriction of the optical axis orientations, allowing $0 < \theta_{\mathrm{L}} < 90°$ and $0 < \theta_{\mathrm{R}} < 90°$, makes negative refraction and zero reflection possible even if both ε and $\boldsymbol{\mu}$ tensors are positive definite. When ε is positive definite, we have $\gamma > 0$ and $\beta > 0$, and in this case $\mu > 0$ is necessary to have propagating modes. The condition for zero reflection can be readily found to be $\gamma^{\mathrm{L}} = \gamma^{\mathrm{R}}$, and $(\beta\mu)^{\mathrm{L}} = (\beta\mu)^{\mathrm{R}}$. For the case of the interface being that of a pair of twinned crystals [31], these requirements are automatically satisfied for any angle of incidence. The twinned structure assures that the zero-reflection condition is valid for any wavelength, despite the existence of dispersion; however, for the more-general case using two different materials, the condition can at best be satisfied at discrete wavelengths because the dispersion effect may break the matching condition, similar to the case of $\varepsilon = \mu = -1$. The negative-refraction condition can be derived from (1.8H) (since $\gamma > 0$, S_x^+ should be used). Note that $S_x^+ = 0$ at $k_{x0} = \delta\sqrt{\beta\mu}/\sqrt{4\gamma + \delta^2}$. If $k_{x0}^{\mathrm{L}} < k_{x0}^{\mathrm{R}}(k_{x0}^{\mathrm{L}} > k_{x0}^{\mathrm{R}})$, S_x changes sign across the interface or negative refraction occurs in the region $k_{x0}^{\mathrm{L}} < k_x < k_{x0}^{\mathrm{R}}(k_{x0}^{\mathrm{R}} < k_x < k_{x0}^{\mathrm{L}})$. For the crystal twin with $\theta_{\mathrm{L}} = \pi/4$ and $\theta_{\mathrm{R}} = -\pi/4$, $k_{x0}^{\mathrm{L}} = -k_{x0}^{\mathrm{R}} = (\varepsilon_1 - \varepsilon_3)/\sqrt{2(\varepsilon_1 + \varepsilon_3)}$). When $\varepsilon_3 > \varepsilon_1$ (i.e., positive birefringence) in the region of $k_{x0}^{\mathrm{L}} < k_x < k_{x0}^{\mathrm{R}}$, $S_x^{\mathrm{L}} > 0$ and $S_x^{\mathrm{R}} < 0$. For any given θ_{L}, ε_1, and ε_3, the maximum bending of the light beam or the strongest negative refraction occurs when $k_x = 0$ and $\sin^2 \theta_{\mathrm{L}} = \varepsilon_3/(\varepsilon_1 + \varepsilon_3)$, where the propagation direction of the light is given by $\phi = \arctan(S_x/S_z) = \arctan[-\delta/(2\beta)]$ for each side, and the amount of bending is measured by $\phi_{\mathrm{L}} - \phi_{\mathrm{R}} = 2\arctan[-\delta_{\mathrm{L}}/(2\beta)]$. For any given θ_{L} (as defined in Fig. 1.2), the maximum amount of bending is $2\theta_{\mathrm{L}}$ for positive crystal ($\varepsilon_3 > \varepsilon_1$ and $0 < \theta_{\mathrm{L}} < \pi/2$) or $2(\theta_{\mathrm{L}} - \pi/2)$ for negative crystal ($\varepsilon_1 > \varepsilon_3$ and $\pi/2 < \theta_{\mathrm{L}} < \pi$), when either $\varepsilon_1/\varepsilon_3 \to \infty$ or $\varepsilon_3/\varepsilon_1 \to \infty$. Figure 1.3 shows an experimental demonstration of amphoteric refraction with minimal reflection loss realized with a YVO_4 bicrystal [31], and Fig. 1.4 compares the experimental and theoretical results for the relationship between the angles of incidence and refraction (note that $\theta_{\mathrm{L}} = -\pi/4$ and $\theta_{\mathrm{R}} = \pi/4$ are assumed) [31].

