1

BASIC CONCEPTS

Satellite image of Tarbela Dam on the Indus River in Pakistan
(Courtesy, Johnson Space Center, NASA)

1-1 Introduction

Liquids are transported from one location to another using natural or constructed conveyance structures. The cross section of these structures may be open or closed at the top. The structures with closed tops are referred to as *closed conduits* and those with the top open are called *open channels*. For example, tunnels and pipes are closed conduits whereas rivers, streams, estuaries etc. are open channels. The flow in an open channel or in a closed conduit having a free surface is referred to as *free-surface flow* or *open-channel flow*. The properties and the analyses of these flows are discussed in this book.

In this chapter, commonly used terms are first defined. The classification of flows is then discussed, and the terminology and the properties of a channel section are presented. Expressions are then derived for the energy and momentum coefficients to account for nonuniform velocity distribution at a channel section. The chapter concludes with a discussion of the pressure distribution in a channel section.

1-2 Definitions

The terms open-channel flow or free-surface flow (Fig. 1-1) are used synonymously in this book. The free surface is usually subjected to atmospheric pressure. Groundwater or subsurface flows are excluded from the present discussions. If there is no free surface and the conduit is flowing full, then the flow is called *pipe flow,* or *pressurized flow.* (Fig. 1-2)

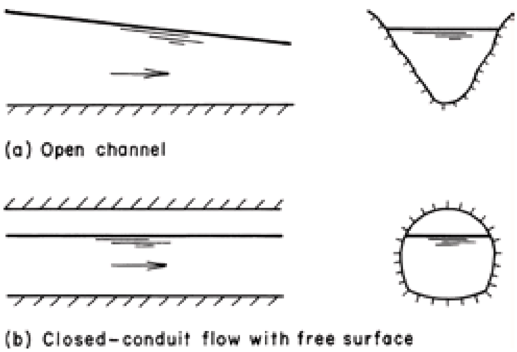


Fig. 1-1. Free-Surface flow

In a closed conduit, it is possible to have both free-surface flow and pressurized flow at different times. It is also possible to have these flows at a given time in different reaches of a conduit. For example, the flow in a storm sewer

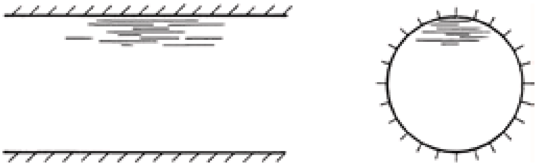


Fig. 1-2. Pipe or pressurized flow

may be free-surface flow at a certain time. Then, due to large inflows produced by a sudden storm, the sewer may flow full and pressurize it. Similarly, the flow in a closed conduit may be free flow in part of the length and pipe flow in the remaining length. This type of combined free-surface, pressurized flow usually occurs in a closed conduit when the downstream end of the conduit is submerged (Fig. 1-3).

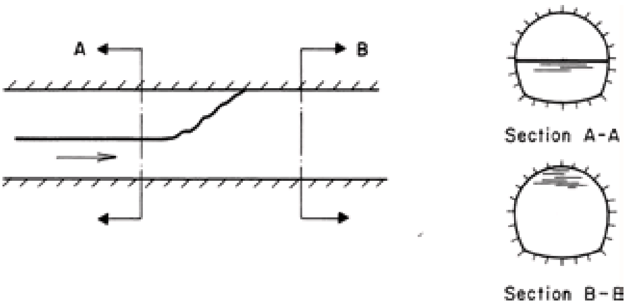
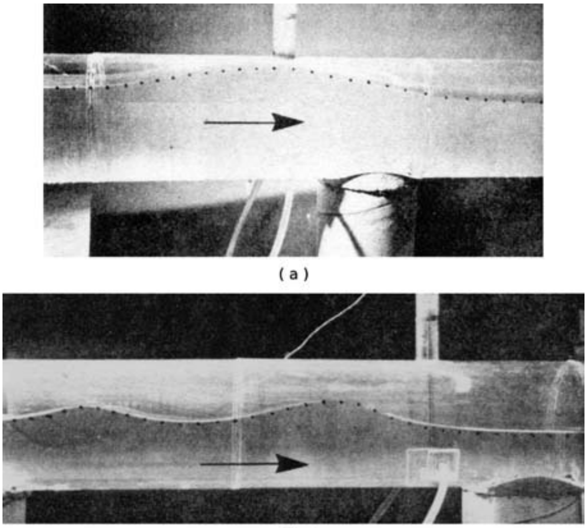


Fig. 1-3. Combined free-surface and pressurized flow

The photographs of Fig. 1-4 show unsteady flow in the 1:84-scale hydraulic model of the tailrace tunnel of Mica Power Plant, located on the Columbia River in Canada. The flow in the two unlined, horseshoe tailrace tunnels, each 18.3 m high and 14.6 m wide, is normally free-surface flow. However, during periods of high tailwater levels, the tunnels may be pressurized following major load changes on the turbogenerators that produce large changes in the inflow to the tunnels. The transient flow conditions shown in Fig. 1-4 are produced by increasing or decreasing in 9 seconds the discharge of three turbines on tunnel no. 2 while the discharge from the three turbines on tunnel no. 1 remains constant. The discharge increase in Fig. 1-4a is from zero to 850 m^3/s and the discharge reduction in Fig. 1-4b is from 850 m^3/s to zero. The free-surface and pressurized flows in a laboratory experiment are shown in Fig. 1-5. The



(a)

(b)

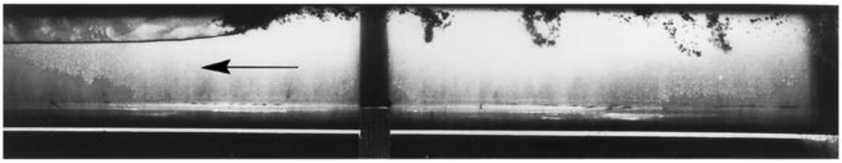
Fig. 1-4. Transient flow in the hydraulic model of Mica Tailrace Tunnel (Courtesy, British Columbia Hydro and Power Authority, Canada)

initial steady state flow is from left to right and thus the upstream end is located at the left-hand side of the photographs.

