

Contents

Part I Mathematical Prerequisites	
<hr/>	
1	Introduction 5
1.1	Introduction to Soliton Theory 5
1.2	Algebraic and Geometric Approaches 6
1.3	A List of Useful Derivatives 8
2	Mathematical Prerequisites 11
2.1	Elements of Topology 11
2.1.1	Separation Axioms 12
2.1.2	Compactness 15
2.1.3	Weierstrass–Stone Theorem 17
2.1.4	Connectedness, Connectivity, and Homotopy 18
2.1.5	Separability and Basis 20
2.1.6	Metric and Normed Spaces 20
2.2	Elements of Homology 21
3	The Importance of the Boundary 23
3.1	The Power of Compact Boundaries: Representation Formulas 23
3.1.1	Representation Formula for $n = 1$: Taylor Series 24
3.1.2	Representation Formula for $n = 2$: Cauchy Formula 24
3.1.3	Representation Formula for $n = 3$: Green Formula 25
3.1.4	Representation Formula in General: Stokes Theorem 26
3.2	Comments and Examples 28
4	Vector Fields, Differential Forms, and Derivatives 31
4.1	Manifolds and Maps 32
4.2	Differential and Vector Fields 35
4.3	Existence and Uniqueness Theorems: Differential Equation Approach 39

4.4	Existence and Uniqueness Theorems: Flow Box Approach . . .	45
4.5	Compact Supported Vector Fields	47
4.6	Lie Derivative and Differential Forms	47
4.7	Invariants	52
4.8	Fiber Bundles	54
4.9	Poincaré Lemma	57
4.10	Tensor Analysis, Covariant Derivative, and Connections	58
4.11	The Mixed Covariant Derivative	60
4.12	Curvilinear Orthogonal Coordinates	62
	4.12.1 Gradient	64
	4.12.2 Divergence	64
	4.12.3 Curl	65
	4.12.4 Laplacian	65
	4.12.5 Special Two-Dimensional Nonlinear Orthogonal Coordinates	66
4.13	Problems	67
5	Geometry of Curves	69
5.1	Elements of Differential Geometry of Curves	69
5.2	Closed Curves	76
5.3	Curves Lying on a Surface	78
5.4	Problems	79
6	Motion of Curves and Solitons	81
6.1	Nonlinear Kinematics of Two-Dimensional Curves and Solitons	82
	6.1.1 The Time Evolution of Length and Area in General . .	94
6.2	Kinematics of Curve Motion: Three Dimension	101
6.3	Problems	102
7	Geometry of Surfaces	103
7.1	Elements of Differential Geometry of Surfaces	105
7.2	Covariant Derivative and Connections	112
7.3	Geometry of Parametrized Surfaces Embedded in \mathbf{R}_3	116
	7.3.1 Christoffel Symbols and Covariant Differentiation for Hybrid Tensors	118
7.4	Compact Surfaces	120
7.5	Surface Differential Operators	122
	7.5.1 Surface Gradient	123
	7.5.2 Surface Divergence	125
	7.5.3 Surface Laplacian	126
	7.5.4 Surface Curl	127
	7.5.5 Integral Relations for Surface Differential Operators . .	129
	7.5.6 Applications	131
7.6	Problems	134

8	Theory of Motion of Surfaces	137
8.1	Coordinates and Velocities on a Fluid Surface	137
8.2	Geometry of Moving Surfaces	143
8.3	Dynamics of Moving Surfaces	145
8.4	Boundary Conditions for Moving Fluid Interfaces	148
8.5	Dynamics of the Fluid Interfaces	149
8.6	Problems	151

Part II Solitons and Nonlinear Waves on Closed Curves and Surfaces

9	Kinematics of Hydrodynamics	157
9.1	Lagrangian vs. Eulerian Frames	157
9.1.1	Introduction	158
9.1.2	Geometrical Picture for Lagrangian vs. Eulerian	159
9.2	Fluid Fiber Bundle	161
9.2.1	Introduction	161
9.2.2	Motivation for a Geometrical Approach	164
9.2.3	The Fiber Bundle	167
9.2.4	Fixed Fluid Container	168
9.2.5	Free Surface Fiber Bundle	172
9.2.6	How Does the Time Derivative of Tensors Transform from Euler to Lagrange Frame?	174
9.3	Path Lines, Stream Lines, and Particle Contours	178
9.4	Eulerian–Lagrangian Description for Moving Curves	184
9.5	The Free Surface	184
9.6	Equation of Continuity	186
9.6.1	Introduction	186
9.6.2	Solutions of the Continuity Equation on Compact Intervals	192
9.7	Problems	198
10	Dynamics of Hydrodynamics	201
10.1	Momentum Conservation: Euler and Navier–Stokes Equations	201
10.2	Boundary Conditions	204
10.3	Circulation Theorem	206
10.4	Surface Tension	212
10.4.1	Physical Problem	212
10.4.2	Minimal Surfaces	214
10.4.3	Application	216
10.4.4	Isothermal Parametrization	219
10.4.5	Topological Properties of Minimal Surfaces	222