As a special case of the general discussion with $0 < \theta_{\mathrm{L}} < 90°$ and $0 < \theta_{\mathrm{R}} < 90°$, zero reflection and/or negative refraction can also be realized at an isotropic–anisotropic interface [32–36]. Assuming $\mu = 1$, zero reflection occurs when $\varepsilon^{\mathrm{L}} = \sqrt{\varepsilon_1{}^{\mathrm{R}}\varepsilon_3{}^{\mathrm{R}}}$, which actually is the condition for the so-called perfectly matched layer [67]. The interface of air and a uniaxial crystal with

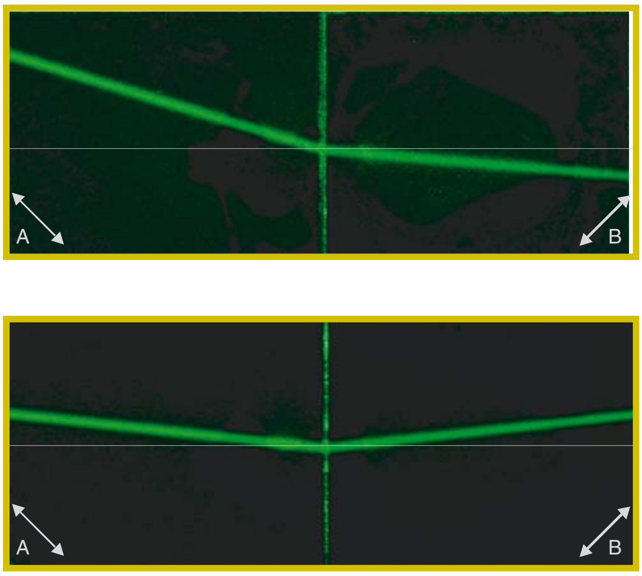

Fig. 1.3. Images of light propagation in a YVO_4 bicrystal. The *upper panel* shows an example of normal (positive) refraction, whereas the *lower panel* shows abnormal (negative) refraction. Note that no reflection at the bicrystal interface is visible to the naked eye. The interface is illuminated by inadvertently scattered light. The *arrows* indicate the orientations of the optical axes (A – *left*, B – *right*)

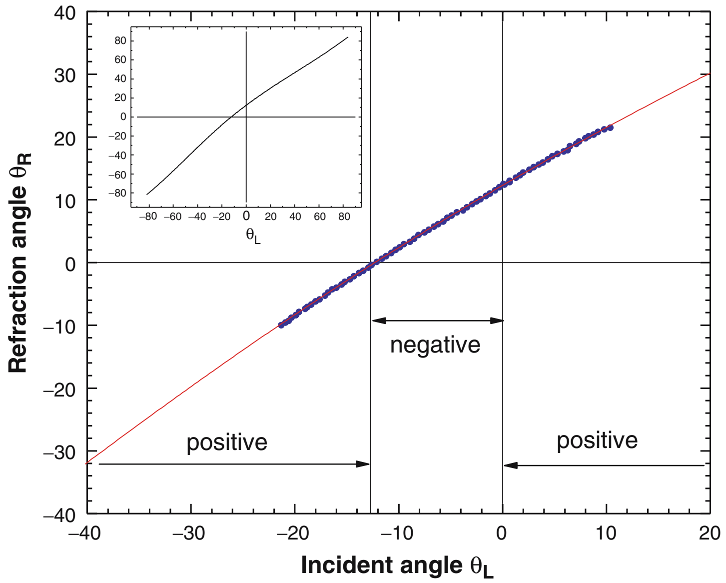

Fig. 1.4. Comparison of theoretical predictions with experimental data. Amphoteric refraction in a YVO_4 bicrystal is divided into three regions: one negative ($\theta_R/\theta_L < 0$) and two positive ($\theta_R/\theta_L > 0$). The data points are measured with a 532-nm laser light, and the curve is calculated with the refractive index of the material ($n_o = 2.01768$ and $n_e = 2.25081$). Inset: the full operation range of the device

its optical axis oriented at a nonzero angle to the interface normal is perhaps the simplest interface to facilitate negative refraction, as illustrated in Fig. 1.1. However trivial it might be, it is a genuine phenomenon of negative refraction.