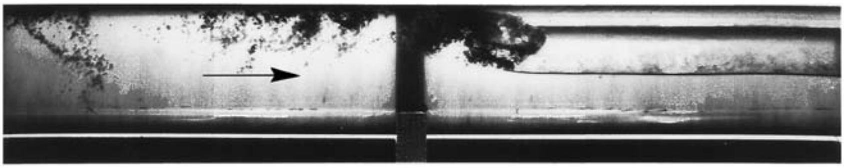
The height to which liquid rises in a small-diameter piezometer inserted in a channel or a closed conduit depends upon the pressure at the location of the piezometer. A line joining the top of the liquid surface in the piezometers is called the *hydraulic-grade line* (Fig. 1-6). In pipe flow, the height of hydraulic-grade line above a specified datum is called the *piezometric head* at that location. In free-surface flow, the hydraulic grade line usually, but not always, coincides with the free surface (see Section 1-6). If the velocity head, $V^2/(2g)$, in which V = mean flow velocity for the channel cross section, and g = acceleration due to gravity, is added to the top of the hydraulic grade line and the resulting points are joined by a line, then this line is called the *energy-grade line*. This line represents the total head at different sections of a channel.

1-3 Classification of Flows

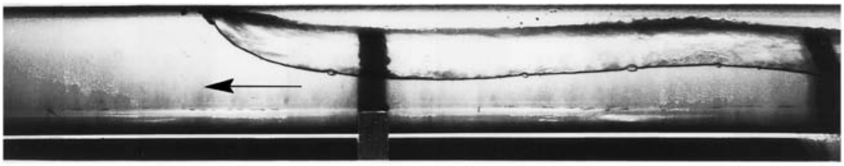
Based on different criteria, free-surface flows may be classified into various types (Fig. 1-7), as discussed in the following paragraphs.



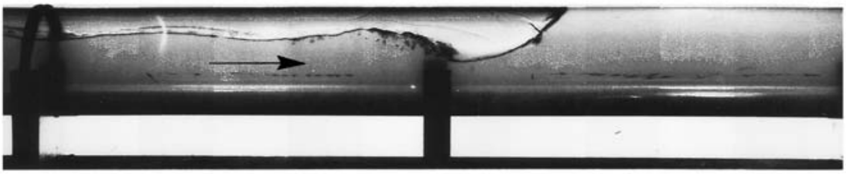
(a) Positive surge from downstream



(b) Positive surge from upstream



(c) Negative surge from downstream



(d) Negative surge from upstream

Fig. 1-5. Free-surface and pressurized flows (Courtesy, Professor C. S. Song [1984])

Steady and Unsteady Flows

If the flow velocity at a given point does not change with respect to time, then the flow is called *steady flow.* However, if the velocity at a given location changes with respect to time, then the flow is called *unsteady flow.*

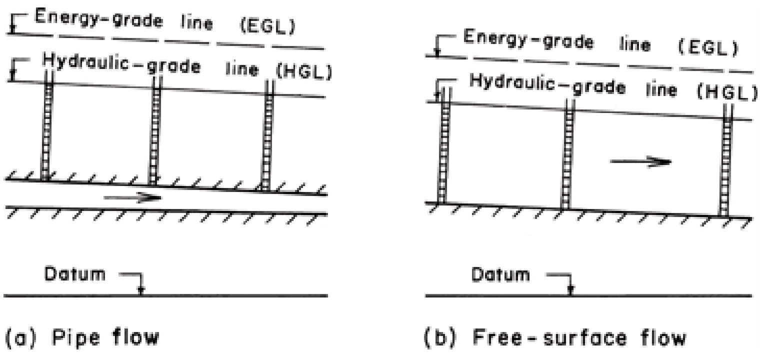


Fig. 1-6. Hydraulic- and energy-grade lines

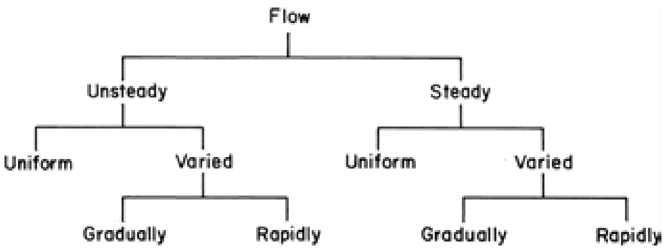


Fig. 1-7. Classification of flows

Note that this classification is based on the time variation of velocity v at a specified location. Thus, the local acceleration, $\partial v/\partial t$, is zero in steady flows. In two- or three-dimensional steady flows, the time variation of all components of velocity is zero.

It is possible in some situations to transform unsteady flow into steady flow by having coordinates with respect to a moving reference. This simplification is helpful in the visualization of flow and in the derivation of governing equations. Such a transformation is possible only if the wave shape does not change as the wave propagates. For example, the shape of a surge wave moving in a smooth channel does not change and consequently the propagation of a surge wave in an otherwise unsteady flow may be converted into steady flow by moving the reference coordinates at the absolute surge velocity. This is equivalent to an observer traveling beside the surge wave so that the surge wave appears to the observer to be stationary; thus the flow may be considered as steady. If the wave shape changes as it propagates, then it is not possible to transform such a wave motion into steady flow. Typical example of such a situation is the movement of a flood wave in a natural channel, where the shape of the wave is modified as it propagates in the channel.

Uniform and Nonuniform flows

If the flow velocity at a given instant of time does not vary within a given length of channel, then the flow is called *uniform flow.* However, if the flow velocity at a time varies with respect to distance, then the flow is called *nonuniform flow,* or *varied flow.*

This classification is based on the variation of flow velocity with respect to space at a specified instant of time. Thus, the convective acceleration in uniform flow is zero. In mathematical terms, the partial derivatives of the velocity components with respect to x, y, and z direction are all zero. However, many times this strict restriction is somewhat relaxed by allowing a nonuniform velocity distribution at a channel section. In other words, a flow is considered uniform as long as the velocity in the direction of flow at different locations along a channel remains the same.