10.4.6	General Condition for Minimal Surfaces	224
10.4.7	Surface Tension for Almost Isothermal Parametrization	225
10.5	Special Fluids	228
10.6	Representation Theorems in Fluid Dynamics	228
10.6.1	Helmholtz Decomposition Theorem in \mathbb{R}_3	228
10.6.2	Decomposition Formula for Transversal Isotropic Vector Fields	231
10.6.3	Solenoidal–Toroidal Decomposition Formulas	234
10.7	Problems	234
11	Nonlinear Surface Waves in One Dimension	237
11.1	KdV Equation Deduction for Shallow Waters	237
11.2	Smooth Transitions Between Periodic and Aperiodic Solutions	242
11.3	Modified KdV Equation and Generalizations	246
11.4	Hydrodynamic Equations Involving Higher-Order Nonlinearities	249
11.4.1	A Compact Version for KdV	249
11.4.2	Small Amplitude Approximation	252
11.4.3	Dispersion Relations	254
11.4.4	The Full Equation	255
11.4.5	Reduction of GKdV to Other Equations and Solutions	257
11.4.6	The Finite Difference Form	261
11.5	Boussinesq Equations on a Circle	264
12	Nonlinear Surface Waves in Two Dimensions	267
12.1	Geometry of Two-Dimensional Flow	267
12.2	Two-Dimensional Nonlinear Equations	275
12.3	Two-Dimensional Fluid Systems with Boundary	278
12.4	Oscillations in Two-Dimensional Liquid Drops	281
12.5	Contours Described by Quartic Closed Curves	283
12.6	Surface Nonlinear Waves in Two-Dimensional Liquid Nitrogen Drops	284
13	Nonlinear Surface Waves in Three Dimensions	289
13.1	Oscillations of Inviscid Drops: The Linear Model	291
13.1.1	Drop Immersed in Another Fluid	293
13.1.2	Drop with Rigid Core	295
13.1.3	Moving Core	301
13.1.4	Drop Volume	305
13.2	Oscillations of Viscous Drops: The Linear Model	307
13.2.1	Model 1	308

13.3 Nonlinear Three-Dimensional Oscillations of Axisymmetric Drops	322
13.3.1 Nonlinear Resonances in Drop Oscillation	330
13.4 Other Nonlinear Effects in Drop Oscillations	340
13.5 Solitons on the Surface of Liquid Drops	344
13.6 Problems	353
14 Other Special Nonlinear Compact Systems	355
14.1 Nonlinear Compact Shapes and Collective Motion	355
14.2 The Hamiltonian Structure for Free Boundary Problems on Compact Surfaces	359

Part III Physical Nonlinear Systems at Different Scales

15 Filaments, Chains, and Solitons	367
15.1 Vortex Filaments	367
15.1.1 Gas Dynamics Filament Model and Solitons	372
15.1.2 Special Solutions	375
15.1.3 Integration of Serret–Frenet Equations for Filaments	377
15.1.4 The Riccati Form of the Serret–Frenet Equations	380
15.1.5 Soliton Solutions on the Vortex Filament	381
15.1.6 Vortex Filaments and the Nonlinear Schrödinger Equation	384
15.2 Nonlinear Dynamics of Stiff Chains	387
15.3 Problems	390
16 Solitons on the Boundaries of Microscopic Systems	391
16.1 Field Theory Model on a Closed Contour and Instantons	392
16.1.1 Quantization: Excited States	394
16.1.2 Quantization: Instantons and Tunneling	394
16.2 Clusters as Solitary Waves on the Nuclear Surface	396
16.3 Solitons and Quasimolecular Structure	404
16.4 Soliton Model for Heavy Emitted Nuclear Clusters	406
16.4.1 Quintic Nonlinear Schrödinger Equation for Nuclear Cluster Decay	408
16.5 Contour Solitons in the Quantum Hall Liquid	411
16.5.1 Perturbative Approach	414
16.5.2 Geometric Approach	417
17 Nonlinear Contour Dynamics in Macroscopic Systems	423
17.1 Plasma Vortex	423
17.1.1 Effective Surface Tension in Magnetohydrodynamics and Plasma Systems	423
17.1.2 Trajectories in Magnetic Field Configurations	424
17.1.3 Magnetic Surfaces in Static Equilibrium	433

17.2 Elastic Spheres	440
17.3 Nonlinear Evolution of Oscillation Modes in Neutron Stars . .	441
18 Mathematical Annex	445
18.1 Differentiable Manifolds	445
18.2 Riccati Equation	446
18.3 Special Functions	446
18.4 One-Soliton Solutions for the KdV, MKdV, and Their Combination	448
18.5 Scaling and Nonlinear Dispersion Relations	450
References	453
Index	461

<http://www.springer.com/978-3-540-72872-6>

Nonlinear Waves and Solitons on Contours and Closed
Surfaces

Ludu, A.

2007, XX, 466 p. 140 illus., Hardcover

ISBN: 978-3-540-72872-6