Solutions Manual

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Open–Channel Flow

Second edition

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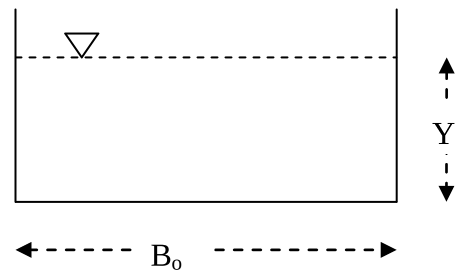
CONTENTS

Chapter 1
BASIC CONCEPTS

1.1

(i) Rectangular section



$A = B_0Y$

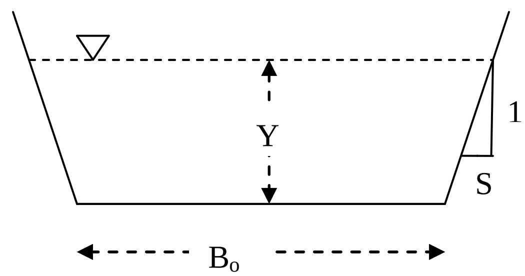
$P = 2Y+B_0$

$B = B_0$

$R = A/P = B_0Y/(2Y+B_0)$

$D = A/B = B_0Y/ B_0 = Y$

(ii) Trapezoidal section



$A = B_0Y+ 2Y(SY/2)$

$= Y(B_0 + SY)$

$P = B0 +2Y\, S(S^2+1)$

$R = A/P = Y(B_0 + SY)/ [B0 +2Y\, S(S^2+1)]$

$B = B_0 +2SY$

$D = A/B = Y(B_0 + SY)/(B_0 + 2SY)$

(iii) Triangular section

We may use the same equation as that in the case of trapezoidal section with $B_0 = 0$.

Thus, $D = Y/2$

(iv) Partially full circular section

$A = r^2\theta/2 +2\ (r\cos\alpha)/2\ (Y-D_0/2)$

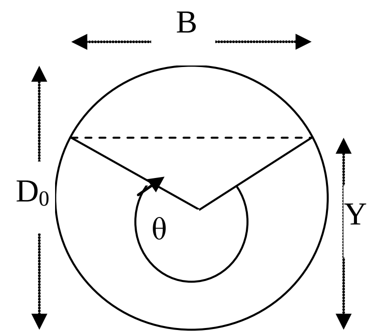
$= D_0^2\theta/8 + (Y - D_0/2)(r\cos\alpha)$

$Y = D_0/2 + (D_0/2) \sin\alpha$

$A = D_o^2\ \theta/8 + D_o^2 / 4 \sin\alpha \cos\alpha$

$$A = \frac{D_o^2\theta}{8} + \frac{D_o^2}{4}\sin\alpha\cos\alpha$$

$$= \frac{D_o^2}{8}\left(\theta + \sin 2\alpha\right)$$



But $\theta = 2\alpha + \pi$

and $\sin\theta = \sin(2\alpha + \pi) = -\sin 2\alpha$

$$A = \frac{D_o^2}{8}\left(\theta - \sin\theta\right) \qquad 0 \le \theta \le 2\pi$$

$$P = r\theta = \frac{D_o\theta}{4}\left(1 - \frac{\sin\theta}{\theta}\right)$$

$$B = 2r\cos\alpha = 2r\cos\left(\frac{\theta}{2} - \frac{\pi}{2}\right) = D_o\sin\frac{\theta}{2}$$

$$D = \frac{A}{B} = \frac{D_o}{8}\left(1 - \frac{\theta - \sin\theta}{\operatorname{Sin}\theta/2}\right)$$

(v) Standard horseshoe section:

Length KB

$$\overline{OM} = \overline{MC} - \overline{OM}$$

$$\overline{OM} = d_0/2\sqrt{2}$$

$$\overline{MC} = \sqrt{\left(\overline{CG}\right)^2 - \left(\overline{GM}\right)^2} = \sqrt{d_0^{\,2} - \left(\frac{d_0}{2\sqrt{2}}\right)^2} = d_0\left(\sqrt{1 - \frac{1}{8}}\right)$$

$$\overline{MC} = \sqrt{\frac{7}{8}}\; d_0$$

$$\overline{OC} = d_0\left(\sqrt{7/8} - \frac{1}{2\sqrt{2}}\right) = 0.58186\,d_0 \text{ --------- } (1)$$

$$\overline{KC} = \overline{FC}^2 - \overline{KF}^2 = d_0^{\,2} - \left(d_0 - \overline{KB}\right)^2 \text{ ------- } (2)$$

$$\overline{KC}^2 = \overline{FC}^2 - \overline{OK}^2 = \overline{OC}^2 - \left(\overline{OB} - \overline{KB}\right)^2$$

$$\overline{KC}^2 = \left(0.58186\,d_0\right)^2 - \left(\frac{d_0}{2} - KB\right)^2 \text{ ---------- } (3)$$

$$(2)\ \ and\ \ (3) \qquad d_0^{\ 2} - \left(d_0 - \overline{KB}\right)^2 = 0.33856\ d_0^{\ 2} - \left(\frac{d_0}{2} - \overline{KB}\right)^2$$

$$d_0^{\ 2} - d_0^{\ 2} + 2d_0\,\overline{KB} - \overline{KB}^2 = 0.38856\ d_0^{\ 2} - \frac{d_0^{\ 2}}{4} + d_0\,\overline{KB} - \left(\overline{KB}\right)^2$$

$$\overline{KB} = 0.088562\ d_0 \qquad \overline{OK} = \frac{d_0}{2} - 0.088562\ d_0 = 0.411438\ d_0$$

$$\overline{KC}^2 = d_0^{\ 2} - \left(d_0 - 0.088562 d_0\right)^2 = 0.169281\ d_0^{\ 2}$$

$$KC = 04114377\ d_0 \qquad CC = 0.822875\, d_0$$

$$Sin\frac{\theta_L}{2} = 0.4114377 \qquad \frac{\theta_L}{2} = 24.295^0 \qquad \theta_L = 48.59^0$$

The standard horse shoe section is divided into three sections, i.e., upper section, middle section and lower section.

(a) Upper section

$\pi \leq \theta_u \leq 2\pi$

Flow area, $A = \dfrac{D_0^2}{8}\left(\theta_u - Sin\theta_u\right) - \dfrac{\pi D_0^2}{8}$

Wetted perimeter, $P = \dfrac{D_0\theta}{2}$

Hydraulic radius, $R = A/P = \dfrac{D_0}{4}\left(1 - \dfrac{Sin\theta}{\theta}\right)$

Top water surface, $B = D_0\ Sin\ (\theta/2)$

Hydraulic depth, $D = A/B = \dfrac{D_0}{8}\left(\dfrac{\theta - Sin\theta}{Sin(\theta/2)}\right)$

(b) Lower section

$0 \leq \theta_L \leq 48.59^0$

Flow area, $A = \dfrac{D_0^2}{8}\left(\theta_L - Sin\theta_L\right) = \dfrac{d_0^2}{2}\left(\theta - Sin\theta\right)$

Wetted perimeter, $P = \dfrac{D_0\theta}{2} = \theta d_0$

Hydraulic radius, $R = A/P = \dfrac{d_0}{2}\left(1-\dfrac{Sin\theta}{\theta}\right)$

Top water surface, $B = 2d_0\ Sin\ (\theta/2)$

Hydraulic depth, $D = A/B = \dfrac{d_0}{4}\left(\dfrac{\theta - Sin\theta}{Sin(\theta/2)}\right)$

(c) Middle section

Assume trapezoidal section, S = 0.215

Area, $A = Y(B_0 + SY)$

$A = Y(0.8229\ d_0 + 0.215Y)$

$P = B_0 + 2Y\ \sqrt{S^2 - 1}$

$$P = 0.8229\,d_0 + 2Y\sqrt{0.215^2 + 1} = 0.8229\,d_0 + 2.05Y$$

$$R = A/P = \frac{Y(0.8229d_0 + 0.2154)}{0.8229\,d_0 + 2.05Y}$$

$$B = B_0 - 254 = 0.8229d_0 + 2\times 0.215Y = 0.8229d_0 + 0.43Y$$

$$D = A/B = \frac{Y(0.8229d_0 + 0.215Y)}{0.8229d_0 + 0.43Y}$$

1.2

$Q = K\ A\ R^{2/3}$

$A = (\theta - Sin\theta)\ D^2/8$

$R = A/P$

$P = D\theta/2$

$$Q = \frac{KD^{\frac{10}{3}}}{8^{\frac{5}{3}}}\ \frac{(\theta - Sin\theta)^{\frac{5}{3}}}{(D/2)^{\frac{2}{3}}\theta^{\frac{2}{3}}} = C\,\frac{(\theta - Sin\theta)^{\frac{5}{3}}}{\theta^{2/3}}$$

Where $C = \dfrac{KD^{10/3}}{8^{5/3}(D/2)^{2/3}}$

$$\frac{dQ}{d\theta} = 0\ \ will\ \ give\ the\ angle\ \theta\ corresponding\ to\ Q_{\max}.$$

$$\frac{dQ}{d\theta} = C\left[-\frac{2}{3}\theta^{-5/3}\ (\theta - Sin\theta)^{5/3} + \frac{5}{3}\theta^{-2/3}(\theta - Sin\theta)^{2/3}(1 - Cos\theta)\right] = 0$$

$$\frac{dQ}{d\theta} = C\frac{\theta^{-2/3}}{3}(\theta - Sin\theta)^{2/3}\left[-2\theta^{-1}(\theta - Sin\theta) + 5(1 - Cos\theta)\right] = 0$$

$\therefore\ 2(\theta - Sin\theta) = 5\theta(1 - Cos\theta)$

θ – Sinθ = 5/2θ – 5/2θ Cosθ

5/2 Cos θ – Sin θ / θ – 1.5 = 0

Solving by trial and error or numerically θ = 302.41

From the figure $Y = \frac{D}{2} + \frac{D}{2} Sin\alpha$ and $\alpha = \frac{\theta}{2} - \pi/2$

$$Y = \frac{D}{2} + \frac{D}{2} Sin\left(\frac{\theta}{2} - \pi/2\right)$$

$$Y = \frac{D}{2}\left[1 - Cos\frac{\theta}{2}\right]$$

Substituting *θ* value for the Q_{max}

$$Y = \frac{D}{2}\left[1 - Cos\frac{302.41}{2}\right]$$

Y= 0.938 D

1.3

$$(AR^{2/3})_F = \frac{\pi D^2}{4}\left(\frac{D}{4}\right)^{2/3} = \frac{\pi}{4^{5/3}} D^{8/3}$$

$$AR^{2/3} = \frac{D^2}{8}(\theta - Sin\theta)\left(\frac{D}{4}\right)^{2/3}\left(\frac{\theta - Sin\theta}{\theta^{2/3}}\right)^{2/3}$$

$$= \frac{2^{2/3}}{8^{5/3}}\frac{D^{8/3}}{\theta^{2/3}}(\theta - Sin\theta)^{5/3}$$

$$\frac{AR^{2/3}}{\left(AR^{2/3}\right)_F} = \frac{1}{2\pi}\left(\frac{\theta - Sin\theta}{\theta^{2/3}}\right)^{5/3}$$

$$\frac{R}{R_F} = \frac{\theta - Sin\theta}{\theta}$$

$$Y = \frac{D}{2} + \frac{D}{2} Sin\alpha \qquad \alpha = \frac{\theta}{2} - \pi/2$$

$$Y = \frac{D}{2} + \frac{D}{2} Sin\left(\frac{\theta}{2} - \frac{\pi}{2}\right)$$

$$Y = \frac{D}{2}\left[1 - Cos\frac{\theta}{2}\right]$$

$$\frac{Y}{D} = \frac{1}{2}\left[1 - Cos\frac{\theta}{2}\right]$$

Using trial and error

$\frac{Y}{D} = 0.938$ gives maximum value for $\frac{AR^{2/3}}{\left(AR^{2/3}\right)_F}$

$\frac{Y}{D} = 0.81$ gives maximum value for $\frac{R^{2/3}}{\left(R^{2/3}\right)_F}$

1.4

$$V = 5.75\, V_0 \log\frac{30y}{K}$$

$$V_m = \frac{\int VdA}{dA} = \frac{\int Vdy}{dy}$$

$$V_m = \frac{5.75\int_0^{y_0} V_0 \log 30y/K\, dy}{\int_0^{y_0} dy} = \frac{5.75V_0}{y_0}\int_0^{y_0}\log\frac{30y}{k}dy$$

Let x = 30y/K dx = $\frac{30}{K}dy$ dy = $\frac{K}{30}dx$

$V_m = 5.75\ V_0/Y_0\ \frac{K}{30}\int \log\times dx$

$= 5.75\ V_0/Y_0\ \frac{K}{30}\left[x\log x - x\right]_0^{y_0}$

$= 5.75\ V_0/Y_0\ \frac{K}{30}\left[\frac{30y}{K}\log\frac{30y}{K} - \frac{30y}{K}\right]_0^{y_0}$

$= 5.75V_0/Y_0\ \left[y_0\log\frac{30y_0}{K} - y_0\right]$

$V_m = 5.75\ V_0\ \left[\log\frac{30y_0}{K} - 1\right]$

Energy coefficient, α :

$$\alpha = \frac{\int V^3 dA}{V_m^3 A} = \frac{\int V^3 B dy}{V_m^3 B y_0} = \frac{\int \left[5.75 V_0 \log \frac{30y}{K}\right]^3 dy}{V_m^3 y_0}$$

$$\alpha = \frac{(5.75V_0)^3 \int \left(\log \frac{30y}{K}\right)^3 dy}{V_m^3 y_0}$$

$$V_m^3 \alpha = \frac{b}{y_0} \int (\log x)^3 dx \qquad \text{where, b} = (5.75\ \text{V}_0)^3\text{K}/30$$

But, $\int (\log x)^n dx = x(\log x)^n - n\int (\log x)^{n-1} dx$

$$V_m^3 \alpha = \frac{b}{y_0}\left[\frac{30y}{K}\left(\log\frac{30y}{K}\right)^3 - 3\left(\frac{30y}{K}\left(\log\frac{30y}{K}\right)^2 - 2\frac{30y}{K}\log\frac{30y}{K} + 2\frac{30y}{K}\right)\right]_0^{y_0}$$

$$= \frac{(5.75V_0)^3}{y_0}\frac{K}{30}\left[\frac{30y_0}{K}\left(\log\frac{30y_0}{K} - 3\left\{\frac{30y_0}{K}\left(\log\frac{30y_0}{K} - 2\frac{30y_0}{K}\log\frac{30y_0}{K} + 2\frac{30y_0}{K}\right)\right\}\right)\right]$$

$$\alpha = \frac{(5.75V_0)^3\left[\left(\log\frac{30y_0}{K}\right)^3 - 3\left(\log\frac{30y_0}{K}\right)^2 + 6\log\left(\frac{30y_0}{K}\right) - 6\right]}{(5.75V_0)^3\left[\log\frac{30y_0}{K} - 1\right]^3}$$

Let, $(V_m)^3 = \left[\log\frac{30y_0}{K} - 1\right]^3 = \left(\log\frac{30y_0}{K}\right)^3 - 3\left(\log\frac{30y_0}{K}\right)^2 + \log(30y_0) - 1$

$$\alpha = \frac{V_m^{*3} + 3\left(\log\frac{30y_0}{K} - 1\right) - 2}{V_m^{*3}} = 1 + \frac{3}{V_m^{*2}} - \frac{2}{V_m^{*3}}$$

$$\alpha = 1 + 3\in^2 - 2\in^3 \qquad \text{where } \in = \frac{1}{V_m^*}$$

Momentum coefficient, β:

$$\beta = \frac{\int V^2 dA}{V_m^2 A} = \frac{\int V^2 B dy}{V_m^2 B y_0} = \frac{\int V^2 dy}{V_m^2 y_0}$$

$$= \frac{\int \left[5.75V_0 \log\frac{30y}{K}\right]^2 dy}{V_m^2 y_0}$$

$$V_m^2 \beta = \frac{b}{y_0}\int (\log x)^2 dx \qquad \text{where b} = (5.75\text{V}_0)^2\text{K}/30$$

$$\int (\log \pi)^n dx = x(\log x)^n - n\int (\log x)^{n-1} dx$$

$$V_m^2\beta = \frac{b}{y_0}\left[\frac{30y}{K}\left(\log\frac{30y}{K}\right)^2 - 2\frac{30y}{k}\log\frac{30y}{k} + 2\frac{30y}{K}\right]_0^{y_0}$$

$$\beta = \frac{\frac{(5.75V_0)^2}{y_0}\frac{K}{30}\left[\frac{30y_0}{K}\left(\log 30\frac{y_0}{K}\right)^2 - 2\frac{30y_0}{K}\log\frac{30y_0}{K} + 2\frac{30y_0}{K}\right]}{(5.75V_0)^2\left[\log\frac{30y_0}{K} - 1\right]^2}$$

Let $V_m^{*2} = \left(\log\frac{30y_o}{K} - 1\right)^2 = \left(\log\frac{30y_0}{K}\right)^2 - 2\log\frac{30y_0}{K} + 1$

$$\beta = \frac{\left(\log\frac{30y_0}{K}\right)^2 - 2\log\frac{30y_0}{K} + 1 + 1}{\left(\log\frac{30y_0}{K}\right)^2 - 2\log\frac{30y_0}{K} + 1}$$

$$\beta = \frac{V_m^{*2} + 1}{V_m^{*2}} = 1 + \frac{1}{V_m^{*2}}$$

$$\beta = 1 + \in^2 \qquad where \in = \frac{1}{V_m^*}$$

1.6

$$\alpha = \frac{\sum V_i^3 A_i \quad \sum A_i^2}{\left(\sum V_i A_i\right)^3}$$

For i = 3

$$\alpha = \frac{\left(V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3\right)\left(A_1 + A_2 + A_3\right)^2}{\left(V_1 A_1 + V_2 A_2 + V_3 A_3\right)^3}$$

$$\beta = \frac{\left(V_1^2 A_1 + V_2^2 A_2 + V_3^2 A_3\right)\left(A_1 + A_2 + A_3\right)}{\left(V_1 A_1 + V_2 A_2 + V_3 A_3\right)^2}$$

Table 1

	A	V	VA	V^2A	V^3A
1	40	3	120	360	1080
2	80	3	240	720	2160
3	80	3.1	248	768.8	2383.28
4	80	3.2	256	819.2	2621.44
5	80	3.3	264	871.2	2874.96
6	80	3.3	264	871.2	2874.96
7	80	3.2	256	819.2	2621.44
8	80	3.1	248	768.8	2383.28
9	40	3	120	360	1080
$\sum$	640		2016	6358.4	20079.36

The calculation is shown in Table -1.

