
Contents

Preface	v
1 What is a Sequential Dynamical System?	1
1.1 Sequential Dynamical Systems: A First Look.....	1
1.2 Motivation	4
1.3 Application Paradigms	7
1.3.1 TRANSIMS	7
1.3.2 Task Scheduling and Transport Computations	13
1.4 SDS: Characteristics and Research Questions	16
1.4.1 Update Order Dependencies	16
1.4.2 Phase-Space Structure	17
1.5 Computational and Algorithmic Aspects	18
1.6 Summary.....	20
Problems	20
Answers to Problems	22
2 A Comparative Study	23
2.1 Cellular Automata.....	23
2.1.1 Background	23
2.1.2 Structure of Cellular Automata	24
2.1.3 Elementary CA Rules	27
2.2 Random Boolean Networks	33
2.3 Finite-State Machines (FSMs).....	34
Problems	35
Answers to Problems	37
3 Graphs, Groups, and Dynamical Systems	39
3.1 Graphs.....	39
3.1.1 Simple Graphs and Combinatorial Graphs	41
3.1.2 The Adjacency Matrix of a Graph	44
3.1.3 Acyclic Orientations	46

3.1.4	The Update Graph	47
3.1.5	Graphs, Permutations, and Acyclic Orientations	48
3.2	Group Actions	50
3.2.1	Groups Acting on Graphs	51
3.2.2	Groups Acting on Acyclic Orientations	52
3.3	Dynamical Systems	56
3.3.1	Classical Continuous Dynamical Systems	57
3.3.2	Classical Discrete Dynamical Systems	59
3.3.3	Linear and Nonlinear Systems	61
	Problems	63
	Answers to Problems	66
4	Sequential Dynamical Systems	
	over Permutations	69
4.1	Definitions and Terminology	69
4.1.1	States, Vertex Functions, and Local Maps	69
4.1.2	Sequential Dynamical Systems	71
4.1.3	The Phase Space of an SDS	73
4.1.4	SDS Analysis — A Note on Approach and Comments	76
4.2	Basic Properties	77
4.2.1	Decomposition of SDS	77
4.2.2	Fixed Points	78
4.2.3	Reversible Dynamics and Invertibility	80
4.2.4	Invertible SDS with Symmetric Functions over Finite Fields	84
4.3	Equivalence	88
4.3.1	Functional Equivalence of SDS	90
4.3.2	Computing Equivalence Classes	91
4.3.3	Dynamical Equivalence	93
4.3.4	Enumeration of Dynamically Nonequivalent SDS	97
4.4	SDS Morphisms and Reductions	103
4.4.1	Covering Maps	104
4.4.2	Properties of Covering Maps	104
4.4.3	Reduction of SDS	105
4.4.4	Dynamical Equivalence Revisited	109
4.4.5	Construction of Covering Maps	110
4.4.6	Covering Maps over Q_α^n	111
4.4.7	Covering Maps over Circ_n	119
	Problems	121
	Answers to Problems	122
5	Phase-Space Structure of SDS and Special Systems	129
5.1	Fixed Points for SDS over Circ_n and $\text{Circ}_{n,r}$	129
5.2	Fixed-Point Computations for General Graphs	137

5.3	Threshold SDS	139
5.4	SDS over Special Graph Classes	140
5.4.1	SDS over the Complete Graph	141
5.4.2	SDS over the Circle Graph	143
5.4.3	SDS over the Line Graph	145
5.4.4	SDS over the Star Graph	146
5.5	SDS Induced by Special Function Classes	146
5.5.1	SDS Induced by $(\text{nor}_k)_k$ and $(\text{nand}_k)_k$	147
5.5.2	SDS Induced by $(\text{nor}_k + \text{nand}_k)_k$	154
	Problems	158
	Answers to Problems	160
6	Graphs, Groups, and SDS	165
6.1	SDS with Order-Independent Periodic Points	165
6.1.1	Preliminaries	166
6.1.2	The Group $G(Y, \mathbf{F}_Y)$	167
6.1.3	The Class of w -Independent SDS	171
6.2	The Class of w -Independent SDS over Circ_n	174
6.2.1	The Groups $G(\text{Circ}_4, \mathbf{F}_{\text{Circ}_4})$	176
6.3	A Presentation of S_{35}	178
	Problems	179
	Answers to Problems	182
7	Combinatorics of Sequential Dynamical Systems over Words	185
7.1	Combinatorics of SDS over Words	187
7.1.1	Dependency Graphs	187
7.1.2	Automorphisms	189
7.1.3	Words	192
7.1.4	Acyclic Orientations	193
7.1.5	The Mapping \mathcal{O}_Y	195
7.1.6	A Normal Form Result	197
7.1.7	The Bijection	198
7.2	Combinatorics of SDS over Words II	199
7.2.1	Generalized Equivalences	199
7.2.2	The Bijection (P1)	201
7.2.3	Equivalence (P2)	204
7.2.4	Phase-Space Relations	206
	Problems	209
	Answers to Problems	210
8	Outlook	213
8.1	Stochastic SDS	213
8.1.1	Random Update Order	214
8.1.2	SDS over Random Graphs	217

8.2	Gene-Regulatory Networks	217
8.2.1	Introduction	217
8.2.2	The Tryptophan-Operon	218
8.3	Evolutionary Optimization of SDS-Schedules	220
8.3.1	Neutral Networks and Phenotypes of RNA and SDS . . .	220
8.3.2	Distances	223
8.3.3	A Replication-Deletion Scheme	226
8.3.4	Evolution of SDS-Schedules	227
8.3.5	Pseudo-Codes	228
8.4	Discrete Derivatives	229
8.5	Real-Valued and Continuous SDS	231
8.6	L-Local SDS	233
8.7	Routing	234
8.7.1	Weights	234
8.7.2	Protocols as Local Maps	235
References		237
Index		245

An Introduction to Sequential Dynamical Systems

Mortveit, H.; Reidys, C.

2008, XII, 248 p. 73 illus., Softcover

ISBN: 978-0-387-30654-4