If $\mu^R < 0$ and the ε^R tensor is indefinite or not positive definite, while allowing $0 < \theta_L < 90°$ and $0 < \theta_R < 90°$, we will have more unusual situations of refraction. Again, all parameters on the left-hand side are assumed positive, and, for simplicity, the left medium is assumed to be isotropic. If both $\varepsilon_1^R < 0$ and $\varepsilon_3^R < 0$, then the results are qualitatively similar to that of the isotropic case discussed above. However, when $\varepsilon_1^R < 0$ but $\varepsilon_3^R > 0$, or when ${\varepsilon_1}^R > 0$ but $\varepsilon_3^R < 0$, we thus have $\gamma^R < 0$; and by appropriately choosing θ_R to have $\beta^R > 0$, we have $\sqrt{\gamma^R(\beta^R\mu^R - k_x^2)} = \sqrt{|\gamma^R|\,(\beta^R\,|\mu^R| + k_x^2)}$, which indicates that all the real k_x components are propagating modes, and therefore, there will be no evanescent wave. For these cases, $(S_z^-)^R > 0$, and it is always possible to choose a value of θ_R (e.g., $\theta_R = 45°$ when ${\varepsilon_3}^R > |\varepsilon_1^R|$ or $\theta_R = -45°$ when ${\varepsilon_1}^R > |\varepsilon_3^R|$) such that $\delta^R < 0$; and thus, $(S_x^-)^R > 0$ for any k_x, which means that there will be no negative refraction for $k_x > 0$, in spite of the medium being left handed (because of $\mu^R < 0$), although refraction is negative for $k_x < 0$. Zero reflection only occurs at $k_x = 0$, when $\varepsilon^L = \sqrt{|\varepsilon_1^R\varepsilon_3^R|}$ and $(\varepsilon\mu)^L = |\beta\mu|^R$.

In summary, having an LHM is neither a necessary nor a sufficient condition for achieving negative refraction. The left-handed behavior does not always lead to evanescent wave amplification. It may not always be possible to match the material parameters to eliminate the interface reflection with an LHM. The double-negativity lens proposed by Veselago and Pendry represents the most-stringent material requirement to achieve negative refraction, zero reflection, and evanescent wave amplification. For a uniform medium, the left-handed behavior can only be obtained with at least one component of the ε or μ tensor being negative: ε_1 for the **E**-polarized wave and μ_1 for the **H**-polarized wave, if limited to materials with uniaxial symmetry [63]. However, once one of the components of either the ε or μ tensor becomes negative so as to have left-handed behavior, then at least one of the components of the other tensor needs to be negative to have propagating modes in the medium, and possibly to have evanescent wave amplification (as discussed above for the **H**-polarized wave and in the literature for the **E**-polarized wave [65]).

Analogous to the discussion of negative refraction in the photonic crystal, one could consider the propagation of a ballistic electron beam in a real crystal. It is perceivable that one could discuss how various types of electronic band structures might bend the electron beam negatively, in a manner similar to the negative "refraction" discussions for the photonic crystal [40]. Again, a domain twin interface, as the one shown in Fig. 1.5 for example, appears to be a simple case that can give rise to negative refraction and zero reflection for a ballistic electron beam [31]. It is a genuine refraction when light goes through such an interface; but for the electron beam, it is fundamentally a phenomenon of diffraction. Complex structures of this type of domain twin could be of great interest for both optics and electronics. Examples of such super structures created by stacking domain twins in a linear manner