$\alpha = (20079.36)(640)^2/2016^3 = 1.0038$

$\beta = (6358.4)(640)/2016^2 = 1.00126$

1.9

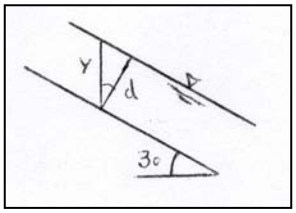
Using hydrostatic pressure distribution and depth = $y = 5$m

$T = \gamma y^2/2 = 9810\ (5^2/1000)2 = 122.6$ KN

$M = TY/3 = 122.6x5/3 = 204.3$ KN.m

But, $d = yCos\theta = 5Cos30 = 4.33$ m

In this case the pressure distribution is not hydrostatic



Correction:

$T_t = Pd/2 = \gamma d^2 Cos\theta/2 = 79.65$KN

$M_t = T_t\ d/3 = 79.65\ (4.33)/3 = 114.96$ KN m

% error in the shearing force:

$100(T_t - T)/T_t = (79.65 - 122.6)/79.65 = 53.9\ \%$

% error in the moment:

$100(M_t - M)/M_t = (114.96 - 204.38) / 114.96 = 77\ \%$

1.10

Centrifugal acceleration = V^2/R

Centrifugal force = $\rho y_s \Delta A\ V^2/R$

Pressure head due to centrifugal acceleration = $(1/g) y_s\ V^2/R$

Total pressure head, $y_t = y_s + (1/g) y_s\ V^2/R = y_s\ (1 + V^2/gR)$

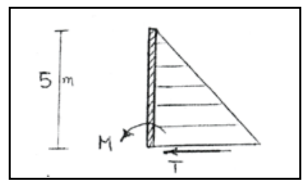
$y_t = 5\ (1 + 20^2/9.81/20) = 15.19$ m

Pressure intensity at point $C = \gamma\ y_t = (9.81)\ (1000)(15.19/1000) = 149$ kPa

1.11

$Q = K\, A\, R^{2/3}$
$A = [B - (h/\sqrt{3})]h$
$P = B + (4h/\sqrt{3})$
$Q = K\ [B - h/\sqrt{3}]^{5/3} h^{5/3} [B + 4\ h/\sqrt{3}]^{-2/3}$
$\quad = K[hB - h^2/\sqrt{3}]^{5/3} [B + 4\ h/\sqrt{3}]^{-2/3}$
Q is maximum or minimum if $dQ/dh = 0$

$$dQ/dh = \frac{5}{3} K \left[hB - h^2/\sqrt{3}\right]^{2/3} \left[B - 2h/\sqrt{3}\right] \left[B + 4h/\sqrt{3}\right]^{-2/3}$$



1.12

(i) nonuniform

(ii) nonuniform

(iii) nonuniform

(iv) uniform

1.13

(i) unsteady

(ii) unsteady

(iii) steady

(iv) unsteady

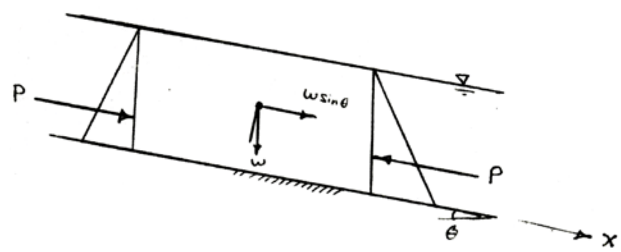
1.14

(i) $R_e = Vy/\nu = (1)(1)/(0.11 * 10^{-5}) = 9.0 * 10^5$ Turbulent

(ii) $R_e = Vy/\nu = \dfrac{0.1\times 2\times 10^{-3}}{0.11\times 10^{-5}}$ = 181.8 Laminar

1.15

It is not possible to have uniform flow in a frictionless sloping channel. The forces in the x direction will not be balanced.



1.16

It is not possible to have uniform flow in a horizontal channel.

There is no acceleration force.

1.17

y = flow depth measured vertically; θ = angle between water surface and horizontal; φ = angle between channel bed and horizontal. Let d = perpendicular on channel bed.

Thus, pressure at channel bed is

$p = \rho g d Cos\phi$

From the geometry, $y = dCos\phi + dSin\phi \tan\theta$

Or, $dCos\phi = \dfrac{y}{1+Tan\phi Tan\theta}$

Or, $p = \rho g \dfrac{y}{1+Tan\phi Tan\theta}$

1.18

$V = 5.75V_f \log (30y/K)$ (1)

$$V_m = \frac{\int V dA}{\int dA} = \frac{\int V dy}{\int dy} = \frac{5.75\int_0^{y_0} V_f \log(30y/K)\,dy}{\int_0^{y_0} dy} = \frac{5.75V_f}{y_0}\int_0^{y_0}\log(30y/K)\,dy$$

Let $x = 30y/K$, $dx = (30/K)dy$ or, $dy = (K/30)dx$

$$V_m = \frac{\int V dA}{\int dA} = \frac{5.75V_f}{y_0}\frac{K}{30}\int_0^{y_0}\log x\,dx = \frac{5.75V_f}{y_0}\frac{K}{30}\left[x\log x - x\right]_0^{y_0}$$

$$V_m = \frac{5.75V_f}{y_0}\frac{K}{30}\left[\frac{30y}{K}\log\frac{30y}{K} - \frac{30y}{K}\right]_0^{y_0}$$

$$V_m = 5.75V_f\left[\log(30y_0/K) - 1\right]$$

In Eq 1, at $y = y_0$, $V = V_{max}$

$V_{max} = 5.75V_f\ \log(30y_0/K)$

Let $\gamma = (V_{max}/V_m)\text{-}1 = 5.75V_f\ \log(30y_0/K)/\ 5.75V_f\log(30y_0/K\text{-}1)\text{-}1$

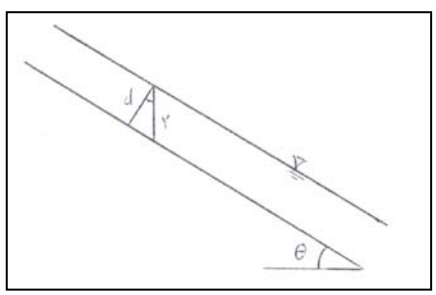
$\gamma = 1/[\log(30y_0/K)\text{-}1]$

Similar to the solution of problem 1.4 this will lead to

$\alpha = 1 + 3\gamma^2 - 2\gamma^3$

$\beta = 1 + \gamma^2$

1.19



$d = y\,Cos\theta$

$p = (\gamma d^2/2)(Cos\theta)$

$p = (\gamma y^2/2)\ (Cos^2\theta\ Cos\theta)$

$p = (\gamma y^2/2)\ (Cos^3\theta\ )$

$M = pd^3/3 = (py\ Cos\theta)/3$

$\quad = (\gamma/2)(y^2Cos^3\theta)(y\ Cos\theta/3)$

$M = (\gamma y^3/6)(Cos^4\theta\ )$

Shearing force = $p = (\gamma y^2/2)(Cos^3\theta\ )$

Chapter 2
CONSERVATION LAWS

2.1

$$d = y\cos\theta$$

$$H = Z + \frac{p}{\gamma} + \frac{\alpha Q^2}{2gA^2}$$

$$p = \gamma d\cos\theta$$

$$p = \gamma y\cos^2\theta$$

$$E = y\cos^2\theta + \frac{\alpha Q^2}{2gA^2}$$

Assume α=1 and rectangular channel and width = B. q=Q/B

$$E = y\cos^2\theta + \frac{q^2}{2gA^2}$$

$$\left[E - y\cos^2\theta\right]y^2 = \frac{q^2}{2g} = \text{ const.}$$

The E-y curve has two asymptotes

$$E - y\cos^2\theta = 0 \;\&\; y = 0$$

$$\frac{E}{y} = \frac{1}{\cos^2\theta}$$

$$\tan^{-1}\phi = \frac{1}{\cos^2\theta}$$

2.2

$$E = y + \frac{Q^2}{2gA^2}$$

$$A = (B + 2y)y = (20 + 2y)y$$

$$E = y + \frac{Q^2}{2g\left[(20 + 2y)y\right]^2}$$

2.3

Applying Newton second law

$$\frac{\gamma Q}{g}(V_2 - V_1) = P_1 - P_2 + W\sin\theta - F_R$$

Assume θ=0 and no friction F_R =0

$$P_1 = \gamma \overline{z_1} A_1$$
$$P_2 = \gamma \overline{z_2} A_2$$
$$\frac{Q^2}{gA_2} - \frac{Q^2}{gA_1} = \overline{z_1}A_1 - \overline{z_2}A_2$$
$$\frac{Q^2}{gA_1} + \overline{z_1}A_1 = \frac{Q^2}{gA_2} + \overline{z_2}A_2$$
$$F_s = \frac{Q^2}{gA} + \overline{z}A$$

Force / Unit weight

2.4

Q=4 x 5 x 5 = 100 m^3/s

$$F = \frac{V}{\sqrt{gy}} = \frac{4}{\sqrt{9.81\times 5}} = 0.57 \quad \text{Sub critical}$$

Energy equation neglecting losses

$$H_1 = E_1 = E_2 - \Delta z$$
$$y_1 + \frac{V_1^2}{2g} = E_2 + 0.2$$
$$5 + \frac{4^2}{2\times 9.81} = y_2 + \frac{Q^2}{2gA^2} + 0.2$$
$$5.61 = y_2 + \frac{100^2}{2\times 9.81\times (5\times y_2)^2}$$
$${y_2}^3 - 5.61\times {y_2}^2 + 20.387 = 0$$

Trial and error will give y_2 = 2.6 m.

$$H = y_1 + \frac{V_1^2}{2g} + 0.2 = y_2 + \frac{V_2^{\ 2}}{2g}$$

$$5 + \frac{4^2}{2\times 9.81} + 0.2 = y_2 + \frac{100^2}{2\times 9.81\times (5\times y_2)^2}$$

$$6.015 = y_2 + \frac{20.387}{y_2^{\ 2}}$$

$$y_2^{\ 3} - 6.015\times y_2^{\ 2} + 20.387 = 0$$

y_2 = 5.285 m rise with water level.

2.5

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{Q/A_1}{\sqrt{gy_1}} = \frac{4}{\sqrt{9.81\times 5}} = 0.571$$

Energy equation neglecting losses

$$H_1 = E_1 = E_2 + \Delta z$$

q = Q/B = 20 m^3/m

$$y_1 + \frac{V_1^2}{2g} = 5 + \frac{4^2}{2\times 9.81} = 5.8155 = E_2 + 0.3$$

$$5.8155 - 0.3 = y_2 + \frac{q^2}{2gy_2^{\ 2}}$$

$$y_2^{\ 3} - 5.5155 y_2^{\ 2} + 20.387 = 0$$

Using trial and error y_2 = 4.52 m

(ii)

$$F_1 = 0.571$$

$$H_1 = y_1 + \frac{V_1^{\ 2}}{2g} + 0.2 = y_2 + \frac{V_2^{\ 2}}{2g} = E_2$$

$$5 + \frac{4^2}{2\times 9.81} + 0.2 = y_2 + \frac{q^2}{2gy_2^{\ 2}}$$

$$y_2{}^3 - 6.015 y_2{}^2 + 20.387 = 0$$

Using trial and error, $y_2 = 5.285$ m

2.6
Q = 400 m^3 / s
V_1 = 400 / 5 x 10 = 8 m / sec

$$F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{8}{\sqrt{9.81 \times 5}} = 1.14 > 1 \quad \text{Super critical}$$

If the step is a rise water surface rises

If the step is a drop water surface drops

2.7
Q = 96 m^3 / s
Section1
B_1 = 8 m Y_1 = 4 m V_1 = 3 m / sec

$H_1 = H_2 = H_3 = H_4 = H_5$
$H_i = E_i$

$$H_1 = y_1 + \frac{V_1^2}{2g} = 4.459 m$$

$$F_1 = \frac{3}{\sqrt{9.81 \times 4}} = 0.479 < 1 \quad \text{Sub critical}$$

$q_1 = Q/B_1 = 96/8 = 12$ m^3/s / m

Section 2
B_2 = 7.5 m
$q_1 = Q/B_1 = 96/7.5 = 12.8$ m^3/s / m

$$E_2 = E_1 = y_2 + \frac{Q^2}{(B_2 y_2)^2 \times 2 \times 9.81} = 4.459$$

$$y_2{}^3 - 4.459 y_2{}^2 + 8.35 = 0$$

y_2 = 3.91 m

Section 3

B_3 = 7 m
$q_1 = Q/B_1$= 96/7 = 13.71 m^3/s / m

$$E_1 = E_3 = y_3 + \frac{q_3^2}{y_3^2 \times 2 \times 9.81} = 4.459$$

$$y_2{}^3 - 4.459 y_2{}^2 + 9.58 = 0$$

y_2 = 3.79 m

Section 4

B_4 = 6.5 m
$q_1 = Q/B_1$= 96/ 6.5= 14.77 m^3/s / m

$$E_1 = E_4 = y_4 + \frac{q_4^2}{y_4^2 \times 2 \times 9.81} = 4.459$$

$$y_2{}^3 - 4.459 y_2{}^2 + 11.12 = 0$$

y_2 = 3.6 m

Section 5

B_4 = 6 m
$q_1 = Q/B_1$= 96/ 6= 16 m^3/s / m

$$E_1 = E_5 = y_5 + \frac{q_5^2}{y_5^2 \times 2 \times 9.81} = 4.459$$

$$y_2{}^3 - 4.459 y_2{}^2 + 13.048 = 0$$

y_2 = 3.11 m

2.8
(i)
h =20 m Q= 40 m^3/s b= 4 m
$E_1 = E_2$, V_1 = 0, $V_2 = Q/By_2 = q/y_2$, q= 10 m^3/s / m

$$h + \frac{V_1}{2 \times g} = y_2 + \frac{q^2}{2 \times g \times y_2^2}$$

$$y_2{}^3 - 20 y_2{}^2 + 5.097 = 0$$

y_2= 0.511 m
V_2=10/0.511=19.57 m/s

$$F_2 = \frac{V}{\sqrt{g \times y}} = 8.47 > 1 \text{ Supercritical}$$

$$y_3 = \frac{y_2}{2}\left[\sqrt{1+8F_2^2}-1\right]$$

$$y_3 = \frac{0.511}{2}\left[\sqrt{1+8\times 8.74^2}-1\right] = 6.066m$$

(ii)

Thrust on the gate

$$F_{th} = F_{s1} - F_{s2}$$

$$F_{s1} = \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{40^2}{9.8\times(20\times 4)} + 10\times(20\times 4) = 802.038m^3$$

$$F_{s2} = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 = \frac{40^2}{9.8\times(0.511\times 4)} + \frac{0.511}{2}\times(0.511\times 4) = 80.31m^3$$

$$F_{th} = 802.038 - 80.31 = 721.72m^3$$

Thrust force = γF_{th}=9.81 x 721.72 = 7080.07 kN

(iii)

Energy losses

$$E_2 = E_3 + H_L$$

$$H_L = E_2 - E_3$$

$$H_L = y_2 + \frac{q^2}{2gy_2^2} - y_3 - \frac{q^2}{2gy_3^2}$$

$$H_L = 0.511 + \frac{10^2}{2\times 9.8\times 0.511^2} - 6.066 - \frac{10^2}{2\times 9.8\times 0.66^2} = 13.85m$$

2.9

$$E = y + \frac{q^2}{2gy^2}$$

$$q^2 = 2gy^2(E-y)$$

$$q = (2gy^2E - 2gy^3)^{1/2}$$

$$\frac{dq}{dy} = \frac{1}{2}(2gy^2E - 2gy^3)^{-1/2}\left[4gyE - 6gy^2\right] = 0$$

Maximum q for critical depth

$$4gyE - 6gy^2 = 0$$

$$y_c = \frac{2}{3}E$$

$$\tan\theta = \frac{y_c}{E} = \frac{2}{3}$$

2.10

(i) Channel width remains constant

Use a step and determine if it is a rise or down

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{4}{\sqrt{9.81}} = 0.639 < 1 \text{ Sub critical flow}$$

$E_1 > E_2$ Δz is positive (step rise)

$$E_1 = E_2 + \Delta z$$

$$q = \frac{Q}{b} = V_y = 16m^3/\text{sec}/m$$

and

$$y_1 = y_2 + \Delta z + (704 - 703.54)$$

$$\Delta z = 3.54 - y_2$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z$$

$$4 + \frac{16}{2\times 9.81} = y_2 + \frac{16^2}{2\times 9.81\times y_2^2} + (3.54 - y_2)$$

$$y_2 = 3.198m$$

$$\Delta z = 3.54 - 3.198 = 0.34m$$

Bottom elevation = 700+0.34=700.34m

(ii)

Channel bottom level 0.5 of transition at elevation 700.2 m

b_1= 8m

y_1 = 4 m; y_2 = 3.34 m

Q= 32 x 4 =128 m^3/s

$$E_1 = E_2 + \Delta z$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z$$

$$4+\frac{16}{2\times 9.81}=3.34+\frac{128^2}{2\times 9.81\times(b_2\times 3.34)^2}+0.2$$

$$4.815=3.54+\frac{74.856}{b_2^2}$$

$$b_2=7.66m$$

2.11

(i)

$$V_1A_1=V_2A_2$$

$$V_2=\frac{0.8\times 2\times b}{y_2\times b}$$

$$V_2=\frac{1.6}{y_2}$$

$$F_1=\frac{0.8}{\sqrt{2\times 9.81}}=0.18<1 \qquad \text{Sub critical}$$

$$E_1=E_2+\Delta z$$

$$y_1+\frac{V_1^{\ 2}}{2g}=y_2+\frac{V_2^{\ 2}}{2g}+\Delta z$$

$$2+\frac{0.8^2}{2\times 9.81}=y_2+\frac{(1.6/y_2)^2}{2\times 9.81}+0.15$$

$$y_2^{\ 3}-1.882y_2^{\ 2}+0.1305=0$$

$$y_2=1.84m$$

(ii)

$y_2 = 2$ m; $V_1 = 0.8$ m; q = 1.6 m^3/sec/m

$$E_1=y_1+\frac{V_1^{\ 2}}{2g}=2.033$$

Maximum Δz occurs at minimum E and F_r=1

$$y_2=\sqrt[3]{\frac{q^2}{g}}=\sqrt[3]{\frac{1.6^2}{9.81}}=0.64$$

$$E_2=y_2+\frac{V_2^{\ 2}}{2g}=0.64+\frac{1.6^2}{2\times 9.81\times 0.64^2}=0.96$$

$$\Delta z_{\max}=E_1-E_2=2.033-0.96=1.073\,\text{m}$$

2.12
Assume b_1=10 m
Q_1= Q_2= 10 x 2 x 0.8 = 16 m^3/s

$$E_1 = y_2 + \frac{q_2^2}{2gy_2^2}$$

$$E_1 y_2^2 = y_2^3 + \frac{q_2^2}{2g}$$

$$q_2 = (2gE_1 y_2^2 - 2gy_2^3)^{1/2}$$

$$q_2 = \frac{Q}{b_2}$$

Q is constant. Therefore, maximum q_2 corresponds to minimum b_2.

$$\frac{dq_2}{dy_2} = \frac{1}{2}(2gy_2^2 E - 2gy_2^3)^{-1/2}\left[4gyE - 6gy^2\right] = 0$$

$$y_2 = \frac{2}{3}E_1$$

From problem 2.11, E_1 = 2.033 m

$$y_2 = \frac{2}{3}\times 2.033 = 1.355\,\text{m}$$

$$q_2 = (2\times 9.81\times 2.033\times 1.355^2 - 2\times 9.81\times 1.355^3)^{1/2}$$

$$= 4.941\,\text{m}^3/\text{sec/m}$$

$$b_2 = 16/4.941 = 3.238\,\text{m}$$

$$\frac{b_2}{b_1} = \frac{3.238}{10} = 0.32$$

2.13
(i)
Q = 80 ft^3 / sec

$$E_1 = E_2$$

$$y_1 + \frac{V_1^{\,2}}{2g} = y_2 + \frac{V_2^{\,2}}{2g}$$

$$y_1 + \frac{Q^2}{2g(b_1 y_1)^2} = y_2 + \frac{Q^2}{2g(b_2 y_2)^2}$$

$$8+\frac{80^2}{2\times 32.2\times (5\times 8)^2}=y_2+\frac{80^2}{2\times 32.2\times (4\times y_2)^2}$$

$$8+0.0621=y_2+\frac{6.211}{{y_2}^2}$$

$${y_2}^3-8.0621{y_2}^2+6.211=0$$

$$y_2=7.964\ \text{ft}$$

(ii)

$$V_1=\frac{80}{5\times 8}=2\ \text{ft/sec}$$

$$F_r=\frac{V}{\sqrt{gy}}=\frac{2}{\sqrt{9.81\times 8}}=0.125<1\,\text{subcritical}$$

$$E_1=E_2+\Delta z$$

$$y_1+\frac{Q^2}{2g(b_1y_1)^2}=y_2+\frac{Q^2}{2g(b_2y_2)^2}+\Delta z$$

$$8+\frac{80^2}{2\times 32.2\times (5\times 8)^2}=7.964+\frac{80^2}{2\times 32.2\times (5\times 7.964)^2}+\Delta z$$

$$8.0621=8.0266++\Delta z$$

$${y_2}^3-8.0621{y_2}^2+6.211=0$$

$$\Delta z=0.035\ \text{ft}$$

2.14

$$V_1=\frac{80}{0.5\times 20}=8\ \text{m/sec}$$

$$F_r=\frac{8}{\sqrt{9.81\times 0.5}}=3.612$$

$$y_2=\frac{y_1}{2}\left[\sqrt{1+8{F_1}^2}-1\right]$$

$$y_2=\frac{0.5}{2}\left[\sqrt{1+8\times (3.162)^2}-1\right]=2.316\ \text{m}$$

Losses $H_L=E_1-E_2$

$$H_L=y_1+\frac{{V_1}^2}{2g}-y_2-\frac{{V_2}^2}{2g}$$

$$H_L = 0.5 + \frac{8^2}{2\times 9.81} - 2.316 - \frac{1}{2\times 9.81}\left(\frac{80}{20\times 23.6}\right)^2$$
$$= 1.294\ \text{m}$$