Depending upon the rate of variation with respect to distance, flows may be classified as *gradually varied flow* or *rapidly varied flow.* As the name implies, the flow is called gradually varied flow, if the flow depth varies at a slow rate with respect to distance, whereas the flow is called rapidly varied flow if the flow depth varies significantly in a short distance.

Note that the steady and unsteady flows are characterized by the variation with respect to time at a given location, whereas uniform or varied flows are characterized by the variation at a given instant of time with respect to distance. Thus, in a steady, uniform flow, the total derivative $dV/dt = 0$. In one-dimensional flow, this means that $\partial v/\partial t = 0$, and $\partial v/\partial x = 0$. In two- and three-dimensional flow, the partial derivatives of the velocity components in the other two coordinate directions with respect to time and space are also zero.

Laminar and Turbulent Flows

The flow is called *laminar flow* if the liquid particles appear to move in definite smooth paths and the flow appears to be as a movement of thin layers on top of each other. In *turbulent flow,* the liquid particles move in irregular paths which are not fixed with respect to either time or space.

The relative magnitude of viscous and inertial forces determines whether the flow is laminar or turbulent: The flow is laminar if the viscous forces dominate, and the flow is turbulent if the inertial forces dominate.

The ratio of viscous and inertial forces is defined as the *Reynolds number,*

$$\mathbf{R}_e = \frac{VL}{\nu} \qquad (1-1)$$

in which $\mathbf{R}_e$ = Reynolds number; V = mean flow velocity; L = a characteristic length; and ν = kinematic viscosity of the liquid. Unlike pipe flow in which the pipe diameter is usually used for the characteristic length, either hydraulic depth or hydraulic radius may be used as the characteristic length in free-surface flows. *Hydraulic depth* is defined as the flow area divided by the top water-surface width and the *hydraulic radius* is defined as the flow area divided by the wetted perimeter. The transition from laminar to turbulent flow in free-surface flows occurs for R_e of about 600, in which R_e is based on the hydraulic radius as the characteristic length.

In real-life applications, laminar free-surface flows are extremely rare. A smooth and glassy flow surface may be due to surface velocity being less than that required to form capillary waves and may not necessarily be due to the fact that the flow is laminar. Care should be taken while selecting geometrical scales for the hydraulic model studies so that the flow depth on the model is not very small. Very small depth may produce laminar flow on the model even though the prototype flow to be modelled is turbulent. The results of such a model are not reliable.

Subcritical, Supercritical, and Critical Flows

A flow is called *critical* if the flow velocity is equal to the velocity of a gravity wave having small amplitude. A gravity wave may be produced by a change in the flow depth. The flow is called *subcritical flow,* if the flow velocity is less than the critical velocity, and the flow is called *supercritical flow* if the flow velocity is greater than the critical velocity. The *Froude number,* $\mathbf{F}_r$, is equal to the ratio of inertial and gravitational forces and, for a rectangular channel, it is defined as

$$\mathbf{F}_r = \frac{V}{\sqrt{gy}} \tag{1-2}$$

in which y = flow depth. General expressions for $\mathbf{F}_r$ are presented in Section 3-2. Depending upon the value of $\mathbf{F}_r$, flow is classified as *subcritical* if $\mathbf{F}_r < 1$; *critical* if $\mathbf{F}_r = 1$; and *supercritical* if $\mathbf{F}_r > 1$.

1-4 Terminology

Channels may be natural or artificial. Various names have been used for the artificial channels: A long channel having mild slope usually excavated in the ground is called a *canal.* A channel supported above ground and built of wood, metal, or concrete is called a *flume.* A *chute* is a channel having very steep bottom slope and almost vertical sides. A *tunnel* is a channel excavated through a hill or a mountain. A short channel flowing partly full is referred to as a *culvert.*

A channel having the same cross section and bottom slope throughout is referred to as a *prismatic channel,* whereas a channel having varying cross section and/or bottom slope is called a *non-prismatic channel.* A long channel may be comprised of several prismatic channels. A cross section taken *normal* to the direction of flow (e.g., Section BB in Fig. 1-8) is called a *channel section.* The depth of flow, y, at a section is the *vertical* distance of the lowest point of the channel section from the free surface. The *depth of flow section, d*, is the depth of flow *normal* to the direction of flow. The *stage, Z*, is the elevation or vertical distance of free surface above a specified datum (Fig. 1-8). The *top width, B*, is the width of channel section at the free surface. The *flow area, A*, is the cross-sectional area of flow *normal* to the direction of flow. The *wetted perimeter, P* is defined as the length of line of intersection of channel wetted surface with a cross-sectional plane normal to the flow direction. The *hydraulic radius, R*, and *hydraulic depth, D,* are defined as

$$R = \frac{A}{P}$$

$$D = \frac{A}{B} \tag{1-3}$$

Expressions for A, P, D and R for typical channel cross sections are presented in Table 1-1.

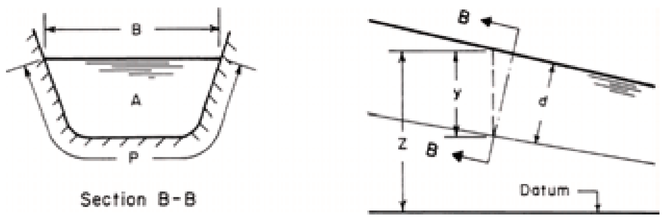


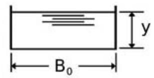
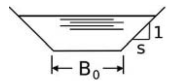
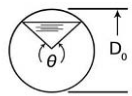
Fig. 1-8. Definition sketch

1-5 Velocity Distribution

The flow velocity in a channel section varies from one point to another. This is due to shear stress at the bottom and at the sides of the channel and due to the presence of free surface. Fig. 1-9 shows typical velocity distributions in different channel cross sections.

The flow velocity may have components in all three Cartesian coordinate directions. However, the components of velocity in the vertical and transverse directions are usually small and may be neglected. Therefore, only the flow velocity in the direction of flow needs to be considered. This velocity component varies with depth from the free surface. A typical variation of velocity with depth is shown in Fig. 1-10.

