

# Preface

In the years 1948 to 1950, one of us (J.S.) had the unique opportunity of attending lecture courses on elliptic differential equations given by Professors Eberhard Hopf and David Gilbarg at Indiana University. These exemplary lectures first awakened his interest in this theory and in particular in the subject of the maximum principle. The other of us (P.P.) began the study of partial differential equations at the Universities of Perugia and of Michigan with Professor Lamberto Cesari, who instilled in her a deep regard for clarity and rigor, as well as for the importance of dealing with concrete problems. This combination is the background of the present work.

The maximum principle enables us to obtain information about solutions of differential equations and inequalities without any explicit knowledge of the solutions themselves, and thus can be a valuable tool in scientific research. In particular, this book should prove useful not only to professional mathematicians and students primarily interested in mathematics, but also to physicists, chemists, engineers and economists. The maximum principle moreover occurs in so many places and in such varied forms that anyone learning about it becomes acquainted with the classically important partial differential equations and, at the same time, discovers the reason for their importance.

We consider classical linear and quasilinear elliptic inequalities as well as divergence structure and variational operators, with emphasis on the important topics of comparison results and tangency theorems. This work ultimately applies also to weak solutions in appropriate Sobolev spaces.

In order that the book may serve the purposes of reference and as a basis for further developments, the proofs are given in detail. This has led, at a number of points, to results either not found elsewhere, or not readily accessible. Many of the proofs and derivations, even of the standard parts of the theory, are new, along with the first book presentation of the modern compact support principle and the general theory of structured elliptic

inequalities. The proofs here, though difficult, make the subject available for the first time to the general reader.

Problems are introduced in the conviction that no mastery of a mathematical subject is possible without working with it. They are designed primarily to illustrate or extend the theory, although the desirability of occasional concrete easy examples has not been ignored.

The most relevant related works are the classical monographs of Gilbarg and Trudinger [43] and the earlier work of Protter and Weinberger [76]. While both these books remain of essential importance and have been invaluable as background for the present work, neither contains an up-to-date modern treatment of the maximum principle itself.

Readers should find the work valuable not only for its detailed presentation, but also as a reference work and possible graduate text material.

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