2.15
Q = 80 m^3 / s

$$V_1 = \frac{54}{8\times 0.6} = 11.25\ \text{m/s}$$

$$F_r = \frac{11.25}{\sqrt{9.81\times 0.6}} = 4.63$$

$$y_2 = \frac{y_1}{2}\left[\sqrt{1+8F_1^{\,2}} - 1\right]$$

$$y_2 = \frac{0.6}{2}\left[\sqrt{1+8\times(4.63)^2} - 1\right] = 3.646\ \text{m}$$

2.16

$$\text{Losses at junction} = \frac{0.2V_2^2}{2g}$$

$$V_1 = \frac{10}{10\times 1.5} = 0.67\ \text{m/s}$$

$$F_r = \frac{0.67}{\sqrt{9.81\times 1.5}} = 0.173 \quad \text{subcritical}$$

$$E_1 = E_2 + \Delta\text{z} + \text{Losses}$$

$$y_1 + \frac{V_1^{\,2}}{2g} = y_2 + \frac{V_2^{\,2}}{2g} + 0.1 + \frac{0.2V_2^2}{2g}$$

$$1.5 + \frac{0.67^2}{2\times 9.81} = y_2 + 1.2\times\frac{10^2}{2\times 9.81\times(8y_2)^2} + 0.1$$

$$y_2^{\,3} - 1.432y_2^{\,2} + 0.0956 = 0$$

$$y_2 = 0.29\ \text{m}$$

2.17

$$B = 8\ \text{m}$$

$$\text{Q} = 8\times 2\times 3 = 48\ \text{m}^3\ /\ \text{s}$$

$$E_1 + \Delta\text{z} = E_2$$

$$2 + \frac{3^2}{2\times 9.81} + 0.15 = y_2 + \frac{48^2}{2\times 9.81\times(8y_2)^2}$$

$$2.609 = y_2 + \frac{1.835}{y_2^2}$$

$$y_2^{\ 3} - 2.609 y_2^{\ 2} + 1.835 = 0$$

$$y_2 = 2.245 \text{ m}$$

2.18

$$F = \frac{V}{\sqrt{g \times y}} = \frac{3}{\sqrt{9.81 \times 3}} = 0.533$$

$$V_1 y_1 = V_2 y_2$$

$$3 \times 3 = V_2 y_2$$

$$V_2 = 9 / y_2$$

$$E_1 = E_2 + \Delta \text{z}$$

$$3 + \frac{3^2}{2 \times 9.81} = y_2 + \frac{1}{2 \times 9.81}\left(\frac{9}{y_2}\right)^2 + 0.3$$

$$y_2^{\ 3} - 3.16 y_2^{\ 2} + 4.128 = 0$$

$$y_2 = 1.66 \text{ m}$$

2.19

$$V_1 = \sqrt{2g(160 - 120)}$$

$$= 28 \text{ m/sec}$$

$$\text{Q} = \text{BV}_1 y_1$$

$$y_1 = \frac{1200}{20 \times 28} = 2.14 \text{ m}$$

$$\text{F}_{\text{r1}}^2 = \frac{V^2}{g y_1} = \frac{28^2}{9.81 \times 2.14} = 37.35$$

$$y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8F_2^{\ 2}} - 1\right]$$

$$y_2 = \frac{2.14}{2}\left[\sqrt{1 + 8 \times 37.35} - 1\right] = 17.46 \text{ m}$$

$$\text{V}_2 = \frac{1200}{20 \times 17.46} = 3.44 \text{ m/s}$$

$$\text{Energy losses} = \text{E}_1 \text{ -E}_2$$

$$= 2.14 + \frac{28^2}{2 \times 9.81} - 17.46 - \frac{3.44^2}{2 \times 9.81} = 24 \text{ m}$$

2.20
Q= 18 m^3/sec; B= 6 m; q= 18/6 = 3 m^3 / s / m

$$V_1 = \frac{18}{6 \times 0.3} = 10 \text{ m/s}$$

$$\mathrm{F}_{r1}^2 = \frac{V^2}{gy_1} = \frac{10^2}{9.81 \times 0.3} = 33.97$$

$$y_2 = \frac{0.3}{2}\left[\sqrt{1+8\times 33.97} - 1\right] = 2.328 \text{ m} \ \text{ sequential depth}$$

$$\mathrm{E} = y + \frac{q^2}{2gy^2}$$

at $\mathrm{y} = 0.3$

$$E = 0.3 + \frac{3^2}{2 \times 9.81 \times 0.3^2} = 5.397$$

$$5.397 = y + \frac{0.4587}{y^2}$$

$$y_2{}^3 - 5.397 y_2{}^2 + 0.4587 = 0$$

$$y_2 = 5.38 \text{ m alternate depth}$$

$$\text{Head loss} = \mathrm{E}_1 \text{-} \mathrm{E}_2$$

$$= y_1 + \frac{q_1^2}{2gy_1^2} - y_2 - \frac{q_2^2}{2gy_2^2}$$

$$= 0.3 + \frac{3^2}{2 \times 9.81 \times 0.3^2} - 2.328 - \frac{3^2}{2 \times 9.81 \times 2.328^2}$$

$$= 5.397 - 2.413 = 2.98 \text{ m}$$

2.21

(i)
q= 15 ft^3/sec/ft
Q= 6 x 5 x 3 = 90 ft^3/sec

$$F_{r1} = \frac{V}{\sqrt{g \times y}} = \frac{3}{\sqrt{32.2 \times 5}} = 0.2364 \quad \text{subcritical}$$

$$E_1 = y_1 + \frac{V_1^{\,2}}{2g} = 5 + \frac{3^2}{2 \times 32.2} = 5.14$$

$$E_1 = E_2 = E_3 = E_4 = E_5 = E_6$$

Section 2 :

$$E_1 = E_2$$

$$5.14 = y_2 + \frac{90^2}{2 \times 32.2 \times 5.8^2 \times y_2^2}$$

$$y_2^{\,3} - 5.14 y_2^{\,2} + 3.74 = 0$$

$$y_2 = 4.99 \text{ ft}$$

Section 3 :

$$E_1 = E_3$$

$$5.14 = y_3 + \frac{90^2}{2 \times 32.2 \times 5.6^2 \times y_3^2}$$

$$y_3^{\,3} - 5.14 y_3^{\,2} + 4.01 = 0$$

$$y_3 = 4.977 \text{ ft}$$

Section 4 :

$$E_1 = E_4$$

$$5.14 = y_4 + \frac{90^2}{2 \times 32.2 \times 5.4^2 \times y_4^2}$$

$$y_4^{\,3} - 5.14 y_4^{\,2} + 4.31 = 0$$

$$y_4 = 4.965 \text{ ft}$$

Section 5 :

$$E_1 = E_5$$

$$5.14 = y_5 + \frac{90^2}{2 \times 32.2 \times 5.2^2 \times y_5^2}$$

$$y_5^{\,3} - 5.14 y_5^{\,2} + 4.65 = 0$$

$$y_5 = 4.95 \text{ ft}$$

Section 6 :

$$E_1 = E_6$$

$$5.14 = y_6 + \frac{90^2}{2\times 32.2\times 5^2 \times y_6^2}$$

$$y_6{}^3 - 5.14 y_6{}^2 + 5.031 = 0$$

$$y_6 = 4.933\,\text{ft}$$

(ii)

$$H_1 = H_2$$

$$E_1 + \Delta z = E_2$$

$$y_1 + \frac{V_1^2}{2g} + \Delta z = y_2 + \frac{V_2{}^2}{2g}$$

for the water surface to be horizontal

$$y_1 + \Delta z = y_2$$

$$\frac{V_1^2}{2g} = \frac{V_2{}^2}{2g}$$

$$9 = \frac{90^2}{5^2 y_2^2}$$

$$y_2 = 6\,\text{ft}$$

$$\Delta z = y_2 - y_1 = 6 - 5 = 1\,\text{ft}$$

2.22

Q= 50 ft^3/sec

$$V_1 = \frac{50}{3\times 5} = 3.33\,\text{ft/sec}$$

$$V_2 = \frac{50}{2.5\times 4} = 5\,\text{ft/sec}$$

$$H_1 = H_2$$

$$E_1 = E_2 + \Delta z$$

$$y_1 + \frac{V_1{}^2}{2g} = y_2 + \frac{V_2{}^2}{2g} + \Delta z$$

$$3 + \frac{3.33^2}{2\times 32.2} = 2.5 + \frac{5^2}{2\times 32.2} + \Delta z$$

$$\Delta z = 0.28\,\text{ft}$$

2.23

$$Y_2 = \frac{1.5}{2}\left[\sqrt{1+8\times 25}-1\right] = 9.88 \text{ ft}$$

$$\mathrm{F_r} = 5 = \frac{V_1}{\sqrt{32.2\times 1.5}}$$

$$V_1 = 34.75 \text{ ft/sec}$$

$$V_2 = \frac{34.75\times 1.5}{9.88} = 5.275 \text{ ft/sec}$$

$$\text{head loss} = E_1 - E_2$$

$$1.5+\frac{34.75^2}{2\times 32.2}-9.88-\frac{5.275^2}{2\times 32.2} = 9.939 \text{ ft}$$

2.24

Assume rectangular cross section and horizontal channel bed

$$\mathrm{F_{r2}} = \frac{V_2}{\sqrt{gy_2}} = \frac{4.3}{\sqrt{32.2\times 5.2}} = 0.3323$$

$$y_1 = \frac{y_2}{2}\left[\sqrt{1+8F_2^2}-1\right]$$

$$y_2 = \frac{5.2}{2}\left[\sqrt{1+8\times(0.3323)^2}-1\right] = 0.968 \text{ ft}$$

$$q = 5.2\times 4.3 = 22.36 \text{ ft}^3/\text{sec/ ft}$$

$$V_1 = \frac{22.36}{0.968} = 23.099 \text{ ft/sec}$$

$E_1 = E_3$ neglecting losses through the gate

$$y_1 + \frac{V_1^{\,2}}{2g} = y_3 + \frac{V_3^{\,2}}{2g}$$

$$y_3 = 0.968+\frac{23.099^2}{2\times 32.2} = 9.25 \text{ ft}$$

$$V_3 = \frac{22.36}{9.25} = 2.41 \text{ ft/sec} \qquad \therefore \mathrm{y_3} = 9.16 \text{ ft}$$

$$V_3 = \frac{22.36}{9.16} = 2.44 \text{ ft/sec} \qquad \therefore \mathrm{y_3} = 9.157 \text{ ft}$$

2.25

$$A = (8+2y)y$$

$$E = y+\frac{Q^2}{2gA^2} = y+\frac{Q^2}{2\times 9.8\times(8+2y)^2 y^2}$$

Y	E (Q=10)	E (Q=20)	E (Q=40)	E (Q=50)
0.3	1.066	3.363	12.55	19.44
0.5	0.7515	1.506	4.527	6.79
0.7	0.8177	1.171	2.58	3.64
0.8	0.886	1.145	2.183	2.96
0.9	0.9655	1.162	1.948	2.54
1	1.051	1.204	1.815	2.27
1.1	1.14	1.262	1.7477	2.11
1.2	1.233	1.33	1.723	2.018
1.4			1.756	1.957
1.5			1.8	1.968
1.6			1.85	1.997
2			2.142	2.22
4			4.02	4.03

2.26

$D = \text{Hydraulic depth}$

$\alpha = \text{Velocity Coefficient}$

$\text{d} = \text{y}\cos\theta$

$$E = H = Z + \frac{p}{\gamma} + \frac{\alpha Q^2}{2gA^2}$$

$$= \text{Z} + \frac{\gamma d\cos\theta}{\gamma} + \frac{\alpha Q^2}{2gA^2} = \text{Z} + y\cos\theta^2 + \frac{\alpha Q^2}{2gA^2}$$

$$\frac{dE}{dy} = 0 = \cos\theta^2 + \frac{\alpha Q^2}{2g}(-2A^{-3})\frac{dA}{dy} \qquad \text{at critical depth}$$

$$\cos\theta^2 = \frac{\alpha Q^2}{g}\frac{B}{A^3} \qquad \text{where } \frac{dA}{dy} = B$$

$$V^2 = \frac{gD\cos\theta^2}{\alpha}$$

$$\frac{V^2}{\dfrac{gD\cos\theta^2}{\alpha}} = 1 = F_r^2$$

$$F_r = \frac{V}{\sqrt{\dfrac{gD\cos\theta^2}{\alpha}}}$$

2.28

$$V_1 = \sqrt{2 \times 32.2(400 - z)}$$

$$\text{Q} = 80000 = 200 \times y_1 \sqrt{2 \times 32.2(400 - z)}$$

$$y_1 = \frac{80000}{200\sqrt{2 \times 32.2(400 - z)}} = \frac{49.844}{\sqrt{400 - z}}$$

$$\text{F}_{\text{r1}}^2 = \frac{V^2}{gy_1} = \frac{2 \times 32.2 \times (400 - z)}{32.2 \times \frac{49.844}{\sqrt{400 - z}}} = 0.04 \times (400 - z)^{3/2}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1 + 8F_1^2} - 1\right]$$

$$\frac{220 - z}{\frac{49.844}{\sqrt{400 - z}}} = \frac{1}{2}\left[\sqrt{1 + 8 \times 0.04 \times (400 - z)^{3/2}} - 1\right]$$

$$(220 - z) \times \sqrt{400 - z} = 24.922\left[\sqrt{1 + 0.32(400 - z)^{3/2}} - 1\right]$$

By trial and error, z = 166.5 ft.

Chapter 3
CRITICAL FLOW

3-1

$\propto = f(y)$

$$E = d\cos\theta + \propto \frac{Q^2}{2gA^2}$$

$$E = \gamma\cos^2\theta + \propto \frac{Q^2}{2gA^2}$$

For critical flow

$$\frac{dE}{dy} = 0 = \cos^2\theta + \frac{Q^2}{2g}\frac{d}{dy}\left(\frac{\propto}{A^2}\right) \qquad (1)$$

$$\frac{d}{dy}\left(\frac{\propto}{A^2}\right) = \frac{1}{A^2}\frac{d\propto}{dy} - \propto\frac{2}{A^3}\frac{dA}{dy} = \frac{1}{A^2}\left[\frac{d\propto}{dy} - \frac{2\propto}{A}\frac{dA}{dy}\right]$$

$$\frac{d}{dy}\left(\frac{\propto}{A^2}\right) = \frac{1}{A^2}\left[\frac{d\propto}{dy} - \frac{2\propto}{A}B\cos\theta\right] = \frac{1}{A^2}\left[\frac{d\propto}{dy} - \frac{2\propto}{D}\cos\theta\right] \qquad \text{where } D = \frac{A}{B}$$

Equation 1

$$\cos^2\theta + \frac{Q^2}{2g}\left[\frac{1}{A^2}\left(\frac{d\propto}{dy} - \frac{2\propto}{D}\cos\theta\right)\right] = 0$$

$$\cos^2\theta = \frac{V^2}{2g}\left[\frac{2\propto}{D}\cos\theta - \frac{d\propto}{dy}\right]$$

Solving for V^2

$$V^2 = \frac{2g\cos^2\theta}{\left[\frac{2\propto}{D}\cos\theta - \frac{d\propto}{dy}\right]}$$

$$V = \sqrt{\frac{2g\cos\theta}{\left[\frac{2\propto}{D} - \frac{1}{\cos\theta}\frac{d\propto}{dy}\right]}}$$

$$Fr = \frac{V}{\sqrt{\frac{2g\cos\theta}{\left[\frac{2\propto}{D} - \frac{1}{\cos\theta}\frac{d\propto}{dy}\right]}}} = 1$$

Or, $Fr = \dfrac{V}{\sqrt{\dfrac{2g\cos\theta}{\left[\dfrac{2\propto}{D} - \dfrac{d\propto}{dd}\right]}}} = 1$

3.3

$$F = \frac{V}{\sqrt{gD\dfrac{\cos\theta}{\propto}}} = \frac{\dfrac{Q}{A}}{\sqrt{gD\dfrac{\cos\theta}{\propto}}} = 1$$

$$A\sqrt{D} = \frac{Q}{\sqrt{g\dfrac{\cos\theta}{\propto}}} = \frac{60}{\sqrt{\dfrac{9.81}{1.1}}} = 20.09$$

$$A = \left(\frac{T+b}{2}\right)y$$

T = 4y + b

$$A = \frac{4y+2b}{2}y = (2y+b)y$$

$$D = \frac{(2y+b)y}{4y+b}$$

$$A\sqrt{D} = (2y+b)y\sqrt{\frac{(2y+b)y}{4y+b}} = \frac{y_c^{\frac{3}{2}}(2y_c+b)^{\frac{3}{2}}}{(4y_c+b)^{\frac{1}{2}}} = 20.09$$

By trial error, y_c = 0.9835m

3.4

$$F_D = \frac{V}{\sqrt{gD\dfrac{\cos\theta}{\propto}}} = 1$$

For critical depth

$$\frac{\dfrac{Q}{A}}{\sqrt{gD\dfrac{\cos\theta}{\propto}}} = 1$$

$$A\sqrt{D} = \frac{\dfrac{Q}{\sqrt{\cos\theta}}}{\sqrt{\dfrac{g}{\propto}}}$$

For $\theta = 0$ and $\propto = 1$

$$A\sqrt{D} = \frac{Q}{\sqrt{g}} = 25.54$$

$A = (B_o + Sy)y = (8 + 0.5y_c)y_c$ where Bo = 8m, S = 0.5

$B = B_o + 2Sy = 8 + y_c$

$$D = \frac{A}{B} = \frac{(8+0.5y_c)\,y_c}{8+y_c} = 25.54$$

Using trial and error $y_c = 2.074$ m

$$(B_o + 0.5y_c)\,y_c\left[\frac{(B_o+0.5y_c)\,y_c}{8+y_c}\right]^{\frac{1}{2}} = 25.54$$

3.5

Y	A	R	B	D=A/B	$Z = A\sqrt{D}$
0	0	0	0	-	-
2	23.4	1.34	16.6	1.41	27.79
4	58.3	2.68	18.2	3.203	104.34
6	95.7	3.7	19.2	4.984	213.65
8	134.8	4.5	19.8	6.808	351.72
10	174.6	5.15	20	8.73	515.88
12	214.4	5.65	19.6	10.939	709.11
14	252.5	5.99	18.3	13.798	937.93
16	287	6.13	16	17.938	1215.54
18	315.4	6.01	12	26.283	1616.96
20	331.7	5.08	0	∞	∞

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{850}{\sqrt{9.81}} = 271.38$$

Using graphical method $y_c = 6.8$ m

3.6

$$E = y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1 + \frac{Q_1^2}{2gB_1^2y_1^2} = y_2 + \frac{Q_2^2}{2gB_2^2y_2^2}$$

Rectangular Channel

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

$$y_1 - y_2 = \frac{q^2}{2g}\left(\frac{y_1^2 - y_2^2}{y_1^2y_2^2}\right)$$

$$\frac{q^2}{g} = 2\frac{y_1^2 y_2^2}{(y_1+y_2)} \qquad \text{but, } y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$y_c = \sqrt[3]{\frac{2(y_1 y_2)^2}{(y_1+y_2)}}$$