Table 1-1. Properties of typical channel cross sections

Section	Area, A	Wetted Perimeter, P	Hydraulic radius, R	Top width, B	Hydraulic depth, D	
Rectangular	$B_o y$	$B_o + 2y$	$\dfrac{B_o y}{B_o + 2y}$	B_o	y	
Trapezoidal	$(B_o + sy)y$	$B_o + 2y\sqrt{1+s^2}$	$\dfrac{(B_o + sy)y}{B_o + 2y\sqrt{1+s^2}}$	$B_o + 2sy$	$\dfrac{(B_o + sy)y}{B_o + 2sy}$	
Triangular	sy^2	$2y\sqrt{1+s^2}$	$\dfrac{sy}{2\sqrt{1+s^2}}$	$2sy$	$0.5y$	
Circular	$\frac{1}{8}(\theta - \sin\theta)D_o^2$	$\frac{1}{2}\theta D_o$	$\frac{1}{4}\left(1 - \dfrac{\sin\theta}{\theta}\right) D_o$	$D_o \sin \frac{1}{2}\theta$	$\left(\frac{\theta - \sin\theta}{\sin\frac{1}{2}\theta}\right)\frac{D_o}{8}$	

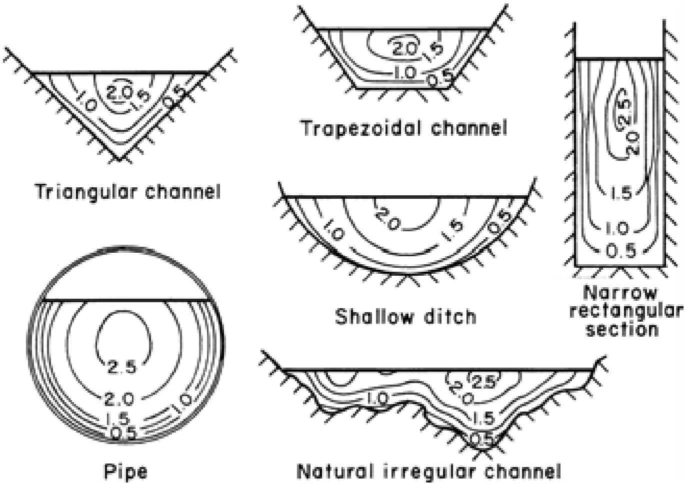


Fig. 1-9. Velocity distribution in different channel sections
(After Chow [1959])

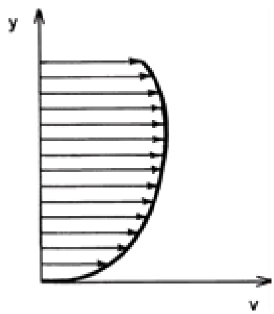


Fig. 1-10. Typical velocity variation with depth

Energy Coefficient

As discussed in the previous paragraphs, the flow velocity in a channel section usually varies from one point to another. Therefore, the mean velocity head in a channel section, $(V^2/2g)_m$, is not the same as the velocity head, $V_m^2/(2g)$, computed by using the mean flow velocity, V_m, in which the subscript m refers to the mean values. This difference may be taken into consideration by introducing an *energy coefficient,* α, which is also referred to as the *velocity-head,* or *Coriolis coefficient.* An expression for this coefficient is derived in the following paragraphs.

Referring to Fig. 1-11, the mass of liquid flowing through area ΔA per unit time $= \rho V \Delta A$, in which $\rho =$ mass density of the liquid. Since, the kinetic energy of mass m traveling at velocity V is $(1/2)mV^2$, we can write

$$\text{Kinetic energy transfer through area } \Delta A \text{ per unit time}$$
$$= \frac{1}{2}\rho V \Delta A V^2$$
$$= \frac{1}{2}\rho V^3 \Delta A \qquad (1-4)$$

Hence,

$$\text{Kinetic energy transfer through area } A \text{ per unit time}$$
$$= \frac{1}{2}\rho \int V^3 \, dA \qquad (1-5)$$

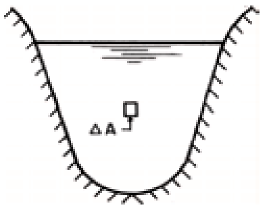


Fig. 1-11. Definition sketch

It follows from Eq. 1-4 that the kinetic energy transfer through area ΔA per unit time may be written as $(\gamma V \Delta A)V^2/(2g) =$ weight of liquid passing through area ΔA per unit time $\times$ velocity head, in which $\gamma =$ specific weight of the liquid. Now, if V_m is the mean flow velocity for the channel section, then the weight of liquid passing through total area per unit time $=\gamma V_m \int dA$; and the velocity head for the channel section $=\alpha V_m^2/(2g)$, in which $\alpha =$ velocity-head coefficient. Therefore, we can write

$$\text{Kinetic energy transfer through area per unit time}$$
$$= \rho \alpha V_m \frac{V_m^2}{2} \int dA \qquad (1-6)$$

Hence, it follows from Eqs. 1-5 and 1-6 that

$$\alpha = \frac{\int V^3 dA}{V_m^3 \int dA} \qquad (1-7)$$

Figure 1-12 shows a typical cross section of a natural river comprising of the main river channel and the flood plain on each side of the main channel. The flow velocity in the floodplain is usually very low as compared to that in the main section. In addition, the variation of flow velocity in each subsection is small. Therefore, each subsection may be assumed to have the same flow velocity throughout. In such a case, the integration of various terms of Eq. 1-7 may be replaced by summation as follows:

$$\alpha = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{V_m^3 (A_1 + A_2 + A_3)} \qquad (1-8)$$

in which

$$V_m = \frac{V_1 A_1 + V_2 A_2 + V_3 A_3}{A_1 + A_2 + A_3} \qquad (1-9)$$

By substituting Eq. 1-9 into Eq. 1-8 and simplifying, we obtain

$$\alpha = \frac{(V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3)(A_1 + A_2 + A_3)^2}{(V_1 A_1 + V_2 A_2 + V_3 A_3)^3} \qquad (1-10)$$

Note that Eq. 1-10 is written for a section which may be divided into three subsections each having uniform velocity distribution. For a general case in which total area A may be subdivided into N such subareas each having uniform velocity, an equation similar to Eq. 1-10 may be written as

$$\alpha = \frac{\sum_{i=1}^{N} (V_i^3 A_i). \left(\sum A_i\right)^2}{(\sum V_i A_i)^3} \qquad (1-11)$$

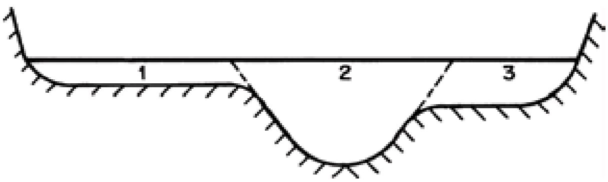


Fig. 1-12. Typical river cross section

Momentum Cefficient

Similar to the energy coefficient, a coefficient for the momentum transfer through a channel section may be introduced to account for nonuniform velocity distribution. This coefficient, also called *Boussinesq coefficient,* is denoted by β. An expression for this may be obtained as follows.