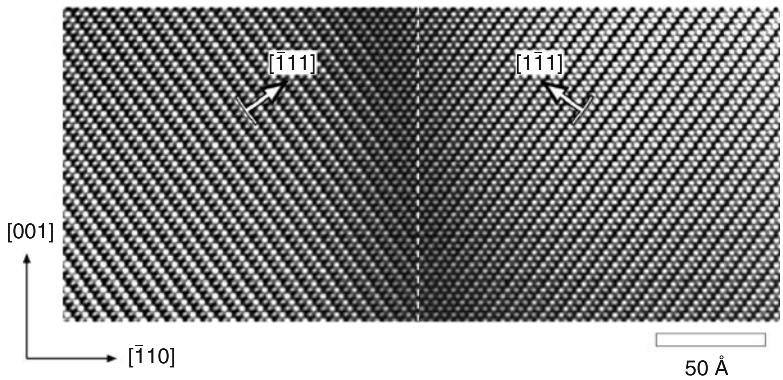

Fig. 1.5. A typical high-resolution cross-sectional transmission electron microscopy (TEM) image of domain twin structures frequently observed in CuPt-ordered III–V semiconductor alloys (e.g., GaInAs). The ordering directions are [$\bar{1}$11] (*left*) and [$1\bar{1}1$] (*right*). The *vertical dashed line* indicates the twin boundary

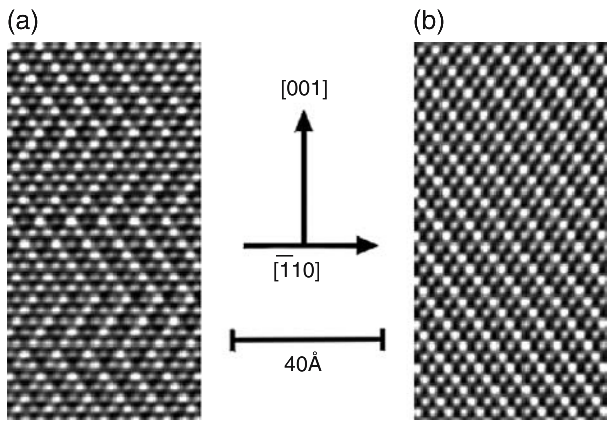

Fig. 1.6. High-resolution cross-sectional TEM images of ordered GaInP alloys: (**a**) double-variant ordered structure with quasiperiodic stacking of domain twins along the [001] direction, and (**b**) single-variant ordered domain

can be found in the literature, though not in the context of negative refraction. For instance, a zig-zag structure found in the so-called "sculptured" thin film is ideally a periodic one-dimensional stacking of the domain twins. Zero reflection for the TM polarized electromagnetic wave was indicated in the literature (for normal incidence [68] and arbitrary angle of incidence [69]). For electronics, an unusual type of semiconductor superlattice, termed an "orientational superlattice," was found in spontaneously ordered semiconductor alloys, and their electronic structures and optical properties were also investigated [70–72]. Figure 1.6 shows a quasiperiodic structure of ordered domain twins, which is an orientational superlattice, in a $Ga_{1-x}In_xP$ alloy [72].

1.3 Conclusion

Negative refraction, as an interesting physical phenomenon, can be observed in a number of circumstances possibly facilitated by very different physical mechanisms. The interest in this field has provided great opportunities for fundamental physics research, material developments, and novel applications.

Acknowledgments

We thank Drs. V.M. Agranovich, C.M. Krowne, B. Fluegel, and S. Smith for helpful discussions. This work was funded by the US Department of Energy, Office of Science, Basic Energy Sciences, under Contract No. DE-AC36-99GO10337 to NREL.

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http://www.springer.com/978-3-540-72131-4

Physics of Negative Refraction and Negative Index Materials
Optical and Electronic Aspects and Diversified Approaches
Krowne, C.M.; Zhang, Y. (Eds.)
2007, XVII, 380 p., Hardcover
ISBN: 978-3-540-72131-4