3.7

For

For $\theta = 0$ and $\propto = 1$ $\qquad z = A\sqrt{D} = \frac{Q}{\sqrt{g}}$

i. Trapezoidal

A = (b+ zy)y $\qquad D = \frac{(b+zy)y}{b+2zy}$

$$\frac{Q}{\sqrt{g}} = \frac{\left[(b+zy_c)y_c\right]^{1.5}}{\sqrt{b+2zy_c}}$$

ii. Triangular

A= Zy^3 $\qquad$ D = 0.5 y

$$\frac{Q}{\sqrt{g}} = \frac{\sqrt{2}}{2} zy_c^{2.5}$$

$$y_c = \left[\frac{Q}{z}\sqrt{\frac{z}{g}}\right]^{\frac{1}{2.5}}$$

iii. circular

$$A = \frac{1}{8}(\theta - \sin\theta) d_o^2 \qquad D = A = \frac{1}{8}\left(\frac{\theta - \sin\theta}{\sin\frac{\theta}{2}}\right) d_o$$

$$\frac{Q}{\sqrt{g}} = \frac{1}{16}\frac{(\theta - \sin\theta)^{1.5}}{\left(2\sin\frac{\theta}{2}\right)^{0.5}} d_o^{2.5}$$

$$y_c = \frac{d_o}{2} + \frac{d_o}{2}\left(\cos\left(180 - \frac{\theta_c}{2}\right)\right) = \frac{d_o}{2}\left(1 - \cos\frac{\theta_c}{2}\right)$$

3.8

i. using a step, with constant width

$$Fr = \frac{V}{\sqrt{gD\frac{\cos\theta}{\propto}}} = \frac{V_1}{\sqrt{gy_1}} = \frac{250/(5\times 50)}{\sqrt{9.81\times 5}} = 0.143 \quad < 1 \text{ subcritical}$$

$E_1 = \Delta z + E_2$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(250/50)^2}{9.81}} = 1.366m$$

$$V_c = 250/(1.366\times 50) = 3.66m/s$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z$$

$$\frac{1}{2\times 9.81} + 5 = \Delta z + \frac{(3.66)^2}{2\times 9.81} + 1.366$$

$\Delta z = 5.051 - 2.049 = 3$ m

ii. Reduction in the channel width

q= 250/50 = 5 m_3/sec/m

$$E_1 = y + \frac{q^2}{2gy^2} = 5 + \frac{5^2}{2\times 9.81\times 5^2} = 5.05m$$

$(E - y)\, 2gy^2 = q^2$

$$q = (2gy^2E - 2gy^3)^{0.5}$$

$$\frac{dq}{dy} = 0.5\left[2gy^2E - 2gy^3\right]^{-0.5}\left[4gEy - 6gy^2\right] = 0 \qquad \text{(Max q)}$$

$4g\, Ey = 6gy^2$ $\qquad$ $y_c = 2/3\, E$

$y_c = 2/3 \times 5.05 = 3.367$ m

$$q_{\max} = \left[2\times 9.81\times 3.367^2\times 5.051 - 2\times 9.81\times 3.367^3\right]^{0.5} = 19.35 \text{ m}^3\text{/sec/m}$$

$B_{min} = Q / q_{max} = 250 / 19.35 = 12.92$ m

Reduction in width = 12.95 / 50 x 100 = 25.8%

iv. Width Reduction and bottom step

- Use $\Delta z < 3.0$ m
- Solve problem (i) to find $y_2 \neq y_c$

- Solve problem (ii) to go from y_2 to y_c by reducing the channel width.

3.9

$E_1 = \Delta z + E_2$ $\qquad V_1 = \frac{Q}{A_1} = \frac{96}{12\times 4.22 = 1.896} = 1.896$ m/s

$$E_1 = 4.22 + \frac{1.896^2}{2\times 9.81} = 4.4m$$

$E_2 = 4.4 - 0.2 = 4.2$ m

$$V_2 = \frac{Q}{A_2} = \frac{96}{\left(\frac{10+10+2y_2}{2}\right)y_2} = \frac{96}{(10+y_2)\,y_2}$$

$$4.2 = y_2 + \frac{96^2}{(10+y_2)^2\, y_2^2 \times 2\times 9.81}$$

$$\sqrt{4.2-y_2} = \frac{21.67}{10y_2 + y_2^2}$$ $y_2 = 4.05$ (by trial and error)

F=1 $\quad \frac{Q}{\sqrt{g}} = A\sqrt{D} = \frac{96}{\sqrt{9.81}} = 30.65$

$$(10+y_c)\left[\frac{(10+y_c)\,y_c}{10+2y_c}\right]^{0.5} = 30.65$$

Or, $$\frac{\left[10y_c + y_c^2\right]^{1.5}}{(10+2y_c)^{0.5}} = 30.65$$

Or, $$\frac{\left[10y_c + y_c^2\right]^{3}}{(10+2y_c)} = 939.45$$

$y_c = 1.97$ m (by trial and error)

3.11

$X = Ky^m$

Triangular $\quad K = S \quad m=1 \quad X = S\,Y$

Rectangular $\quad K = B_o / 2 \quad m = 0$

Parabola $\quad K = \left(\frac{1}{a}\right)^m \qquad m = 1/n \qquad Y = a\,x^n$

$$A_c = 2\int_0^{Y_c} Ky^m dy = 2K\frac{y_c^{m+1}}{m+1}$$

$$B_c = 2x_c = 2Ky_c^m$$

$$\frac{Q^2}{gA_c^2}=\frac{A_c}{B_c} \qquad \frac{Q^2}{g}=\frac{A_c^3}{B_c}=\frac{\left[\frac{2K}{m+1}y_c^{m+1}\right]^3}{2Ky_c^m}$$

$$\frac{Q^2}{g}=\frac{8K^3}{(m+1)^3}y_c^{3m+3}\frac{1}{2Ky_c^m}=\frac{4K^2}{(m+1)^3}y_c^{2m+3}$$

$$y_c=\left[\frac{Q^2}{g}\frac{(m+1)^3}{4K^2}\right]^{\frac{1}{2m+3}}$$

3.12

Y = a x ^{n}

3 = a (4.5) ^{2}

A = 0.14815

Critical depth

$$\frac{Q}{\sqrt{g}}=A\sqrt{D} \qquad (1)$$

$$\frac{50}{\sqrt{32.2}}=\frac{2}{3}TY\sqrt{\frac{2}{3}Y}$$

16.187 = TY$^{1.5}$ (2)

Also $Y=0.148\left(\frac{T}{2}\right)^2$

T = 5.196 Y $^{0.5}$ (3)

Putting Eq 3 in Eq 2,

16.187 = 5.196 Y$^{0.5}$ Y $^{1.5}$

Y $_{c}$ = 1.765

3. 13

Slope = 1 ft / mile = 1 /(1760 x 3)

$$Y=\frac{d_o}{2}\left[1-\cos\frac{\theta}{2}\right]$$

$$\frac{Q}{\sqrt{g}}=A\sqrt{D}$$

$$\frac{100}{\sqrt{32.2}}=\frac{1}{16}\frac{(\theta-\sin\theta)^{1.5}}{\left(2\sin\frac{\theta}{2}\right)^{0.5}}d_o^{2.5}$$

$$2.2028 = \left[\frac{\theta - \sin\theta}{\sin\frac{\theta}{2}} \right]^{0.5} (\theta - \sin\theta)$$

or, $\theta \cong 135.2^o$ (trial and error)

or, $Y_c = \frac{8}{2}\left[1 - \cos\frac{135.2}{2}\right] = 2.48$

This problem may also be solved by using Figure 3.7.

3.14

$$\frac{Q}{\sqrt{g}} = A\sqrt{D} = 17.62$$

A = (B_o + 5y) y = (10 +2y) y

$$D = \frac{(10+2y)\,y}{10+4y}$$

$$17.62 = \left(10y + 2y^2\right)\left(\frac{10y+2y^2}{10+4y}\right)^{0.5}$$

Critical depth y_c = 1.33

3.15

Q =15 ft^3/sec

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{15}{\sqrt{32.2}} = 2.64$$

Using fig. 3.7

$$\frac{Z_c}{D_o^{2.5}} = \frac{2.64}{5^{2.5}} = 0.0472$$

$$\frac{Y}{D_o} = 0.215$$

$$Y_c = 5 \times 0.215 = 1.075$$

3.16

$$\frac{Q}{\sqrt{g}} = A\sqrt{D} = \frac{300}{\sqrt{32.2}} = 52.868$$

Use the notations in problem 1.1.

Assume that the critical depth occurs in the lower portion

$$A = \frac{1}{8}(\theta - \sin\theta)\,d_o^2 \qquad B = 2d_o \sin\frac{\theta}{2}$$

Using θ = 48.59^0

$$A = \frac{30^2}{2}\left(\frac{48.59}{180}\pi - \sin 48.59\right) = 44.125\, ft^2$$

$$B = 2\times 30 \sin\frac{48.59}{2} = 58.99\, ft$$

The critical depth is located in the lower portion.

Trial and error procedure

$\theta_{\text{critical}} = 47.2^o$

$$Y_{critical} = \frac{D}{2}\left[1-\cos\frac{47.2}{2}\right]$$

$D = 2d_o = 60$

$$Y_c = \frac{60}{2}[1-0.916] = 2.52$$

3.17

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{10}{\sqrt{32.2}} = 1.762$$

$$\frac{Z_c}{D_o^{2.5}} = \frac{1.762}{4^{2.5}} = 0.05506$$

Using Fig 3.7,

$Y / D_o = 0.23$

$Y_c = 0.23 \times 4 = 0.92$

3.18

$$Q = C_d A\sqrt{2gH} = 0.7\times 10 y_2\sqrt{2\times 32.2\times 60}$$

$Q = 435.127\ y_2$

$$F_3 = \frac{v_3}{\sqrt{gy_3}} = \frac{435.127 y_2}{10\times 9\sqrt{32.2\times 9}}$$

$F_3 = 0.284\ y_2$

$$y_2 = \frac{y_3}{2}\left[-1+\sqrt{1+8F_3^2}\right]$$

$$y_2 = \frac{9}{2}\left[-1+\sqrt{1+8(0.284)^2\, y_2^2}\right]$$

$$\frac{y_2}{4.5}+1 = \sqrt{1+0.645y_2^2}$$

$$y_2^2 + 9y_2 + 20.25 = 20.25\left(1+0.645y_2^2\right)$$

$12.06\ y_2^2 - 9\ y_2 = 0$ $\qquad$ $y_2 = 0.766$

Q = 0.746 x 435.127 = 324.6 cfs

$$\rho_F = F_{s1} - F_{s2} = \frac{Q^2}{gA_1} + \overline{Z}_1 A_1 - \frac{Q^2}{gA_2} - \overline{Z}_2 A_2$$

$$= \frac{(324.6)^2}{32.2\times 10}\left[\frac{1}{60} - \frac{1}{0.746}\right] + 30\times 60\times 10 - \frac{0.746}{2}\times 0.746\times 10 = 1364.037\ ft^3$$

Thrust = r ρ $_F$ = 62.4 x 1364.037 = 85115.9 lb

3.19

Q = 2 x 4 = 8 ft^3/sec/ft

$$F_r = \frac{v}{\sqrt{gD}} = \frac{2}{\sqrt{32.2\times 4}} = 0.176$$

Subcritical

$E_1 = \Delta z + E_2$

$$\frac{v_1^2}{2g} + y_1 = \frac{q^2}{2gy_2^2} + y_2 + \Delta z$$

$$\frac{2^2}{2\times 32.2} + 4 = \frac{8^2}{32.2\times 2\times y_2^2} + y_2 + 0.5$$

$$3.562 = \frac{0.9938}{y_2^2} + y_2$$

$$y_2^3 - 3.562 y_2^2 + 0.9938 = 0$$

Y_2 = 3.48

v_2 = 8 / 3.48 = 2.99 ft/sec

3.20

q = 250 / 50 = 5 m^3/sec/m

$$E = y + \frac{q^2}{2gy^2} = 5 + \frac{5^2}{2\times 9.81\times 5^2} = 5.05m$$

y_c= 0.67 E = 3.367 m

$$q = \sqrt{gy_c^3} = \sqrt{9.81\times 3.367^3} = 19.35 \quad m^3/sec/m$$

B_{min} = 250 / 19.35 = 12.92 m

3.23

$$F = \frac{1.5}{\sqrt{9.81\times 4}} = 0.2395 \quad \text{subcrtical}$$

Q= 1.5 x 4 =6 m^3/sec/m

Q = 6 x 5 = 30 m^2/s

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{36}{9.81}} = 1.543m$$

if there is no converging transition

$$E = y + \frac{q^2}{2gy^2} = 4 + \frac{6^2}{2\times 9.81\times 4^2} = 4.114$$

y_c= 0.67 E = 2.74 m

$y_c = \sqrt[3]{\frac{q^2}{g}}$ q = 14.2 m^3/sec/m

Min Width = 30 / 14.2 = 2.11 m

Chapter 4
UNIFORM FLOW

4.1

Given: Compute Y_n (Normal Depth)

B= 5m
Q=5m3/s
n=0.013
S_0=0.001

a) Design curves Method

Section Factor: $\frac{nQ}{S_0^{1/2}}$ = $(0.013)(5)/(0.001)^{1/2}$= $AR^{2/3}$

Thus, $AR^{2/3}/B^{8/3}$= $2.055/5^{8/3}$= 0.028

For a rectangular Channel and $AR^{2/3}/B^{2/3}$= 0.028,

Figure 4.5 gives Y_n/B=0.128,

Thus, Y_n= 0.64 m.

b)Trial and error method

We have $A=BY_n=5Y_n$ (Area)

$P=2Y_n+B=2Y_n+5$(Wetted perimeter)

R= $\frac{A}{P}=\frac{5Yn}{2Yn+5}$

Therefore, $AR^{2/3}$= $(5Y_n)\left[\frac{5Y_n}{(2Y_n+5)}\right]^{2/3}$=2.055

or, $14.62Y_n-2.055(2Y_n+5)^{2/3}=0$ ---------------------------------------(1)

Substituting values in Eq 1, we get the following results:

Y	f(y)
0.6	-0.69
0.7	0.984
0.64	-0.05
0.645	0.037

Therefore, Y_n=0.64m

c) Numerical Methods

The programs in Appendix C may be used to compute Y_n.

4.2

Given: Compute: Y_n (normal depth)

Q=50m3/s

n=0.013

B_0=10m

S_0=0.001

a) Design Curves Method:
b) Section Factor: $nQ/S_0^{1/2}$= (0.013)(50) /$(0.001)^{1/2}$=20.55

$AR^{2/3}$ = 20.55 and $AR^{2/3}/ B^{8/3}$= 0.0443

Entering in figure 4.5 with S=2 and $AR^{2/3}/ B^{8/3}$= 0.044 we get

Y_n/B_0=0.145

and, Y_n=(0.145)(10)

or

Y_n= 1.45m

b) Trial and Error method

for Trapezoidal channel we have: $A = (10+2Y_n)Y_n$, and R=$\dfrac{(10+2Y_n)}{\left(10+2Y_n\sqrt{1+2^2}\right)}Y_n$

or,

$AR^{2/3}$=$\dfrac{(10+2Y_n)}{\left(10+2Y_n\sqrt{1+2^2}\right)^{2/3}}Y_n\left[(10+2Y_n)Y_n\right]^{2/3}$=20.55

Rearranging the equation:

$$\left[(10+2Y_n)Y_n\right]^{2/3} - 20.55\left(10+2Y_n\sqrt{1+4}\right)^{2/3} = 0 \text{-----------------}\{1\}$$

By trial and error, the solution to equation 1 is:

Y_n= 1.46m

c) Numerical methods

A similar result will be obtained by using the programs in appendix C to solve for the normal depth

From the graph we get Y_n= 3.6m

For the critical depth we have:

$$\frac{Q}{\sqrt{g}} = A\sqrt{\left(\frac{A}{T}\right)} = \frac{150}{\sqrt{9.81}} = 47.89$$

or; $\left[(b+0.583y)y\right]^{3/2} \big/ [b+y(1.667)]^{1/2}$ = 47.89 for $0< Y_c<4$

Where, b=15, therefore:

$$\frac{\left[15y+0.583y^2\right]^{3/2}}{\left[15+1.667y\right]^{1/2}} = 47.89$$

Solving by trial and error for y, we get

y_c=2.15 m.

4.4

Given Q=150 m^3/s

S_0=2/1000

n=0.03

Compute Y_n and Y_c

Compute section factor: $Q_n/S_0^{1/2}$= (150)(0.03)/(0.002)$^{1/2}$= 100.62

From Manning's equation: $AR^{2/3}$=100.62

For $0< y \leq 4$:

T_T=b+y/1.5+y/2 (Top width)

A_T=(b+T_T)y/2=(b+b+1.1667y)y/2 (flow area)

$P_T = 15 + y\sqrt{(1+1.5^2)} + y\sqrt{(1+2^2)}$

P_T=15+4.0388y (wetted perimeter)

Make a plot of $A_T R_T^{2/3}$ vs. y to compute the normal depth

Y	A_T	P_T	R_T	$R_T^{2/3}$	$A_T R_T^{2/3}$
2	32.33	23.08	1.40	1.25	40.41
4	69.33	31.16	2.22	1.70	117.86
3	50.25	27.12	1.853	1.51	75.81

4.5

Given:

Trapezoidal Channel 1H: 1V

b=10m

S_0=0.0005

Q=60 m^3/s

n=0.013 (for concrete)

Determine flow depth (normal depth)

Solution:

For a trapezoidal channel with 1H: 1V we have:

A=(b+T)y/2=(b+b+2y)/2=(b+y)y

T=b+2y

$P = b + 2\left(\sqrt{2}\right)y$

Manning's equation becomes: $Q_n/S_0^{1/2} = A^{5/3} P^{-2/3}$

Or,

$Q_n/S_0^{1/2} = (by+y2)^{5/3}/\{b+2\left(\sqrt{2}\right)y\}^{2/3}$

Where, $Q_n/S_0^{1/2}$= (60) (0.013)/ (0.0005)$^{1/2}$=34.88

Then, $\left[10y + y^2\right]^{5/3}\left[10 + 2(\sqrt{2})y\right]^{-2/3} - 34.88 = 0$

This equation can be solved by trial and error or by using numerical methods as presented in appendix C. The solution is y_n=2.10m.

4.6

According to Manning's equation:

$AR^{2/3}$=$nQ/S_0^{1/2}$[for n and S_0 constants we have $Q \propto AR^{2/3}$]

For a circular cross – section the section factor is:

$$AR^{2/3} = \frac{\left[\frac{D_0^2}{8}(\theta - Sin\theta)\right]^{2/3}}{\left(\frac{1}{2}\theta D_0\right)^{2/3}} = 0.04496 D_0^{8/3}\frac{(\theta - Sin\theta)^{5/3}}{\theta^{2/3}}$$

See Table 1.1 for geometric factors.