The mass of liquid passing through area ΔA per unit time $= \rho V \Delta A$. Therefore, the momentum passing through area ΔA per unit time $= (\rho V \Delta A)V = \rho V^2 \Delta A$. By integrating this expression over the total area, we get

$$\text{Momentum transfer through area } A \text{ per unit time} = \rho \int V^2 \, dA \tag{1-12}$$

By introducing the momentum coefficient, β, we may write the momentum transfer through area A in terms of the mean flow velocity, V_m, for the channel section, i.e.,

$$\text{Momentum transfer through area } A \text{ per unit time} = \beta \rho V_m^2 \int dA \tag{1-13}$$

Hence, it follows from Eqs. 1-12 and 1-13 that

$$\beta = \frac{\int V^2 \, dA}{V_m^2 \int dA} \tag{1-14}$$

Theoretical values for α and β can be derived from the power law and the logarithmic law for velocity distribution in wide channels. Chen (1992) derived the theoretical values of α and β using the power law distribution. The values of α and β for typical channel sections [Temple 1986; Watts et al. 1967; Chow 1959] are listed in Table 1-2. For turbulent flow in a straight channel having a rectangular, trapezoidal, or circular cross section, α is usually less than 1.15 [Henderson, 1966]. Therefore, it may not be included in the computations since its value is not precisely known and it is nearly equal to unity.

Table 1-2. Values of α and β for typical sections*

Channel section	α	β
Regular channels	1.10-1.20	1.03-1.07
Natural channels	1.15-1.50	1.05-1.17
Rivers under ice cover	1.20-2.00	1.07-1.33
River valleys, overflooded	1.50-2.00	1.17-1.33

* Compiled from data given by Chow [1959]

Example 1-1

The velocity distribution in a channel section may be approximated by the equation, $V = V_o(y/y_o)^n$, in which V is the flow velocity at depth y; V_o is the flow velocity at depth y_o, and n = a constant. Derive expressions for the energy and momentum coefficients.

Solution:

Let us consider a unit width of the channel. Then, we can replace area A in the equations for the energy and momentum coefficients by the flow depth y. Now,

$$V_m = \frac{\int V\,dA}{\int dA}$$

For a unit width, this equation becomes

$$V_m = \frac{\int V\,dy}{\int dy}$$

By substituting the expression for V into this equation, we obtain

$$\begin{aligned} V_m &= \frac{\int_0^{y_o} V_o(\frac{y}{yo})^n\,dy}{\int_0^{y_o} dy} \\ &= \frac{V_o}{y_o^n}\frac{y^{n+1}}{n+1}\Big|_0^{y_o}\frac{1}{y_o} \\ &= \frac{V_o}{n+1} \end{aligned}$$

By substituting $V = V_o(y/y_o)^n$, $V_m = V_o/(n+1)$, and $dA = dy$ into Eq. 1-7, we obtain

$$\begin{aligned} \alpha &= \frac{\int_0^{y_o} V_o^3(y/yo)^{3n}dy}{[V_o/(n+1)]^3\int_0^{y_o} dy} \\ &= \frac{(V_o^3/y_o^{3n})[y^{3n+1}/(3n+1)]}{y_o[V_o/(n+1)]^3} \\ &= \frac{(n+1)^3}{3n+1} \end{aligned}$$

Substitution of $V = V_o(y/y_o)^n$ and $V_m = V_o/(n+1)$ into Eq. 1-14 yields

$$\beta = \frac{\int_0^{y_o} V_o^2 (y/y_o)^{2n}\, dy}{[V_o/(n+1)]^2 \int_0^{y_o} dy}$$
$$= \frac{(V_o^2 y_o)/(2n+1)}{[V_o/(n+1)]^2 y_o}$$
$$= \frac{(n+1)^2}{2n+1}$$

1-6 Pressure Distribution

The pressure distribution in a channel section depends upon the flow conditions. Let us consider several possible cases, starting with the simplest one and then proceeding progressively to more complex situations.

Static Conditions

Let us consider a column of liquid having cross-sectional area ΔA, as shown in Fig. 1-13. The horizontal and vertical components of the resultant force acting on the liquid column are zero, since the liquid is stationary. If $p =$ pressure intensity at the bottom of the liquid column, then the force due to pressure at the bottom of the column acting vertically upwards $= p\Delta A$. The weight of the liquid column acting vertically downwards $= \rho g y \Delta A$. Since the vertical component of the resultant force is zero, we can write

$$p\Delta A = \rho g y \Delta A$$

or

$$p = \rho g y \qquad (1-15)$$

In other words, the pressure intensity is directly proportional to the depth below the free surface. Since ρ is constant for typical engineering applications, the relationship between the pressure intensity and depth plots as a straight line, and the liquid rises to the level of the free surface in a piezometer, as shown in Fig. 1-13. The linear relationship, based on the assumption that ρ is constant, is usually valid except at very large depths, where large pressures result in increased density.

