

Preface

The concept of one-parameter operator semigroups is one of the oldest and most well studied parts of the theory of operators in Banach spaces. It was developed in the middle of the last century as a natural approach to investigation of PDEs and has proved to have many useful and important applications. It is difficult to observe this concept in its entirety, in the sense that there are dozens of textbooks and thousands of research papers directly related to one-parameter operator semigroups. We do not attempt such an overview in this book but devote our attention to some relatively new topics from this theory. Namely, we present and discuss some non-spectral methods that have been developed over the last two decades for the investigation of asymptotic behavior of operator semigroups.

The book has three chapters. The first chapter (with the exception of Section 1.3) contains standard background on operator semigroups. It is addressed to graduate level students. In Section 1.1 we present several basic ergodic theorems for one-parameter semigroups. We discuss here almost periodicity and mean ergodicity of operator semigroups and present such important results as the Jacobs–Deleeuw–Glicksberg splitting theorem and the Eberlein mean ergodic theorem. In Section 1.2 we discuss briefly the spectral theory of C_0 -semigroups. We have only one, but quite difficult problem, which is to find a way to make our explanation short enough for a first introduction. All results presented in Sections 1.1 and 1.2 (as well as many others related to C_0 -semigroups) can be found in standard textbooks on C_0 -semigroups. Then, in Section 1.3 (the content of which is not quite standard), we deal with a very interesting class of one-parameter semigroups — asymptotically finite dimensional semigroups.

Main tools in the investigation of C_0 -semigroups are the notions of a generator and analysis of its spectral properties. Such a “spectral” approach to the study of C_0 -semigroups is well known and it can be found in standard textbooks and research monographs [13], [41], [48], [57], [80], [89], [104], [130], etc. (see also Section 1.2). However, in many important cases, this approach does not work satisfactorily, particularly in the investigation of asymptotic properties of C_0 -semigroups of Markov operators. Recently, essential progress was made in the developing of non-spectral methods in analysis of one-parameter Markov semigroups in L^1 -spaces, motivated by applications to probability theory and dynamical systems. These methods and their applications are reflected in the excellent book of Lasota and Mackey [71].

The subject of the second and the third chapters is completely new and not covered in other books, with the exception, that some theorems on Markov operators in commutative L^1 -spaces are explained in the above mentioned book [71]. In this part of our book, we give very recent results on non-spectral analysis of asymptotic behavior of positive semigroups and discuss open problems on semigroups of positive operators in ordered Banach spaces.

In the second chapter, we develop some non-spectral methods for the asymptotic analysis of positive one-parameter operator semigroups in ordered Banach spaces. We introduce two classes of ordered Banach spaces which include classical L^p spaces for $1 \leq p < \infty$ as well as preduals of von Neumann algebras. Most results of Section 2.1 are about the asymptotic behavior of positive one-parameter operator semigroups in these spaces. Section 2.2 is devoted to positive semigroups in Banach lattices. In this section we present some theorems on inheritance of asymptotic properties of positive semigroups under the domination, and some theorems concerning the mean ergodicity of positive semigroups. Then in Section 2.3 we discuss several problems on the geometry of Banach spaces, related to one-parameter operator semigroups.

In the third chapter, we investigate positive semigroups in L^1 -spaces and in preduals of von Neumann algebras. We study mainly the following two asymptotic problems. The first one is: under what conditions is a one-parameter operator semigroup mean ergodic, almost periodic, or asymptotically stable? The second problem concerns preserving under domination of various asymptotic properties of positive semigroups.

The important aim of the second and the third chapter of the book is an attempt to unify recent results, proofs, and terminology from various sources. We try to present them in a reasonable manner for the potential reader. The author hopes that the bibliography is considerably complete.

The major theorems are usually given with proofs. Only in the case of the proof being too long or involving special methods, the author prefers to send the reader to standard textbooks. At the end of each section we put a supplement, which includes related results, historical notes, exercises, and open problems. In these supplements we usually omit proofs or leave them as exercises for the reader. We assume that the reader is familiar with the basic functional analysis and operator theory, and refer for the more advanced technique to special monographs.

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