The value of θ corresponding to maximum discharge is found by equating the first derivative of $AR^{2/3}$:

$\partial(AR^{2/3})/\partial\theta = 0, or$

$$\frac{0.0496D_0^{8/3}}{\theta^{4/3}}\left[5/3(\theta - Sin\theta)^{2/3}(1 - Cos\theta)\theta^{2/3} - 2/3\theta^{-1/3}(\theta - Sin\theta)^{5/3}\right] = 0$$

Equating the term in brackets to zero and simplifying we get : $\theta(1.5 - 2.5Cos\theta) + Sin\theta = 0$

Solving for $\theta \rangle 0$ by trial and error we get, $\theta = 5.278$

The corresponding depth is : $y_n = 0.5(1 - Cos(5.278/2))$

$or, y_n / D_0 = 0.938$

In terms of flow velocity the Manning's equation gives:

$R^{2/3}$=$nV/S_0^{1/2}$

Then, for constant n and S_0, the velocity is maximum, when $R^{2/3}$ is maximum, or:

$V \propto R^{2/3}$

For a circular section: $R^{2/3}$ =$[1/4(1-Sin\theta/\theta)D_0]^{2/3}$(see Table 1.1). Following the same reasoning as for the discharge, the angle θ Corresponding to the maximum velocity is given by: $\partial(R^{2/3})/\partial\theta$=0

or: $2/3(D_0/4)^{2/3}(1 - \frac{Sin\theta}{\theta})^{-1/3}[(Sin\theta - \theta\, Cos\theta)/\theta^2]$=0

Solving for θ results in: θ_m=4.493

The corresponding depth for maximum velocity is:

y_n=$D_o/2(1-Cos4.493/2)$

or, y_n/D_0=0.812

4.7

$$\text{Max(Q)} \Rightarrow Max(\frac{A^{8/3}}{P^{2/3}}) \Rightarrow Min(P)$$

For a triangular section: P=2y$\sqrt{(1+s^2)}$where s is the lateral slope

Also,
$A=sy^2$
Then,
$P=2y\sqrt{\left(1+A^2/y^4\right)}$
To find the min (P), dP/dy=0 is to be satisfied.
or, $dP/dy=2(1+A^2/y^4) - A^2/y^2=0$
but, $A^2=s^2y^4$, then,
$y=1/s\sqrt{\left(\frac{1+s^2}{2}\right)}$
Substituting this expression in the equation for P we get:
$P=2\sqrt{\left(\frac{1+s^2}{2}\right)}\frac{\sqrt{(1+s^2)}}{s}=\frac{2}{\sqrt{2}}\frac{(1+s^2)}{s}$
$dP/ds=2-1/s^2-1=0$
or, s=1
or, $\theta=45^0x2$
$d^2P/ds^2=2/s>0$ implies P is minimum.

ii. Trapezoidal section :
$A=(b+sy)y$ ---------------------------(1)
$P= b+2\left(\sqrt{1+s^2}\right)y$ ------------------ (2)
$dP/dy=2\{\sqrt{(1+s^2)}-s\}-b/y$
for minimum wetted perimeter dP/dy=0
or:
$b=2y(\sqrt{(1+s^2)}-s)$ ----------------(3)
substituting 3 in 1 we have:
$A=[2y(\sqrt{(1+s^2)}-s+sy]y$
or, $y=\sqrt{\frac{A}{\left(2\{\sqrt{1+s^2}\}-s\right)}}$
from Eq 2, we have,
$P=2y[2\sqrt{(1+s^2)}-s]$
or, $P=2\sqrt{\left(A\left\{2\sqrt{[1+s^2]}-s\right\}\right)}$
To find s corresponding to minimum P, dP/ds=0 is to be satisfied.
for $A\neq 0$, we get $\frac{2s}{\sqrt{(1+s^2)}}-1=0$
or, $s=\frac{\sqrt{3}}{3}=\tan 30^0$. Thus, the most efficient section for a channel is half a hexagon.

4.8

The channel having the least wetted perimeter for a given area has the maximum flow capacity. The area and the wetted perimeter for a circular channel are:

$A=1/8(\theta - Sin\theta)\ D_0^2$, $P=1/2\ \theta\ D_0$

Solving for D_0 in the equation for the area:

$$D_0=\sqrt{\left(\frac{8A}{\theta - Sin\theta}\right)}$$

Substituting in to the perimeter:

$$P=\theta/2\sqrt{\left(\frac{8A}{\theta - Sin\theta}\right)}$$

The wetted perimeter is minimum when $dP/d\theta=0$

$$dP/d\theta=\left[\frac{1}{\sqrt{(\theta - Sin\theta)}}-\frac{\theta}{2}(\theta - Sin\theta)^{-3/2}(1-Cos\theta)\right]\frac{\sqrt{(8A)}}{2}$$

$dP/d\theta=0$, when $\theta=\pi$.

This corresponds to a semicircular section.

4.9

For flow in a pipe flowing partially full that $Q_p=Q_f\dfrac{\left(\theta-\dfrac{Sin2\theta}{2}\right)^{5/3}}{\left(\pi\theta^{2/3}\right)}$ in which Q_p=flow for partially full pipe and Q_f=flow when pipe is full.If D,=pipe diameter, y_n=normal depth and S_0=slope of the energy grade line in the case of full pipe flow (equal to the slope of the pipe bottom in pipe angle is given by:

$$\theta = Cos^{-1}\left(1-\frac{2y_n}{D_0}\right)$$

Christensen's equation for partially full pipes, based on experimental data, is given by:

$$\left(\frac{Q_p}{Q_f}\right)_{exp}=0.46-0.5\cos(\pi y_n/D_0)+0.04(2\pi y_n/D_0)$$

Plot y_n/D_0 vs. Q_p/Q_f using the analytical result and Christensen's equation. Compute and plot n_p/n_f with respect to y/D_0 assuming that n is a function of depth.

Solution:

Prove that $Q_p=Q_f(\theta-0.5Sin2\theta)^{5/3}/(\pi\,\theta^{2/3})$

a)Find an expression for the area of a circular segment for the element showed in the figure we have:

$y=RCos\theta$

$dy=-RSin\theta d\theta$

The area of the segment under the element is given by:

A= $\int 2xdy$

$x^2+y^2=R^2$ therefore:

A= $\int_{-y}^{-R} 2\sqrt{(R^2-y^2)}dy$

Or, in terms of θ, A=$2R^2 \int_0^{\theta} Sin^2\theta d\theta$

Solving by integration by parts we get,

A=$2R^2$ [θ–0.5Sin2θ]--------------------------------------(1)

Put θ in terms of y_n.

When, y_n<R

y_n=R–RCosθ

$2y_n$=2R(1–Cosθ)

$2y_n$/D0=1–Cosθ

or, θ=$Cos^{-1}(1–2y_n/D_0)$

The same equation results when y_n>R.

θ=$Cos^{-1}(1–2y_n/D_0)$ --------------------------------(2)

Use Manning's equation

$Q_p = \frac{1}{n}\frac{\left(A_p^{5/3}S_0^{1/2}\right)}{P_p^{2/3}}$,

where A_p is given by equation1

For full-pipe flow:

$Q_f = \frac{1}{n}\frac{\left(A_f^{5/3}S_0^{1/2}\right)}{P_f^{2/3}}$

Then, $Q_p/Q_f = \frac{\left(A_p^{5/3}P_f^{2/3}\right)}{P_p^{2/3}A_f^{5/3}}$ ---------------------------(3)

Recalling that P_p=2Rθ, $A_f=\pi r^2$, and P_f=2πR and substituting equations 1 and 2 into 3 we get,

$(Q_p/Q_f)_T = (\theta–0.5Sin2\theta)^{5/3}/(\pi\,\theta^{2/3})$------------------(4)

Where T refers to a theoretical result.

2.Compute n_p/n_f

Assuming that the difference between theoretical equation 4 and experimental results $(Q_p/Q_f)_{exp}$ is due to a variation of n with respect to y_n, we have

$Q_{f\ exp} = \frac{1}{n_f}\frac{\left(A_f^{5/3}S_0^{1/2}\right)}{P_f^{2/3}}$ and, $Q_{p\ exp} = \frac{1}{n_p}\frac{\left(A_p^{5/3}S_0^{1/2}\right)}{P_p^{2/3}}$,

Then,

$$(Q_p/\ Q_f)_{exp} = (n_f/n_p)\frac{\left(A_p^{\ 5/3}P_f^{\ 2/3}\right)}{P_p^{\ 2/3}A_f^{\ 5/3}} \text{ ----------------------(5)}$$

The last term on the right hand side of Eq 5 is the same as Eq 3 and the result is given by Eq 4. Therefore,

$(Q_p/\ Q_f)_{exp}=(n_f/n_p)\ (Q_p/\ Q_f)_T$
$(n_p/\ n_f)_{exp}=(Q_f/Q_p)_{exp}\ (Q_p/\ Q_f)_T$

Where, $(Q_p/\ Q_f)_T$ is calculated from Eq 4 and $(Q_f/Q_p)_{exp}$ is calculated with Christensen's equation.

3. Construction of curves

Equations 4, 6 and Christensen's are functions of y_n/D_0. To solve Eq 4, angle θ must be calculated first by using Eq 2.Table1 summarizes this calculation.

Table1: $Q_p/\ Q_f$, $n_p/\ n_f$ for circular pipes as function of y_n/D_0

y_n/D_0	θ (rad)	$(Q_p/\ Q_f)_T$	$(Q_p/\ Q_f)_{exp}$	(n_f/n_p)
0.1	0.6435	0.0209	0.0168	1.2403
0.2	0.9273	0.0876	0.0679	1.2403
0.3	1.1593	0.1958	0.1537	1.2737
0.4	1.3694	0.3370	0.2731	1.2338
0.5	1.5708	0.5000	0.4200	1.1905
0.6	1.7722	0.6719	0.5821	1.1541
0.7	1.9823	0.8373	0.7415	1.1291
0.8	2.2143	0.9775	0.8779	1.1148
0.9	2.4981	1.0658	0.9679	1.1012

4.10

For the horse-shoe section the following graphical method will be used:

1. Compute the value of $nQ/\sqrt{S}$ from the given data.
2. Plot a graph of y verses the section $AR^{2/3}$
3. The normal depth is the value of y corresponding to the ordinate $AR^{2/3}= nQ/\sqrt{S}$.

For this particular problem we have:

1. $nQ/\sqrt{S}$ =0.03x800/(0.0005)$^{1/2}$=1073.31
2. for the horse-shoe section:

y(m)	A(m^2)	R(m)	AR $^{2/3}$(m $^{4/3}$)
5	91.06	3.35	203.87
10	210.63	5.63	666.59
12	260.37	6.28	886.74
13	285.37	6.57	1001.55

13.5	297.87	6.71	1059.44
13.75	304.06	6.77	1087.88
15	334.94	7.06	1232.64
20	448.44	7.67	1744.09

The normal depth is y_n=13.55m. This value was obtained by plotting the first and the last columns of the previous table and taking the value corresponding to $nQ/\sqrt{S}$ =1073.

4.11

Given: n=0.0125
B_0=1.2m
Q=210l/s, 350l/s, 450l/s
Channel gradient: 0.67, 0.07, 0.17, 0.3 m/Km
Compute: Normal Depths.

Case1: S_0=0.00067 and Q=0.21m^3/s, $nQ/S_0^{1/2}$=0.10141
$AR^{2/3}=(B_0y_n)^{5/3}/(2y_n+B_0)^{2/3}$
Solve for y_n by trial and error or by using the computer programs in Appendix C.

$(1.2y_n)^{5/3}/(2y_n+1.2)^{2/3}$=0.10141 or, y_n=0.262m

The other cases are solved in a similar way by using the appropriate values of Q and S_0.The following Table summarizes the results.

Computation of Normal Depths for Nimes Aqueduct

Case	Section	Q_0(m^3/s)	S_0	$nQ/\sqrt{S_0}$	y_n(m)
1	ELEVATED CHAN.	0.21	0.00067	0.10141	0.262
2	ELEVATED CHAN.	0.35	0.00067	0.16902	0.375
3	ELEVATED CHAN.	0.45	0.00067	0.21731	0.448
4	POND DU GARD	0.21	0.00007	0.31375	0.587
5	POND DU GARD	0.35	0.00007	0.5229	0.869
6	POND DU GARD	0.45	0.00007	0.67232	1.062
7	SUBTERRANEAN CH.	0.21	0.00017	0.2013	0.424
8	SUBTERRANEAN CH.	0.35	0.00017	0.3355	0.618
9	SUBTERRANEAN CH.	0.45	0.00017	0.43142	0.748
10	SUBTERRANEAN CH.	0.21	0.00030	0.1515	0.347
11	SUBTERRANEAN CH.	0.35	0.00030	0.2526	0.500
12	SUBTERRANEAN CH.	0.45	0.00030	0.32476	0.603

4.12

Given: B_0=15ft
Q=150cfs
S=1.5

n=0.024
S_0=2.5ft/mi
1mi=5280ft

Solution:
S_0=2.5ft/5280ft/mi=0.000473

For steady–uniform flow the channel flow depth will be the normal depth, then we use Manning's equation to solve for $y=y_n$

$n\,Q/\,S_0^{1/2}=1.49\,(AR^{2/3})$

$AR^{2/3}$=0.024x150/(1.49x90.000473)$^{1/2}$=111.09

1. Design curves method

Use Figure 4.5 for $AR^{2/3}/B_0^{8/3}$=0.081 and s=1.5 to get y_n/B_0=0.21 or y_n=0.21x15=3.15ft

2. Trial and Error

Solve for y_n by trial and error (or using a root finding computer program).

$$AR^{2/3}=\frac{\left[(B_0+sy_n)y_n\right]^{5/3}}{\left[B_0+2y_n\sqrt{1+s^2}\right]^{2/3}}=111.09$$

or
$$\frac{\left[(15+1.5y_n)y_n\right]^{5/3}}{\left[15+2y_n\sqrt{1+1.5^2}\right]^{2/3}}=111.09$$

or y_n=3.17 ft

4.13

Given: S_0=10ft/mi
n=0.045
Q=50ft^3/s

Parabolic cross-section of problem 3.12

$y=\frac{x^2}{4P}$ where, P=distance between the focus and the vertex.

Compute: uniform flow depth

Using the coordinates (4.5, 3), P is computed as $P=\frac{4.5^2}{4(3)}$=1.6875

Then, the equation of the parabolic section is: $y=\frac{x^2}{6.75}$

Also: A=(2/3)T_y, T=2x, P=T+(8/3)y^2/T
or, A=0.1975x^3
P=2x+0.02926x^3
nQ/(1.49$S_0^{1/2}$)=(0.045)(50)/{(1.49)(0.00189$^{1/2}$)}=34.7348 ----------------------(1)

and $\frac{A^{5/3}}{P^{2/3}} = \frac{0.06698x^5}{\left(2x + 0.02926x^3\right)^{2/3}}$ ----------------------------(2)

Equating 1 and 2 and solving for x we get,

x=4.9336 ft

Finally, $y_n = \frac{x^2}{6.75}$ or y_n= 3.606 ft

4.14
Given:
Circular cross section
D_0=8ft
S_0=1ft/m
Q=30ft^3/s
Concrete lined
Compute: flow depth, y

Solution: Assume steady-uniform flow, therefore y=y_n
S_0=1/5280=0.000189
n=0.013(from table4.1)
$nQ/(1.49S_0^{1/2})=(0.013)(30)/(1.49)(0.000189)^{1/2}=19.019$

1. Design curves

Using Figure 4.5 for $AR^{2/3}/D_0^{8/3}=19.019/8^{8/3}=0.0742$ we get $y_n/D_0=0.33$ or $y_n=0.33(8)=2.64$ ft.

2. Numerical solution

Expressing $AR^{2/3}$ in terms of the angle θ we get, $\frac{\left[1/8(\theta - Sin\theta)D_0^{\;2}\right]^{5/3}}{\left((1/2)\theta D_0\right)^{2/3}} = 19.019$

The solution is:

$\theta = 2.46 rad$, which corresponds to $\frac{D_0}{2}\left(1 - Cos\frac{\theta}{2}\right)$=2.64 ft. Therefore, y_n = 2.64 ft

4.15
Given:
Sewer of problem 3-15
D_0=5ft
S_0=2ft/mi=0.00038
Q=15ft^3/s
Concrete lined
Compute: Normal depth

Solution:
$AR^{2/}3= nQ/(1.49S_0^{1/2}) = (0.014)(15)/(1.49)(0.00038)^{1/2} = 7.23$

1. Design curves method

Use Figure 4.5 with $AR^{2/3}/D_0^{8/3}=7.23/5^{8/3}=0.09891$ and get, $y_n/ D_0=0.39$ or $y_n=1.95$ ft

2. Numerical solution

Expressing $AR^{2/3}$ in terms of the angle θ we get $\dfrac{\left[1/8(\theta - Sin\theta)D_0^{\ 2}\right]^{5/3}}{\left((1/2)\theta D_0\right)^{2/3}}$ =7.23

The solution is:

$\theta = 2.6861 rad$, which corresponds $to \dfrac{D_0}{2}\left(1 - Cos\dfrac{\theta}{2}\right)$=1.94 ft. Therefore, y_n =1.94 ft.

4.16

Given:
$Q=28m^3/s$
S=1
$B_0=8m$
$S_0=0.0001$
y=3m
Compute y if Q is doubled.

Solution:
Assuming steady-uniform flow compute Maning's coefficient using data for $Q=28m^3/s$ and Manning's equation.
Assuming constant n for the new flow we can compute the new flow depth.
$n= (AR^{2/3}S_0^{1/2})/Q$
For a trapezoidal channel,Table1.1 gives:
$A=(B_0+sy_n)y_n=(8+3)3=33$

$R=\dfrac{(B_0 + sy_n)y_n}{\left(B_0 + 2y_n\sqrt{1+s^2}\right)}=\dfrac{(8+3)3}{\left(8+2(3)\sqrt{2}\right)}=2.00$

and $n=33(2)^{2/3}(0.0001)^{1/2}/28=0.0187$
Section Factor for $56m^3/s$:

$AR^{2/3}=\dfrac{nQ}{\sqrt{S_0}}=\dfrac{(0.0187)(28x2)}{\sqrt{0.0001}}=104.76$

1. Design Curves method

Use Figure 4.5 for $\dfrac{AR^{2/3}}{B_0^{\ 8/3}} = 0.41\ and\ s = 1.0$ to get $y_n/B_0=0.56$
or, $y_n=0.56x8=4.48m$

2. Numerical solution

Solving for y_n from

$\left[(8+(1)y_n)y_n\right]^{5/3} - 104.76\left(8+(2)y_n\sqrt{2}\right)^{2/3} = 0$, we obtain
$y_n = 4.40m$

4.17
Given:
Long rectangular channel
Change in flow depth from 4ft to 5ft
Determine: Percentage change of rate of discharge.
From Manning's equation: $Q= (1.49/n)AR^{2/3}S_0^{1/2}$

At 4ft: $Q_1= (1.49/n)\dfrac{(By_1)^{5/3}}{(B+2y_1)^{2/3}}\ S_0^{1/2}$

At 5ft: $Q_2= (1.49/n)\dfrac{(By_1)^{5/3}}{(B+2y_1)^{2/3}}\ S_0^{1/2}$

Then $\dfrac{Q_1}{Q_2} = \left(\dfrac{y_1}{y_2}\right)^{5/3}\left(\dfrac{B+2y_2}{B+2y_1}\right)^{2/3}$

Assuming a wide rectangular channel:
$B+2y_2 \approx B$
$B+2y_1 \approx B$,

Then, $Q_1/Q_2= (0.8)^{5/3}=0.689$

or , $Q_1/Q_2=68.9$ % i.e., Q_1 is 68.9 % of Q_2

4.18
Assuming that the flow must be controlled by improving the channel conditions two possible solutions are:

a) Improve Lining
The conveyance of the channel can be increased by reducing the channel resistance. If the channel is lined, for example with concrete, the Manning's coefficient will decrease and the channel capacity will be increased. Different lining processes should be considered.

b) Increasing the flow area.
If the cross-section area is increased, the capacity of the channel will increase. For example, a trapezoidal section could be a good choice. If any case, the cost of excavation and other local conditions will dictate the viability of this option.

4.19
Given
Rectangular Channel
B=4m
$Q=9m^3/s$

S_0=0.005
n=0.014
Determine if the flow is sub-critical or supercritical

Solution:
i) Compute Critical Depth:
$y_c = \sqrt[3]{Q^2/(gB^2)}$ (see problem 3.11)

$y_c = \sqrt[3]{9/(9.81x16)}$ =0.802m
ii) Assume steady-uniform flow and compute the normal depth:
$AR^{2/3} = \dfrac{nQ}{\sqrt{S_0}}$
$AR^{2/3}$=1.065
or, $\dfrac{(By_n)^{5/3}}{(B+2y_n)^{2/3}} = 1.065$
or, y_n=0.5m

3) Compute y_n and y_c
In this case $y_n < y_c$

4.20
Given:
Trapezoidal channel
B_0=20ft
s=1.5
Q=220cfs
S_0=0.00032
n=0.022
Determine if the flow is subcritical or super critical.