Horizontal, Parallel Flow

Let us now consider the forces acting on a vertical column of liquid flowing in a horizontal, frictionless channel (Fig. 1-14). Let us assume that there is no acceleration in the direction of flow and the flow velocity is parallel to the channel bottom and is uniform over the channel section. Thus the

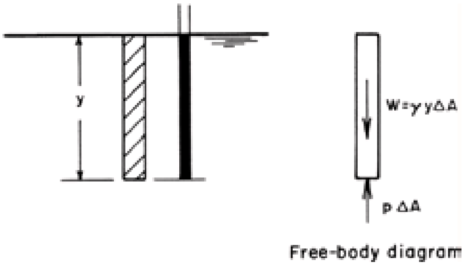


Fig. 1-13. Pressure in stationary fluid

streamlines are parallel to the channel bottom. Since there is no acceleration in the direction of flow, the component of the resultant force in this direction is zero. Referring to the free-body diagram shown in Fig. 1-14 and noting that the vertical component of the resultant force acting on the column of liquid is zero, we may write

$$\rho gy\Delta A = p\Delta A$$

or

$$p = \rho gy = \gamma y \tag{1-16}$$

in which $\gamma = \rho g$ = specific weight of the liquid. Note that this pressure distribution is the same as if the liquid were stationary; it is, therefore, referred to as the *hydrostatic pressure distribution.*

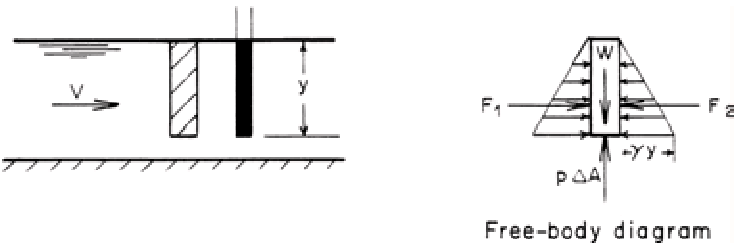


Fig. 1-14. Horizontal, parallel flow

Parallel Flow in Sloping Channels

Let us now consider the flow conditions in a sloping channel such that there is no acceleration in the flow direction, the flow velocity is uniform at a channel cross section and is parallel to the channel bottom; i.e., the streamlines are parallel to the channel bottom. Figure 1-15 shows the free-body diagram of a column of liquid normal to the channel bottom. The cross-sectional area of the column is ΔA. If $\theta =$ slope of the channel bottom, then the component of the weight of column acting along the column is $\rho g d \Delta A \cos\theta$ and the force acting at the bottom of the column is $p\Delta A$. There is no acceleration in a direction along the column length, since the flow velocity is parallel to the channel bottom. Hence, we can write $p\Delta A = \rho g d \Delta A \cos\theta$, or $p = \rho g d \cos\theta = \gamma d \cos\theta$. By substituting $d = y\cos\theta$ into this equation ($y =$ flow depth measured vertically, as shown in Fig. 1-15), we obtain

$$p = \gamma y \cos^2\theta \qquad (1-17)$$

Note that in this case the pressure distribution is not hydrostatic in spite of

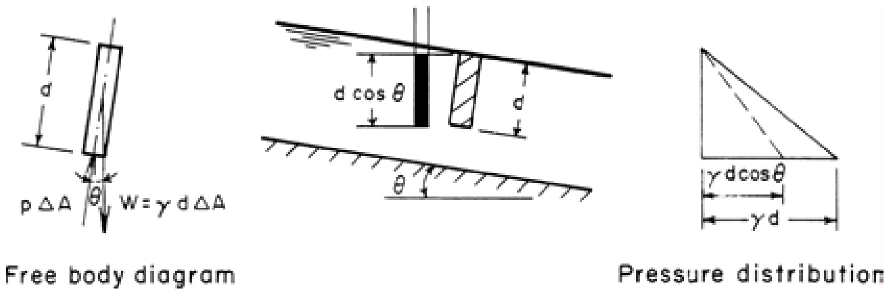


Fig. 1-15. Parallel flow in a sloping channel

the fact that we have parallel flow and there is no acceleration in the direction of flow. However, if the slope of the channel bottom is small, then $\cos\theta \simeq 1$ and $d \simeq y$. Hence,

$$p \simeq \rho g d \simeq \rho g y \qquad (1-18)$$

In several derivations in the subsequent chapters we assume that the slope of the channel bottom is small. With this assumption, the pressure distribution may be assumed to be hydrostatic if the streamlines are almost parallel and straight, and the flow depths measured vertically or normal to the channel bottom are approximately the same.

Curvilinear Flow

In the previous three cases, the streamlines were straight and parallel to the channel bottom. However, in several real-life situations, the streamlines have pronounced curvature. To determine the pressure distribution in such flows, let us consider the forces acting in the vertical direction on a column of liquid with cross-sectional area ΔA, as shown in Fig. 1-16.

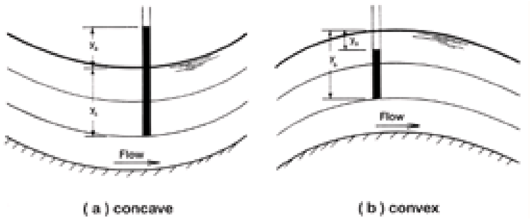


Fig. 1-16. Curvilinear flow

$$\text{Mass of the liquid column} = \rho y_s \Delta A \qquad (1-19)$$

If r = radius of curvature of the streamline and V is the flow velocity at the point under consideration, then

$$\text{Centrifugal acceleration} = \frac{V^2}{r} \qquad (1-20)$$

and

$$\text{Centrifugal force} = \rho y_s \Delta A \frac{V^2}{r} \qquad (1-21)$$

Dividing the centrifugal force by the area of the column and converting the pressure to pressure head, we obtain the following expression for the pressure head, y_a, acting at the bottom of the liquid column due to centrifugal acceleration

$$y_a = \frac{1}{g} y_s \frac{V^2}{r} \qquad (1-22)$$

The pressure due to centrifugal force is in the same direction as the weight of column if the curvature is concave, as shown in Fig. 1-16a, and it is in a direction opposite to the weight if the curvature is convex (Fig. 1-16b). Therefore, the total pressure head acting at the bottom of the column is an algebraic sum of the pressure due to centrifugal action and the weight of the liquid column, i.e.,

$$\text{Total pressure head} = y_s\left(1 \pm \frac{1}{g}\frac{V^2}{r}\right) \qquad (1-23)$$

A positive sign is used if the streamline is concave, and a negative sign is used if the streamline is convex. Note that the first term in Eq. 1-23 is the pressure head due to static conditions while the second term is the pressure head due to centrifugal action. Thus, the liquid in a piezometer inserted into the flow rises, as shown in Fig. 1-16a. In other words, pressure increases due to centrifugal action in concave flows and decreases in convex flows (Fig. 1-16b).