Solution:
i) Compute critical depth:
Section Factor Z=Q / $\sqrt{g}$ =220/ $\sqrt{32.2}$ =38.77

a) Design Curves Method
Use Figure 3.7 for $Z/B_0^{2.5}$=0.0217 and s=1.5
to obtain, y_c/B_0=0.076 or y_c=1.52ft

b) Solving by trial and error (or using numerical methods)
Solve for $A\sqrt{D}$ =38.77 or:

$$\frac{\left[(B_0 + sy_c)y_c\right]^{3/2}}{(B_0 + 2sy_c)^{1/2}} = 38.77$$

$$\frac{\left[(20 + 1.5y_c)y_c\right]^{3/2}}{(20 + 2(1.5)y_c)^{1/2}} = 38.77 \text{ or,} y_c = 1.49\text{ft}$$

ii) Compute Normal Depth:

Section factor: $\dfrac{nQ}{1.49\sqrt{S_0}}$ or, $\dfrac{0.022(220)}{1.49\sqrt{0.00032}}$ =181.59

Design Curves:

Use Figure 4.5 for $AR^{2/3}/B_0^{8/3}$ = 0.0616 and s=1.5 to get y_n/b_0=0.18 or y_n=(0.18)(20)=3.6ft

Trial and Error:

Solve for y_n from

$$\left[(B_0 + sy_n)y_n\right]^{5/3} - 181.59\left(B_0 + 2y_n\sqrt{1 + 1.5^2}\right)^{2/3} = 0$$

or, $\left[(20 + 1.5y_n)y_n\right]^{5/3} - 181.59\left(20 + 2y_n\sqrt{1 + 1.5^2}\right)^{2/3} = 0$

y_n=3.61 ft

iii) Compare y_n and y_c

The flow is subcritical as $y_n > y_c$.

4.21

Given:

Trapezoidal Channel

$Q=15m^3/s$

$B_0=10m$

S=2

$y_n=2m$

Compute the flow depth for $Q=20m^3/s$.

Solution:

Assuming steady-uniform flow and determine Manning's coefficient from the data for $15m^3/s$:

$A=(B_0+sy_n)y_n=28m^2$

$$R = \left(\frac{(B_0 + sy_n)}{B_0 + 2y_n\sqrt{(1 + s^2)}}y_n\right) = 1.478\text{m}$$

$n=AR^{2/3}S_0^{1/2}/Q = n/S_0^{1/2} = 2.422$

for, $Q=20m^3/s$, $AR^{2/3} = nQ/S_0^{1/2} = 2.422 \times 20 = 48.44$ or,

$$\left[(10 + 2y_n)y_n\right]^{5/3} - 48.44(10 + 2\sqrt{(5)}y_n)^{2/3} = 0$$

or, $y_n = 2.34m$

The flow depth y_n for $Q=20m^3/s$ is 2.34m

4.22
Given:
Compound Channel $S_0=0.001$
n=0.021(main channel)
n=0.039 (flood plains)
Compute i) Equivalent n (n_e)
ii) Velocity-head coefficient (α)
iii) Slope of the energy grade line (S_f)
Assuming that the flow is 5m, use Eq 4.35 for n_e and 4.47 for α. Alternatively, Eq 4.36, 4.37 or 4.38 may be used to estimate n_e.

The following Table shows the computations for the compound channel. It has been subdivided into sub-sections 1, 2 and 3.

Computation of n_e and α for compound channel

Section	n_i	P_i	A_i	R_i	K_i	$P_i n_i^{3/2}$	K_i^3/A_i^2
1	0.021	30	106	3.533	11708.88	0.091	$1.429x10^8$
2	0.039	7.236	6	0.829	135.77	0.056	$6.952x10^4$
3	0.039	7.236	6	0.829	135.77	0.056	$6.952x10^4$
$\sum$		44.47	118		11980.42	0.203	$1.430x10^8$

$n_e= (\sum P_i n_i^{3/2}/ \sum P_i)^{2/3}=(0.203/44.47)^{2/3}$
$n_e=0.028$
$\alpha = (\sum \{K_i^3/A_i^2\}) (\sum A_i)^2/ K_i=1.16$
The discharge should be known in order to compute S_f using Eq 4.48. However, if steady-uniform flow is assumed, the compound channel will have $S_f=S_0$

Thus, $S_f = S_0 = 0.001$

4.23
Given: Rectangular channel
B=12ft
y=3ft
n=0.035
So=0.001
Compute: i) Critical depth (yc) for the flow corresponding to y=3 ft
ii) Determine if the flow is critical, subcritical or super critical.
i) Compute the flow rate:
$Q=(1.49/n)AR^{2/3}S_0^{1/2}=(1.49/0.035)(12x3)\{(12x3/(12+6)\}^{2/3}(0.001)^{1/2}$
Q=76.93cfs or q=76.93/12=6.41cfs/ft

The critical depth is $y_c = \sqrt[3]{\frac{q^2}{g}}$

$y_c = \sqrt[3]{\frac{6.41^2}{32.2}}$ =1.085 ft

As $y_c < y$, flow is subcritical.

Chapter 5
GRADUALLY VARIED FLOW

5.1
Given:

Gradually varied flow equation: $\frac{dy}{dx}=\frac{S_o-S_f}{1-F^2}$

Wide rectangular channel
Manning and Chezy formulae
Prove that:

a) $\frac{dy}{dx}=S_o\frac{1-(y_n/y)^{10/3}}{1-(y_c/y)^3}$ using Manning's formula

and

b) $\frac{dy}{dx}=S_o\frac{1-(y_n/y)^3}{1-(y_c/y)^3}$ using Chezy's formula

Solution:
The geometric properties for a wide rectangular channel can be approximated by,
$A = by$
$P = b$
$R = by/b = y$ and
$D = y$
Therefore, $AR^{2/3}=by^{5/3}$ and the Froude number can be expressed as

$$F^2=\frac{Q^2}{2gb^2y^3}$$

a) Using Manning's equation the slope of the energy grade line for gradually varied flow is

$$S_f=\frac{nQ^2}{AR^{2/3}}=\frac{nQ^2}{b^2y^{10/3}} \qquad (1)$$

Also, for uniform flow

$$S_o=\frac{nQ}{b^2y_n^{10/3}}$$

or

$$S_oy_n^{10/3}=\frac{n^2Q^2}{b^2} \qquad (2)$$

When the flow is critical, the Froude number is one. Thus,

$$F_c^2=\frac{Q^2}{2b^2gy_c^3}=1$$

and

$$\frac{F^2}{F_c^2} = F^2 = \left(y_c / y\right)^3 \qquad (3)$$

Substituting Eqs 1, 2 and 3 in the gradually varied flow equation we get,

$$\frac{dy}{dx} = \frac{S_o - S_o\left(y_c / y\right)}{1 - \left(y_c / y\right)^3}$$

or $$\frac{dy}{dx} = \frac{S_o\left(1 - \left(y_n / y\right)\right)^{10/3}}{1 - \left(y_c / y\right)^3}$$

b) If Chezy's equation is used, then

$$AR^{1/2} = \frac{Q}{CS_f^{1/2}} = (by)y^{1/2}$$

Or, $S_f = \dfrac{Q^2}{C^2 b^2 y^3}$ and $S_o = \dfrac{Q^2}{C^2 b^2 y_n^3}$

or, $S_o y_n^3 = \dfrac{Q^2}{C^2 b^2}$

Combining the expressions for S_f and S_o we get

$$S_f = S_o\left(y_n / y\right)^3$$

Therefore, the gradually varied flow equation becomes

$$\frac{dy}{dx} = \frac{S_o\left(1 - \left(y_n / y\right)^3\right)}{1 - \left(y_c / y\right)^3}$$

5.2

Consider the control volume:

The momentum equation establishes that the sum of forces acting on the control volume must be equal to the net change in momentum inside the control volume, For the X-direction this is expressed mathematically as,

$$\rho(Q + dQ)(V + dV) - \rho QV = F_1 - F_2 - F_f + W \sin\theta \qquad (1)$$

Where,

F_1 =Pressure force at section 1

F_2 = Pressure force at section 2

F_f = Force due to friction

W = Weight of the fluid inside the control volume

Assuming a hydrostatic pressure distribution, the pressure forces may be expressed as:

$$F_1 = \gamma \bar{y} A$$

$$F_2 = \gamma\left(\bar{y} + dy\right)A + \gamma \frac{dAdy}{2} \approx \gamma\left(\bar{y} + dy\right)A$$

and the friction force as

$F_f = \tau_o P dx = \gamma R S_f P dx = \gamma A S_f dx$ (see Eq 4.7)

where,
γ = specific weight,
$\bar{y}$ = location of the centroid of the cross section perpendicular to the flow,
τ_o= shear stress at the channel wall,
P=wetted perimeter and,
S_f= slope of energy grade line.

Substituting the expression for F_1,F_2 and F_f into Eq 1, simplifying and neglecting second order terms like $dx\, dV$ we get,

$$\frac{QdV}{g} + \frac{Vq}{g}dx = Ady - AS_f dx + AS_o dx \qquad (2)$$

Where, $W \sin\theta$ was substituted by $\gamma AS_o dx$ and $\frac{Vqdx}{g}$ is the contribution of the lateral flow to the change in momentum inside control volume.

Further simplification leads to:

$$\frac{dy}{dx} = \left(S_o - S_f - \frac{qQ}{gA^2}\right) - \frac{Q}{gA}\frac{dV}{dx} \qquad (3)$$

But, $dV = \left(\frac{AdQ - QdA}{A^2}\right)$; therefore, the last term on the right hand side becomes

$$\frac{Q}{gA}dV = \frac{Qq}{gA^2} - \frac{Q^2}{gA^3}\frac{dA}{Dx} \qquad (4)$$

Recalling that for a prismatic channel

$$\frac{dA}{dx} = \frac{dA}{dy}\frac{dy}{dx} \quad \text{and} \quad \frac{dA}{dy} = B$$

Equation (4) becomes,

$$\frac{Q}{gA}dv = \frac{Qq}{gA} - \frac{Q^2}{gA^3}B\frac{dy}{dx}$$

substituting Eq 5 in Eq 3 and solving for *dy/dx* we get,

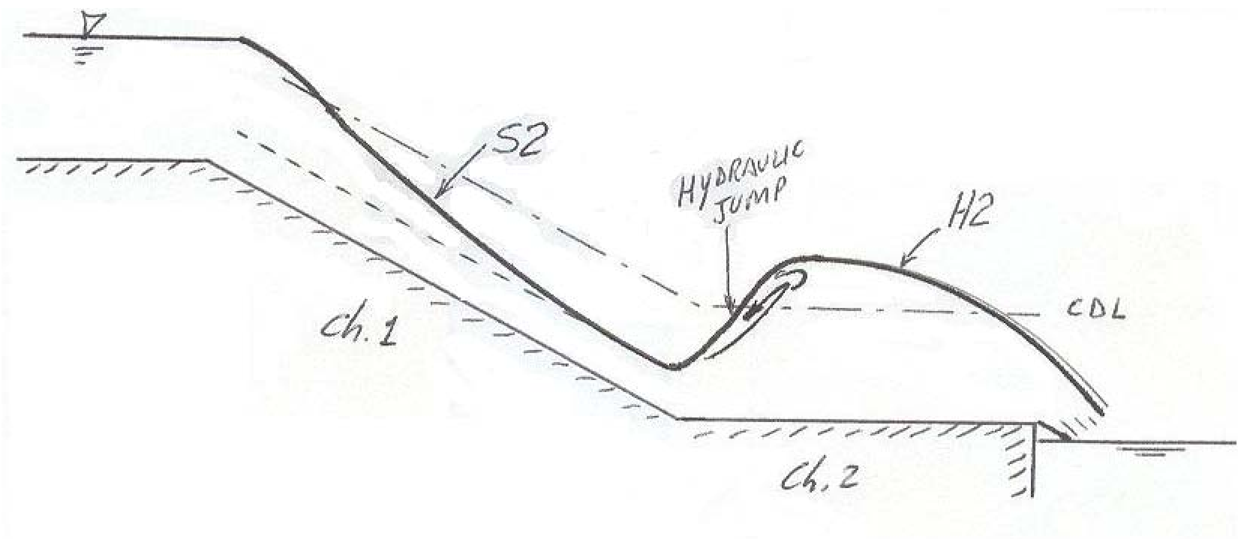
$$\frac{dy}{dx} = \frac{S_o - S_f - 2qQ^2/gA^2}{1 - \frac{Q^2B}{gA^3}} \qquad (6)$$

or, $$\frac{dy}{dx} = \frac{S_o - S_f - 2qQ^2/gA}{1 - F^2} \qquad (7)$$

where, F is the Froude number. When q is zero, Eqs 6 and 7 become the differential equation for gradually varied flow at constant discharge without lateral flow.

5.3

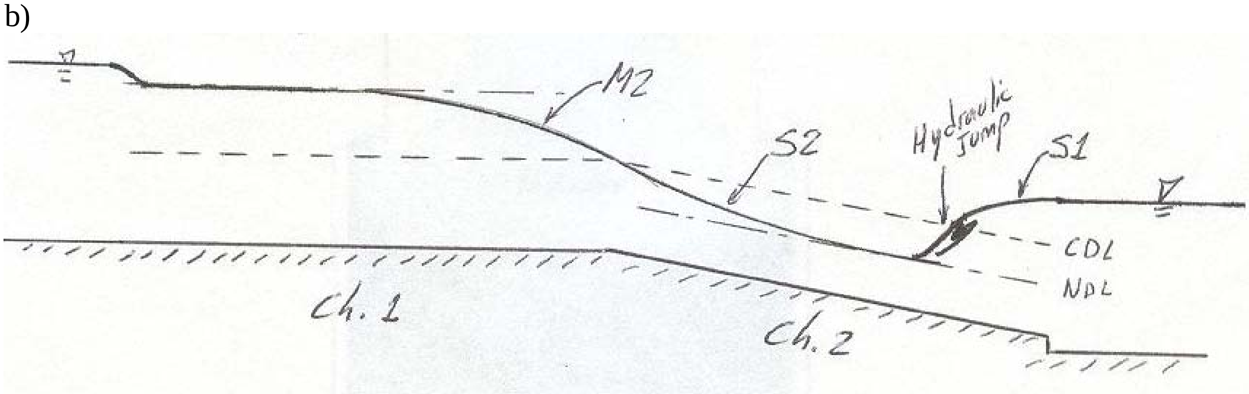
a)



Comment: At the upstream reservoir $y_{Res} > y_c$ and in channel 1 $y < y_c \Rightarrow$ There is a control section near reservoir and channel 1 is steep.

Channel 2 is horizontal $\Rightarrow y_n = \infty$ and $S_o < S_c \Rightarrow$ The flow must go from supercritical to subcritical through a hydraulic jump. At the free flow the critical depth must occur again.

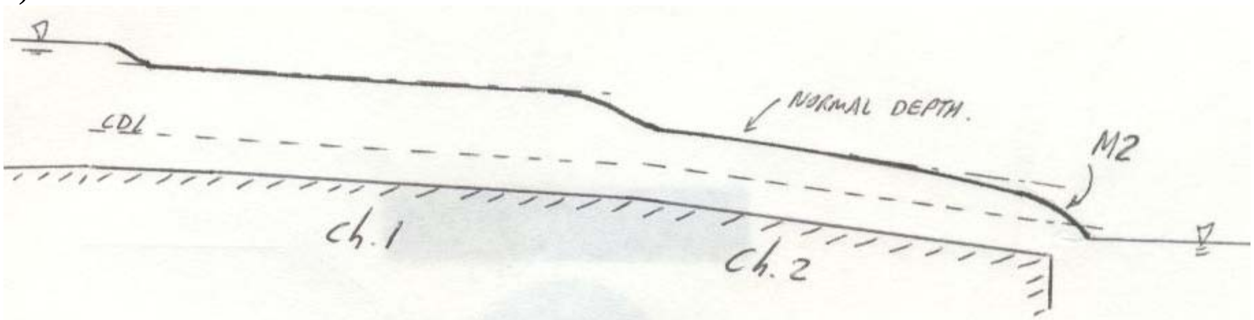
b)



Comment: At the upstream reservoir $y_{Res} > y_n > y_c \Rightarrow$ the control section is downstream and channel 1 is mild with subcritical flow.

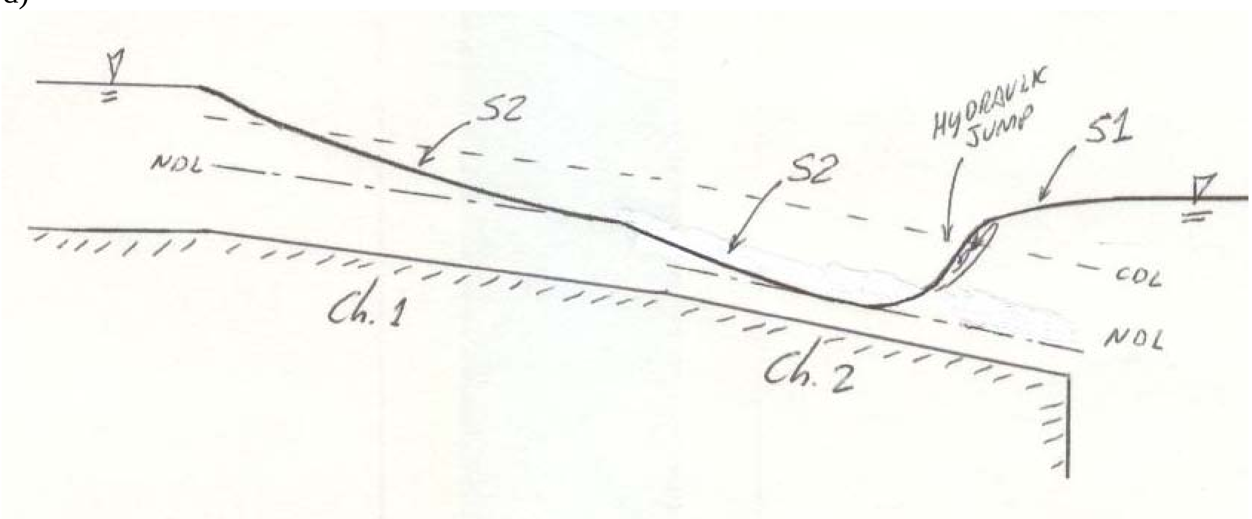
In channel 2, the flow becomes super critical after the slope changes from mild to steep. Because the down stream reservoir is very high a hydraulic jump is formed to raise the water level over the critical depth.

c)



Comment:
Both channels are mild ($y_n > y_c$ always)
The water profile crosses the critical depth just before the downstream reservoir.

d)



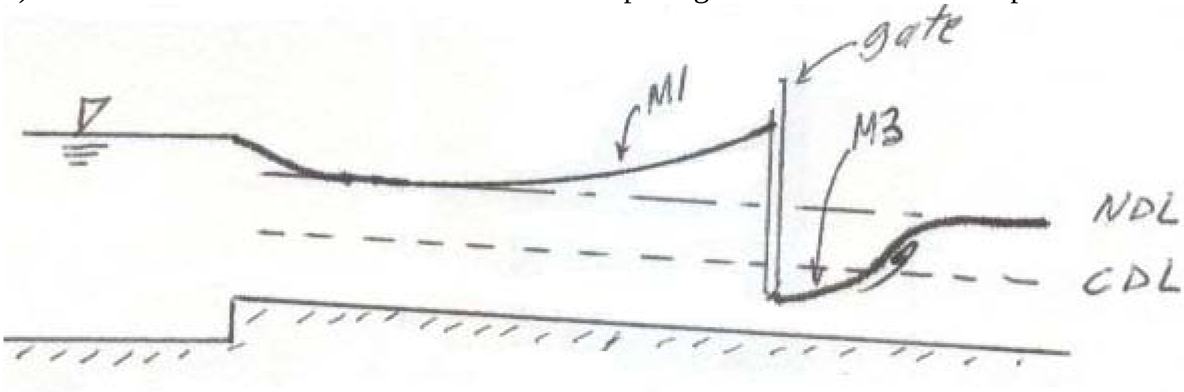
Comment:
Both channels are mild ($y_n < y_c$ always)
There is a control section near the upstream reservoir.
A hydraulic jump forms close to the down stream reservoir.

5.4
Analyze a sluice gate as a flow control device from a lake.
Assume that the channel bottom slope is
i) Mild
ii) Steep

Notation: NDL=Normal depth line, CDL= Critical depth line
The problem will be illustrated with several possible combinations of channel slope and gate openings close and far from the reservoir. The more appropriate location for the gate will be suggested based on the analysis.

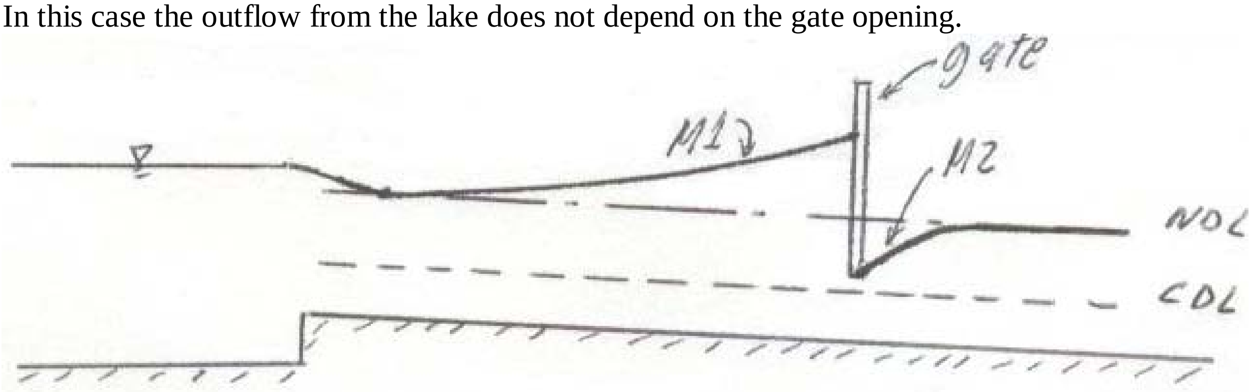
MILD CHANNEL

a) Gate located 'far' from the lake outlet. Gate opening less than the critical depth.



The control section is located at the gate.

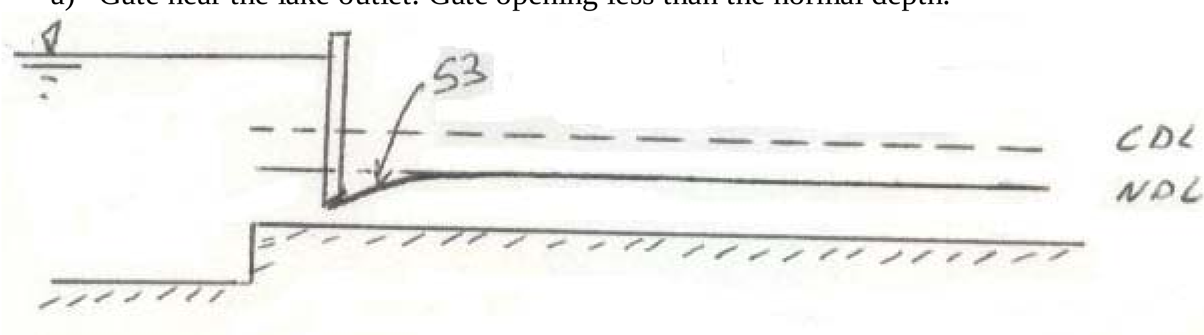
b) Gate far from the lake outlet. Gate opening greater than the critical depth.
In this case the outflow from the lake does not depend on the gate opening.



Cases (a) and (b) will be the same for a gate near the lake. The water level at the gate position is very close to the reservoir level, therefore; if the gate is located very far from the reservoir, the size of it will increase considerably. From this point of view to locate the gate near the reservoir is more convenient.

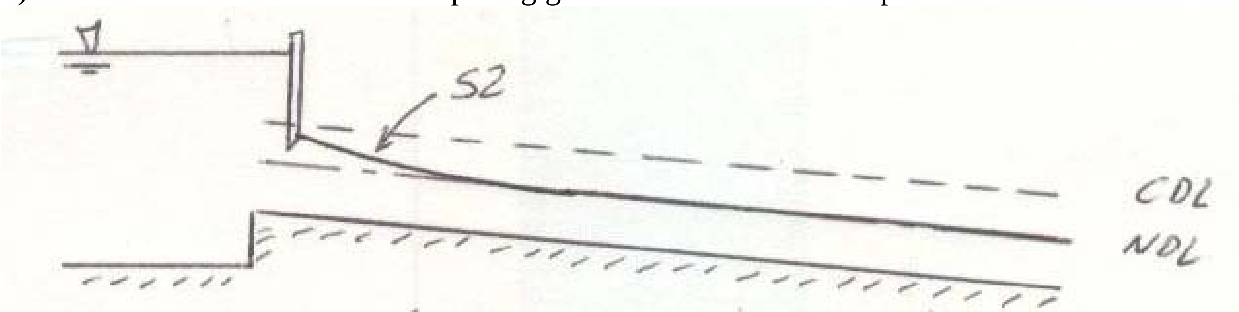
STEEP CHANNEL

a) Gate near the lake outlet. Gate opening less than the normal depth.



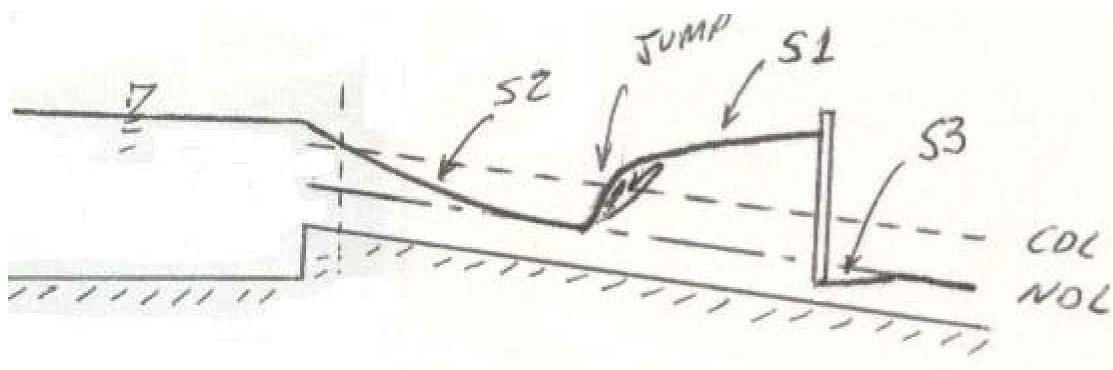
There is a control section at the gate.

b) Gate near the lake outlet. Gate opening greater than the normal depth.



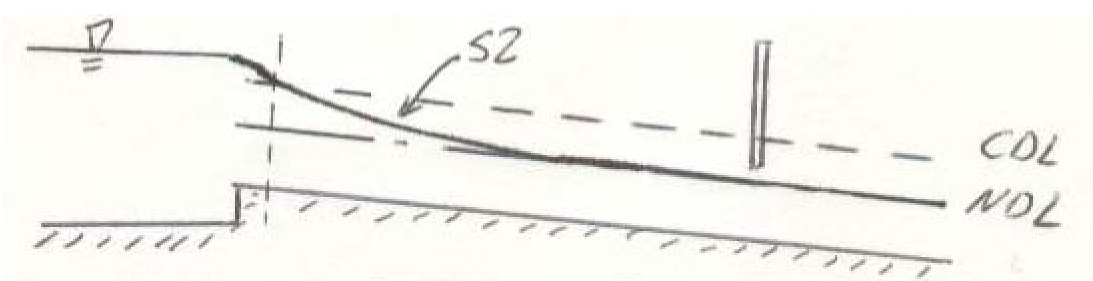
The control section is at the gate.

c) Gate far from channel entrance. Gate opening less than the normal depth.



There is a control section at the channel entrance.

d) Gate far from channel entrance. Gate opening greater than the normal depth.



There is a control section at the channel entrance.
From the cases presented before it can be concluded that the more appropriate location for the gate in order to establish a control section is near the reservoir for both steep and mild channels.

5.5
Given:
Manning's formula
Gradually varied flow equation
Show that $dy/dx \to \infty$ as $y \to 0$

Solution
From,

Manning's equation: $S_f = \dfrac{n^2Q^2}{A^2R^{4/3}}$

Froude: $F^2 = \dfrac{Q^2}{A^2gD} = \dfrac{Q^2B}{A^2g}$

For a wide rectangular channel, we have $R \approx y$ and A=By,
Then,

$$S_f = \frac{n^2Q^2}{B^2y^{10/3}}$$

and $F^2 = \dfrac{Q^2}{B^2y^3g}$

Substituting the expression for S_f and F^2 into the gradually varied flow equation we get:

$$\frac{dy}{dx} = \frac{\left(B^2y^{10/3} - n^2Q^2\right)g}{\left(B^2y^3g - Q^2\right)y^{1/3}}$$

Taking the limit when $y \to 0$ the result is

$$\lim_{y\to 0}\left(dy/dx\right) = \infty$$

5.6
Given:
5m wide rectangular channel (concrete-lined)
S_o =0.004
n = 0.013
H_o= 2m (lake level)
Compute:
i) Q in the canal (neglecting head losses)
ii) Q if S_o is changed to 0.001 and head losses are $0.1V^2/2g$.
i)Compute Q
1) Assume steep or critical slope, then

$y_c = \dfrac{2}{3}\mu_o$ = 4/3 m= 1.33 m

and $q^2 = y_o^3g$

or, $q = \sqrt{y_o^3g}$ = 4.822 m^3/m-s

$Q = qb$ = 24.111 m^3/s

2) Compute the critical slope

$$Q = \frac{1}{n} A_c R_c^{2/3} S_c^{1/2}$$

or, $$S_c = \left[\frac{Q_n (2y_c + b)^{2/3}}{(by_c)^{5/3}} \right]^2$$

$$S_c = \left[\frac{0.3134(8/3 + 5)^{2/3}}{(5 \times 4/3)^{5/3}} \right]^2$$

S_c= 0.00266

3) Compare S_o and S_c

In this case $S_o > S_c$, then the canal is steep and the flow will be,

Q = 24.11m^3/s

ii) If S_o =0.001 and K= 0.1 (minor loss co-efficient)

then,

1) Compute y_c by using the energy equation at the entrance,

$$H_o = y_c + \frac{V^2}{2g} + 0.1\frac{V^2}{2g} \quad \text{(assuming steep channel)}$$

$$H_o = y_c + 1.1 y_c / 2$$

$$y_c = 1.29 \text{ m}$$

2) Compute Q for steep channel,

$$q = \sqrt{y_c^3 g}$$

q = 4.589 m^3/m-s

$Q = qb$= (4.589)(5) =22.945m^3/s

Q=22.945m^3/s →This is the maximum discharge in the canal

3) Compute the critical slope

$$S_C = \frac{n^2 Q^2 (2y_C + b)^{4/3}}{(by_C)^{10/3}}$$

$$S_C = \frac{(0.08897)(12.577)}{499.49}$$

$$S_C = 0.00224$$

4) Compare S_O and S_C

In this case $S_O \langle S_C$, then the channel slope is mild and there is not control section at the entrance. Therefore, the previous analysis does not apply.

We can assume that the flow will reach normal depth near the lake then; Q is given by Manning's uniform flow formula. The flow depth can be obtained combining the energy and Manning's equation.
Doing this we get:

$$H = y + \frac{1+k}{2gn^2} R^{4/3} S_o$$

$$H = y + \frac{1+k}{2gn^2} \left(\frac{by}{2y+b} \right)^{4/3} S_o$$

Substituting $H = 2m,$ $k = 0.1$ $b = 5,$ $n = 0.013$ *and* $g = 9.81$
And solving for y we get the normal depth as
$y_n = 1.668m$
Q is obtained from Manning's equation and the answer is

$$Q = 20.33 m^3/s$$

5.7

Lakes A and B are connected by a 10m-wide rectangular channel.

$n = 0.013$
$S_o = 0.001$
$L = 2000m$

Sketch the water surface profile in the channel if:

i) Lake B is at EL. 155
ii) Lake B is at EL. 161

Computation of critical depth

$$y_c = \frac{2}{3} H_o \qquad \text{(Wide rectangular channel)}$$

$$H_o = 168 - (158 - (0.001)(2000)) = 8m$$

$$y_c = 5.33m$$

Unit discharge for critical flow:

$$q = \sqrt{g{y_c}^3} = \sqrt{9.81(5.33)^3}$$

$$q = 38.58 m^3/s - m$$

Total discharge: $Q_c = Bq = 3.77 m^3/s$

The critical slope is: $S_c = \dfrac{n{Q_c}^2}{A^2 R^{2/3}} = \dfrac{(0.013)(385.77)^2}{(10 \times 5.33)^2 \left(\dfrac{53.3}{10 + 10.67} \right)^{2/3}}$

$$S_c = 0.0025$$

Given that $S_c \rangle S_o$ the channel is classified as MILD.

To get the uniform flow conditions we solve for y_n

From:

$$Q = \frac{1}{n} AR^{2/3} S_o^{1/2} \qquad \text{(Manning's equation)}$$

And $$H_o = y + \frac{V^2}{2g} + k\frac{V^2}{2g} \qquad \text{(energy at the entrance)}$$

Assuming $k = 0$ (zero entrance losses) and using $V = Q/A$

We have

$$H_o = y + \frac{Q^2}{2gA^2} = y + \frac{R^{4/3} S_o}{2n^2 g}$$

Solving for y: $$y = H_o - \frac{R^{4/3} S_o}{2ng}$$

Or

$$y = H_o - \left(\frac{By}{B+2y}\right)^{4/3} \frac{S_o}{2gn^2}$$

Solving for y (by trial and error or by numerical methods) one gets

$y_n = 6.77m$

The flow profile is now sketched knowing the normal and critical depths in the channel.

In both cases, a M2 curve is produced.

For EL. 155 a free-fall condition at the downstream end exists.

5.8.

Given: Concrete-lined channel

N = 0.013

B = 15m (rectangular shape)

L = 15000m

Reservoir Elevations: Water surface = 129.65m

Bottom = 121.4m

i) Determine Q, sketch and label the water surface profile.

$S_o = 0.001$, Water elevation at downstream reservoir (y) is 109m

$y_c = 2/3 H_o$ $\qquad H_o = 129.65 - 121.4 = 8.25m$

$y_c = 5.5m$ (Critical depth)

$q = \sqrt{g\, y_c^{\,3}} = \sqrt{9.81(5.5)^2}$

$q = 40.4\ m^3/s-m$ (unit charge for critical depth)

$Q_c = qB = 606\,m^3/s$ (Critical flow)

The critical slope is

$$S_c = \frac{n^2 Q_c}{AR^{4/3}} = \frac{n^2 Q_c^{\,2}\, p^{4/3}}{A^{10/3}}$$

where, p = wetted perimeter = 26m

A = area = $82.5\,m^2$

$$S_c = \frac{(0.013)^2 (606)^2 (2\times 5.5+15)^{4/3}}{(15\times 5.5)^{10/3}}$$

S_c=0.00196

$S_c \rangle S_o$ = is a mild channel

Combining Manning's equation for uniform flow and the energy equation between the reservoir and the channel entrance we get:

$$H_{o=y_n+}\frac{R^{4/3}S_o}{2n^2 g}$$ (Equation 5.18 neglecting entrance losses)

Or

$$8.25 = y_n + \left(\frac{15y_n}{15+2y_n}\right)^{4/3} \frac{0.001}{2\times 9.81\times 0.013^2}$$

Solving for y_n we get $y_n = 6.634m$

Then, $Q = \frac{1}{n} A(R)^{2/3} S_o^{1/2}$

$$Q = \frac{99.52}{0.013}\left(\frac{99.52}{28.27}\right)^{\frac{2}{3}} (0.001)^{1/2}$$

$$Q = 560\,m^3/s$$

ii). $S_o = 0.008$

In this case the channel slope is STEEP, $S_c \langle S_o$. Thus, the discharge is given by $Q_c = 606\, m^3/s$ the normal depth is obtained directly from Manning's equation

$$AR^{2/3} = \frac{nQ_c}{\sqrt{S_o}}$$

$$15y_n = \left[\frac{15y_n}{15+2y_n}\right]^{2/3} = 88.08$$

Or $y_n = 3.35m$

5.9

Start with the total energy equation:

$$H = Z + y + \frac{\alpha V^2}{2g} \qquad [1]$$

The change in energy along the channel is:

$$\frac{dH}{dX} = \frac{dZ}{dX} + \frac{dY}{dX} + \frac{d\left(\alpha V^2\right)}{2gdX} \qquad [2]$$

Recall that $\frac{dZ}{dX} = -S_o$ (channel bottom slope)

$\frac{dH}{dX} = S_f$ (Energy grade line slope)

And $Q = VA$

Then for L=1, equation 2 becomes:

$$-S_f = -S_o + \frac{dY}{dX} + \frac{d\left(Q^2/A^2\right)}{2gdX} \qquad [3]$$

For constant discharge along the channel we have:

$$\frac{d\left(Q^2/A^2\right)}{2gdX} = \frac{Q^2}{2g}\frac{d\left(1/A^2\right)}{dX}$$

Or

$$\frac{d\left(1/A^2\right)}{dX} = -2A^{-3}\frac{dA}{dX}$$

But $A = f(y,x)$ (the cross-sectional are is also a function of y)

Therefore,

$$\frac{dA}{dX} = \frac{dA}{dX} + \frac{dA}{dY} dY/dX \quad \text{(Chain rule)}$$

Also

$B = dA/dY$ Then, equation 3 becomes:

$$-S_f = -S_o + dY\Big/dX - \frac{Q^2}{gA^3}\left(\frac{dA}{dX} + \frac{dA}{dY}\frac{dY}{dX}\right)$$

Or:

$$\frac{dY}{dX} = \frac{S_o - S_f + \frac{V^2}{gA}\frac{dA}{dX}}{1 - \frac{BV^2}{gA}}$$

5.10

For a wide rectangular, derive expressions for the channel bottom slope to be mild, step and critical.

By definition a mild channel satisfy $S_c \langle S_o$ and a steep channel satisfy $S_c \rangle S_o$

Therefore, we look for an expression for S_c in the terms of the critical flow conditions and determine the channel type (mild or steep) using these inequalities.

For a wide rectangular channel the hydraulic radius can be approximated as:

$R \cong y$

From Manning's equation we get,

$$Q = \frac{1.49}{n} AR^{2/3} S_c^{1/2} \qquad \text{Where} \qquad Q = qB$$

For critical flow conditions

$$q_c B = \frac{1.49}{n}(By_c) y_c^{3/2} S_c^{1/2}$$

Finally, $$S_c = \left[\frac{q_c n}{1.49 y_o^{5/3}}\right]^2$$

The critical flow depth and discharge may be computed using the methods of chapter 3.

5.11

Given:

Chute spillway blasted through rock (not lined) $S_o = 0.075(1.5\,ft/20\,ft)$

Water level at the entrance 10ft above the channel bottom.

Compute: Flow depth and discharge in the chute.

Solution:

Assume control at channel entrance, then:

$y_o = 2/3 H_o = 6.667\,ft$ (Critical depth)

$q_c = \sqrt{g y_c^3} = 97.68\ ft^3/s - ft$

$$Q_c = q_c B = 1.49(By)\left(\frac{By}{2y_c + B}\right)^{2/3} S_c^{1/2}$$

Simplifying and solving for S_c we get:

$$S_c = \left[\frac{q_c n}{1.49 y_c^{5/3}}\right]^2 \quad \text{(See problem 5.10)}$$

$S_c = 0.0094 \langle\langle S_o$ (Using n=0.035 for blasted rock).