Boussinesq derived a formula for solving problems with small water surface curvatures. Detailed derivations are presented in Subramanya [1991] and Jaeger [1957].

1-7 Reynolds Transport Theorem

The Reynolds transport theorem relates the flow variables for a specified fluid mass to that of a specified flow region. We will utilize it in later chapters to derive the governing equations for steady and unsteady flow conditions. To simplify the presentation of its application, we include a brief description in this section; for details, see Roberson and Crowe [1997].

We will call a specified fluid mass the *system* and a specified region, the *control volume.* The boundaries of a system separate it from its *surroundings* and the boundaries of a control volume are referred to as the *control surface.* The three well-known conservation laws of mass, momentum, and energy describe the interaction between a system and its surroundings. However, in hydraulic engineering, we are usually interested in the flow in a region as compared to following the motion of a fluid particle or the motion of a quantity of mass. The Reynolds transport theorem relates the flow variables in a control volume to those of a system.

Let the *extensive property* of a system be B and the corresponding *intensive* property be β. The intensive property is defined as the amount of B per unit mass, m, of a system, i.e.,

$$\beta = \lim_{\Delta m \to 0} \frac{\Delta B}{\Delta m} \qquad (1-24)$$

Thus, the total amount of B in a control volume

$$B_{cv} = \int_{cv} \beta \rho d\mathcal{V} \qquad (1-25)$$

in which ρ = mass density and $d\mathcal{V}$ = differential volume of the fluid, and the integration is over the control volume.

We will consider mainly one-dimensional flows in this book. The control volume will be fixed in space and will not change its shape with respect to time, i.e., it will not stretch or contract. For such a control volume for one-dimensional flow, the following equation relates the system properties to those in the control volume:

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} \beta \rho d\mathcal{V} + (\beta \rho A V)_{out} - (\beta \rho A V)_{in} \qquad (1-26)$$

in which the subscripts *in* and *out* refer to the quantities for the inflow and outflow from the control volume and V = flow velocity. The system is assumed to occupy the entire control volume, i.e., the system boundaries coincide with the control surface.

Let us now discuss the application of this equation to a control volume. As an example, the time rate of change of momentum of a system is equal to the sum of the forces exerted on the system by its surroundings (Newton's second law of motion).To use this equation to describe the conservation of momentum of the water of mass m in a control volume, the extensive property B is the momentum of water $= mV$ and the corresponding intensive property, $\beta = \lim_{\Delta m \to 0} V(\Delta m/\Delta m) = V$. To describe the conservation of mass, B is the mass of water and the corresponding intensive property $\beta = \lim_{\Delta m \to 0}(\Delta m/\Delta m) = 1$.

1-8 Computer Program

A computer program for computing the energy and momentum coefficients is presented in Appendix A. A trapeziodal rule is used for computing the integrals.

1-9 Summary

In this chapter, commonly used terms were defined, classification of flows using several different criteria was outlined, and the properties of a channel section were presented. The distribution of velocity and pressure in a channel section was discussed and two coefficients were introduced to account for the nonuniform velocity distribution. The distribution of pressure was discussed and a brief description of the Reynolds transport theorem was presented to facilitate its application in later chapters.

Problems

1.1. Derive expressions for the flow area, A, wetted perimeter,P, hydraulic radius,R, top-water surface width, B, and hydraulic depth,D, for the following channel cross sections:

i. Rectangular (bottom width $=B_o$)
ii. Trapezoidal (bottom width $= B_o$, side slopes = 1 V : s H)
iii. Triangular (side slopes = 1 V : s H)
iv. Partially-full circular (diameter = D)
v. Standard horseshoe(Fig. 1-17).

1.2. The discharge in a channel is proportional to $AR^{2/3}$ if the flow is uniform. For a circular conduit having an inside diameter D, prove that the discharge is maximum when the flow depth is $0.94D$.

1.3. Compute $(R/R_f)^{2/3}$ and $AR^{2/3}/(AR^{2/3})_f$ for different values of y/D for a circular conduit flowing partially full, in which y = flow depth; D = conduit diameter; and the subscript ‘f’ refers to the values for the full section. At what values of ratio y/D do the curves have maximum values?

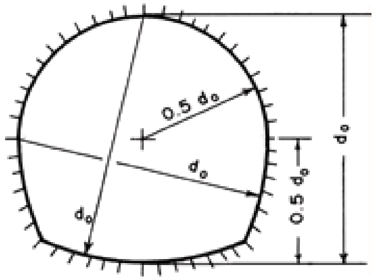


Fig. 1-17. Horseshoe section

1.4. Determine the energy and momentum coefficients for the velocity distribution, $V = 5.75V_o \log(30y/k)$, in which V_o = flow velocity at the free surface; y_o = flow depth, and k = height of surface roughness. Assume the channel is very wide and rectangular.

1.5. The flow velocities measured at various flow depths in a wide rectangular flume are listed in the following table. Write a computer program to determine the values of α and β. Use Simpson's rule for the numerical integration.

Table 1-3. Flow velocities at different depths

y (m)	0.0	0.2	0.4	0.6	0.8	1.0	1.2
V (m/s)	0.0	3.87	4.27	4.53	4.72	4.87	5.0

1.6. At a bridge crossing, the mean flow velocities (in m/s) were measured at the midpoints of various subareas, as shown in Fig. 1-18. Compute the values of α and β for the cross section.

1.7. Write a computer program to compute α and β for the flow in a channel having a general cross section. By using this program, compute α and β for the velocity distributionshown in Fig. 1-19.

1.8. Fig. 1-20 shows the velocity distribution measured on the scale model of a canal. By using the computer program of Problem 1-7, compute the energy and momentum coefficients.

1.9. While computing the bending moment and the shear force acting on the side walls of the spillway chute of Fig. 1-21, a structural engineer assumed

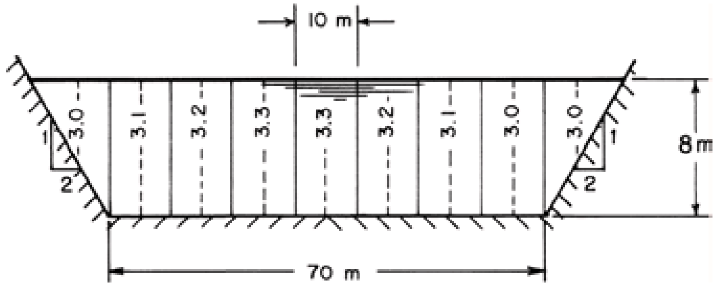


Fig. 1-18. Velocities at bridge crossing

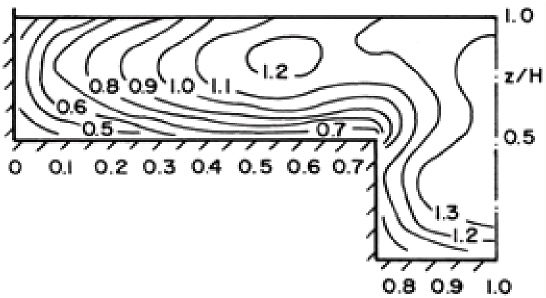


Fig. 1-19. Dimensionless isovels (After Knight and Hamed [1984])

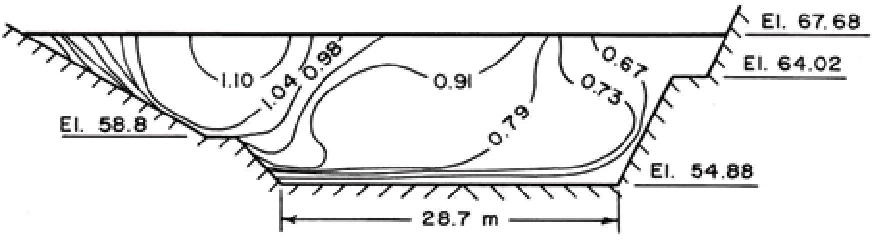


Fig. 1-20. Velocity distribution (After Babb and Amorocho [1965])

that the water pressure varies linearly from zero at the free surface to $\rho g y$ at the invert of the chute, in which y= flow depth measured vertically. What are the computed values for the bending moment and the shear force at the invert level? Are the computed results correct? If not, compute the percentage error.

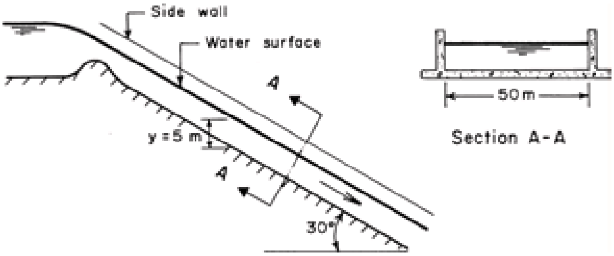


Fig. 1-21. Spillway chute

1.10. A spillway flip bucket has a radius of 20 m (Fig. 1-22). If the flow velocity at section BB is 20 m/s and the flow depth is 5 m, compute the pressure intensity at point C.

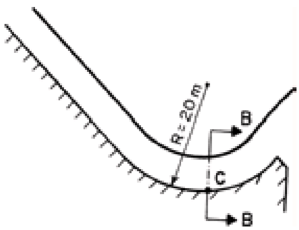


Fig. 1-22. Flip bucket

1.11. In a partially full channel having a triangular cross section (Fig. 1-23), the rate of discharge $Q = kAR^{2/3}$, in which k = a constant; A = flow area, and R = hydraulic radius. Determine the depth at which the discharge is maximum. For the triangular channel section shown, $A = [B - (h/\sqrt{3})]h$, and $P = B + (4h/\sqrt{3})$.

1.12. In the following situations, is the flow uniform or nonuniform?

i. Flow in a channel contraction or expansion
ii. Flow at a channel entrance
iii. Flow in the vicinity of a bridge pier
iv. Flow at the end of a long prismatic channel.

1.13. In the following situations, is the flow steady or unsteady?

i. Flow in a storm sewer during a large storm
ii. Flow in a power canal following shutting down of turbines

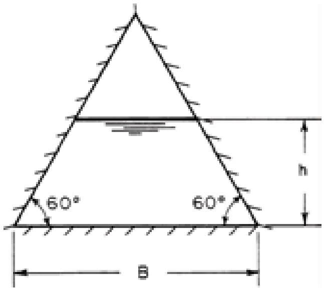


Fig. 1-23. Triangular channel cross section

iii. Flow in a power canal when the turbines have been producing constant power
iv. Flow in an estuary during a tide

1.14. In the following cases, is the flow laminar or turbulent?

i. Flow in a wide rectangular channelat a flow velocity of 1 m/s at 1 m flow depth
ii. Flow in a wide rectangular channelat a flow velocity of 0.1 m/s at 2 mm flow depth

1.15. Is it possible to have uniform flow in a frictionless sloping channel? Give reasons for your answer.

1.16. Is it possible to have uniform flow in a horizontal channel? Justify your answer.

1.17. If the angle between the flow surface and horizontal axis is ϕ and the angle between the channel bottom and horizontal is ϕ, prove that the pressure intensity at the channel bottom is

$$p = \frac{1}{1 + \tan\theta \tan\phi}\rho g y$$

in which y = flow depth measured vertically.

1.18. For an assumed velocity distribution, $V = 5.75\log(30y/k)$, prove that $\alpha = 1 + 3r^2 - 2r^3$ and $\beta = 1 + r^2$. In the above expression, $r = V_{max}/\bar{V} - 1$; $\bar{V}$ = mean velocity; and V_{max} = maximum velocity.

1.19. Show that the bending moment on the side walls of a steep channel with a bottom slope θ for a flow depth of y is $\frac{1}{6}\gamma y^3 \cos^4\theta$. Derive an expression for the shear force.

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