The normal depth is given by Manning's equation as:

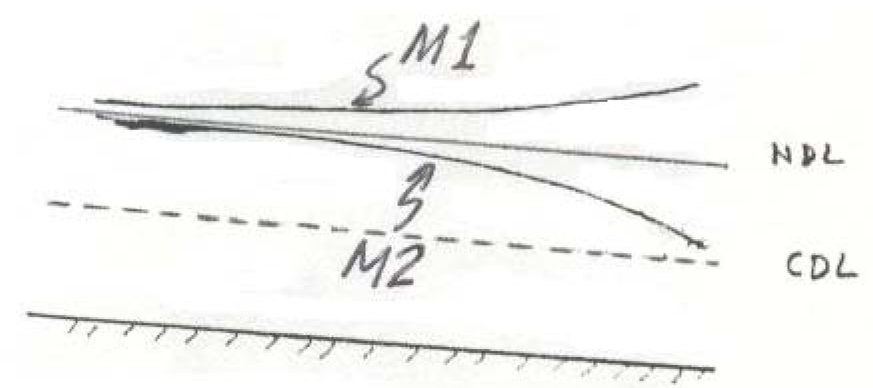
$$\left(\frac{By_n}{2y_n + B}\right)^{2/3}(By) = \frac{nqB}{1.49\sqrt{S_o}}$$

The result is $y_n = 1.147\,f = y_n \langle y \langle y_c \Rightarrow$

The channel is STEEP the flow is supercritical and the profile type is S2.

5.12

a.



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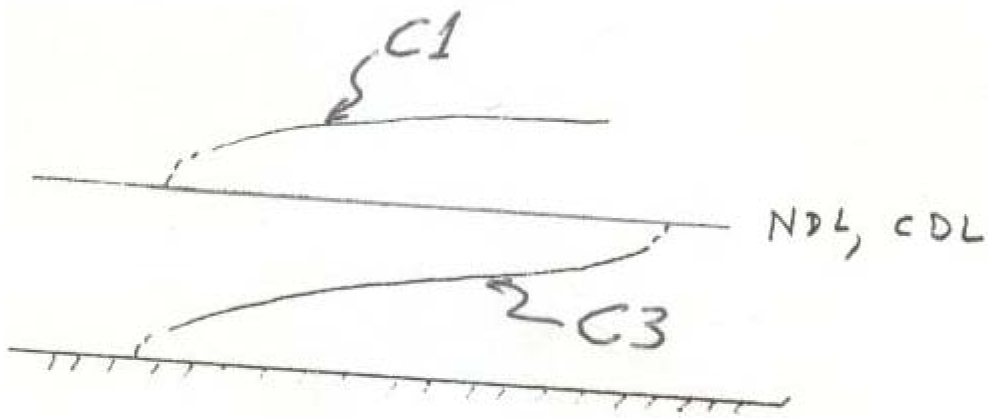
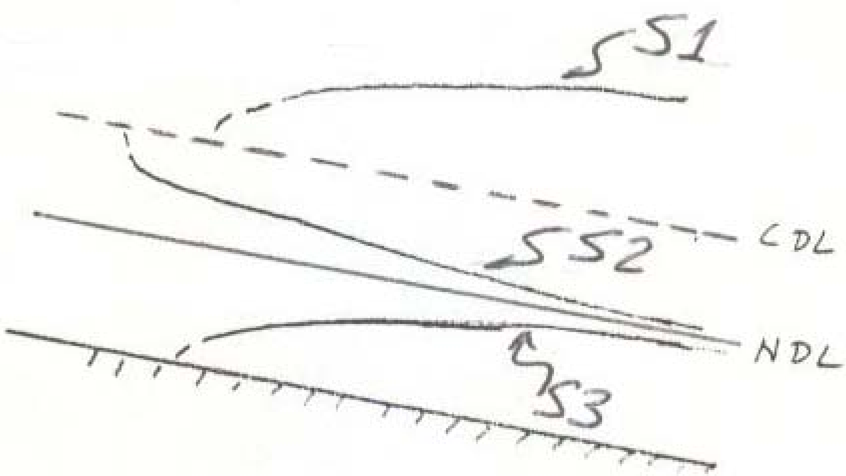
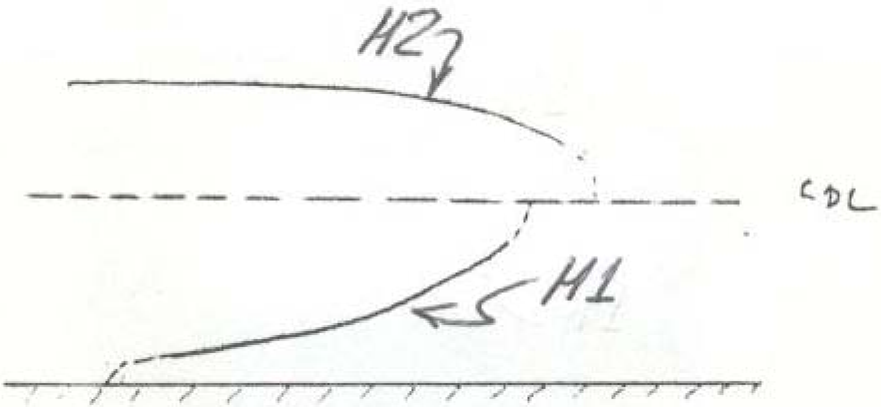


c.



d.



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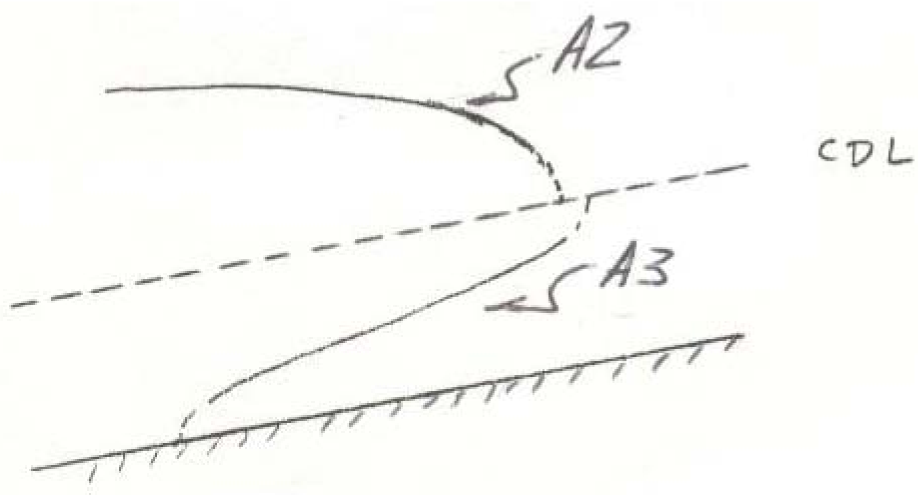
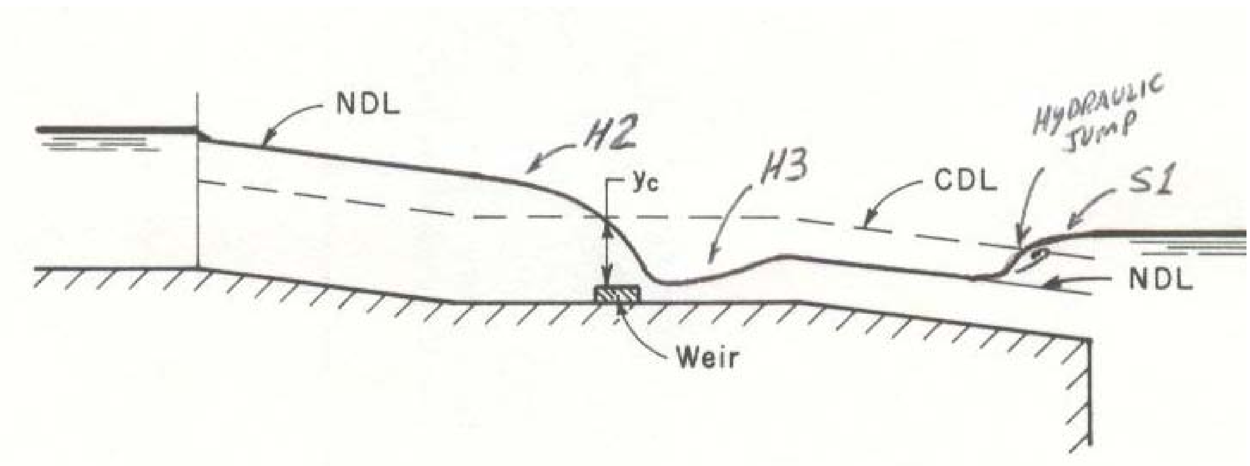


5.13



5.14.
Given:

Trapezoidal channels
Q = 800cfs
N = 0.028
S = 1
B = 15 ft
Compute: y_C and y_n sketch the water surface profile.
For the channel 1: $S_O = 0.0005$

Use Manning's equation to get the normal depth:

$$\frac{nQ}{1.49{S_o}^{1/2}} = AR^{2/3} \Rightarrow \frac{(0.028)(800)}{1.49(0.0005)^{1/2}} = AR^{2/3}$$

Or $\quad AR^{2/3} = 672.32$

Enter the design curve with

$$\frac{AR^{2/3}}{b^{8/3}} = \frac{672.32}{(15)^{8/3}} = 0.491$$

To get $y_n/b = 0.6 \Rightarrow y_n = 0.6 \times 15 = 9\,ft$

Or solve for y_n from:

$$672.32 = [(15 + y_n)y_n]^{5/3} / \left(15 + 2\sqrt{z}\,y_n\right)^{2/3}$$

The result is: $\quad y_n = 9.09\,ft$

To compute the critical depth, solve for y_c from:

$$[(15 + y_c)y_c]^{3/2}[15 + 2y_c]^{-1/2} = \frac{800}{\sqrt{32.2}} = 140.98$$

The solution is: $y_c = 4.05\,ft$

(Alternatively, you can use design curve for critical depth).

For channel 2 $\quad S_o = 0.05$

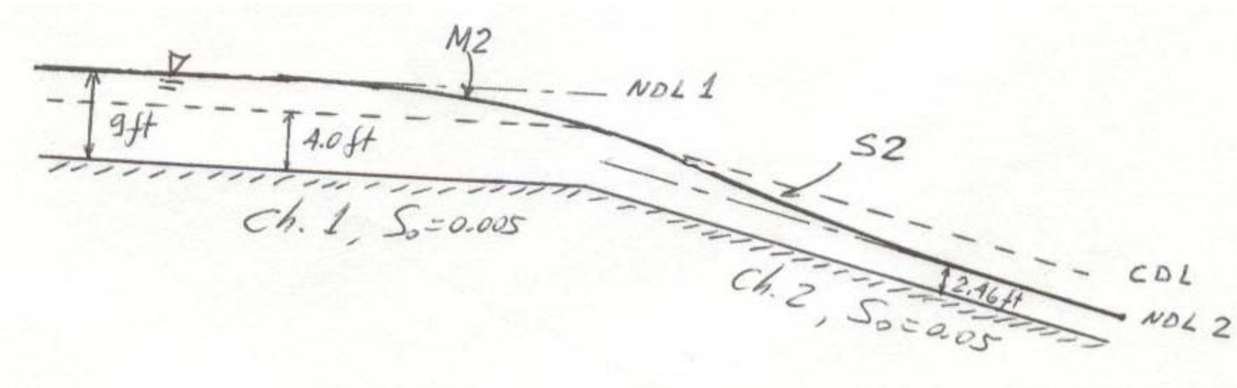
Normal depth: $AR^{2/3} = \frac{nQ}{1.49{S_o}^{1/2}} \Rightarrow \frac{(0.028)(800)}{1.49\sqrt{0.05}} = 67.232$

Solve for y_n from:

$$[(15 + y_n)y_n]^{5/3} / \left(15 + 2\sqrt{2}\,y_n\right)^{2/3} = 67.232$$

$$y_n = 2.46\,ft$$

In channel 1 the flow is sub critical and in channel 2 the flow is supercritical.



5.15

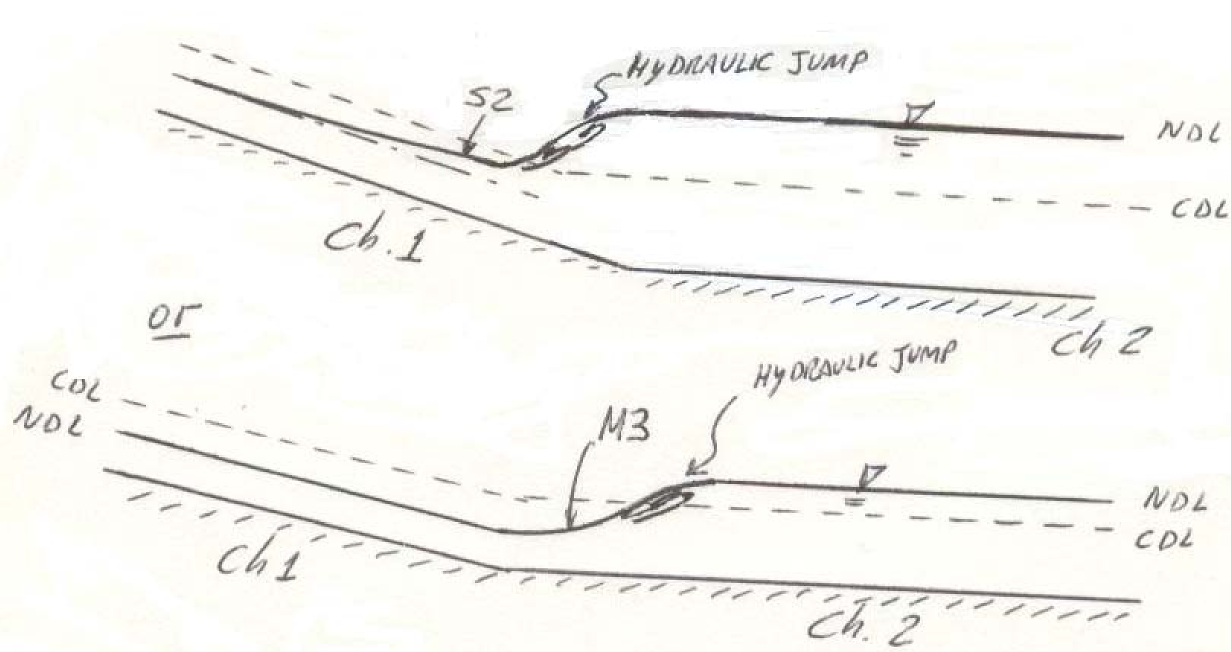
y_n for channel 1 was computed in problem 5-14

y_n For channel 2 is obtained from:

$$\left[(15+y_n)y_n\right]^{5/3} \Big/ \left[15+2\sqrt{2}y_n\right]^{2/3} = \frac{nQ}{1.49\sqrt{S_o}}$$

In this case, $\frac{nQ}{1.49\sqrt{S_o}} = 867.96$ and $y_n = 10.40\,ft.$

Channel 1 has supercritical flow and channel 2 has subcritical flow.



Chapter 6
COMPUTATION OF GRADUALLY VARIED FLOW

6.1

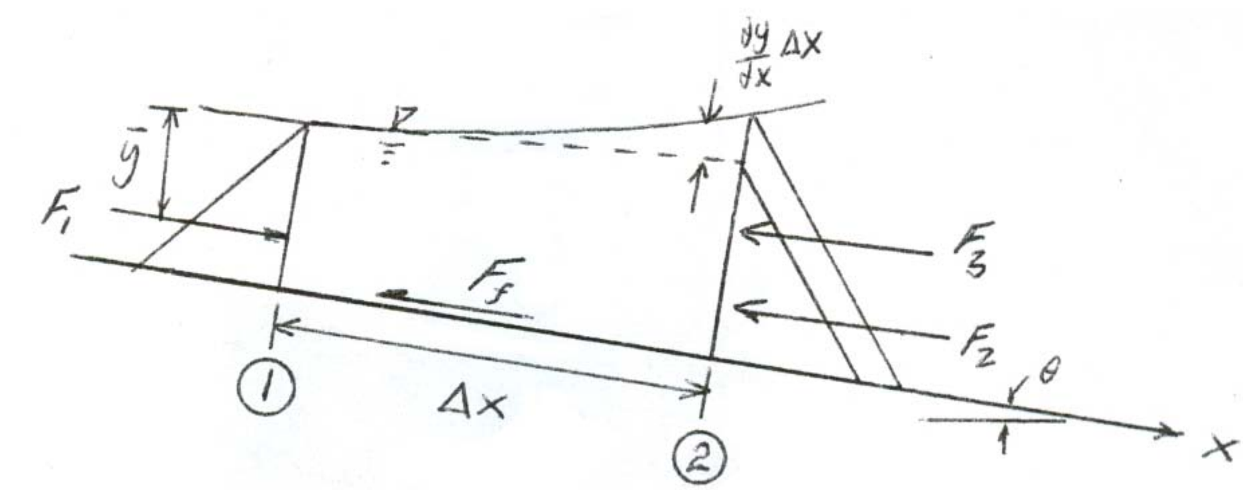
The forces acting on the control volume shown in the next figure are:

$F_1 = \mu A \bar{y}$ → Hydrostatic force at section 1

$F_2 + F_3 = \mu A \bar{y} + \mu A \frac{\partial x}{\partial x} \Delta x$ → Hydrostatic force at Section 2

$F_f = T_0 P \Delta x = \mu A \Delta x S_f$ → Friction force

$W_x = \mu A \Delta x \sin\theta = \mu A \Delta x S0$ → Weight component in the x-direction



The sum of forces gives

$$\sum F_x = F_1 - F_2 - F_3 - F_f + W_x$$

$$\sum F_x = \mu A \Delta x \left(-\frac{dy}{dx} - S_f + S_0\right)$$

According to the principle of conservations of momentum, the sum of the forces acting on the control volume plus the net rate of momentum is flux must be equal to the time rate of change of momentum inside the control volume.

The net rate of momentum influx produces

$$\rho AV^2 - \rho(AV^2 + \frac{d}{dx}(AV^2)\,\Delta x)$$

Or

$$\rho\frac{d}{dx}(AV^2)\,\Delta x$$

The time rate of change of momentum is

$$\frac{\partial}{\partial t}(\rho AV\Delta x)$$

Then, the principle of conservation of momentum becomes:

$$\frac{\partial}{\partial t}(\frac{\mu}{g}AV\Delta x) = \frac{-\mu}{g}\frac{\partial}{\partial x}(AV^2)\Delta x - \mu A\Delta x(\frac{dy}{dx} + S_f - S_0)$$

For steady-state conditions, $\frac{\partial}{\partial t}(\frac{\mu}{g}AV\Delta x) = 0$

Then, $\frac{1}{g}\frac{\partial}{\partial x}(AV^2) = A\,(S_0 - \frac{dy}{dx} - S_f)$--------------(1)

Expanding the derivative $\frac{\partial}{\partial x}(AV^2)$ we get,

$$\frac{\partial(AV^2)}{\partial x} = V^2\frac{\partial A}{\partial y}\frac{dy}{dx} + A\frac{\partial(d^2/A^2)}{\partial x}$$

$$= V2\frac{\partial A}{\partial y}\frac{dy}{dx} - 2V^2\frac{\partial A}{\partial y}\frac{dy}{dx} = -V^2\,B\frac{dy}{dx}$$

Then Eq 1becomes:

$$-\frac{-V^2}{g}B\frac{dy}{dx} = A(S_0 - \frac{dy}{dx} - S_f))$$

Or

$$\frac{-Q^2B}{gA^3}\frac{dy}{dx} = (S_0 - \frac{dy}{dx} - S_f)$$

Solving for dy/dx :

$$\frac{dy}{dx} = \frac{S0 - Sf}{1 - \frac{Q^2 B}{gA^3}}$$

This is same as Eq 6.3 with $\alpha = 1$.

6.2

Given

Rectangular Channel

10 mts. Wide

Concrete Lined

$S_0 = 0.01$

Constant take upstream, neglect entrance losses.

$H_0 = 6$m

Compute : (i) Flow depth 800 m downstream

(ii) Distance where y = 2.5 m

Method of solution : Direct Step

Mannin's coefficient : n = 0.013 for concrete

From example 5.3 : $y_0 = 4$m

Q = 250.6 $m^{3/5}$

Steep Channel

$y_n = 2.37$ m

The water surface profiles have critical depth at the entrance. A S2 curve will develop from the entrance to the normal depth.

Table P6.2 shows the detailed computations of the flow profile by Direct Step method. These computations may be easily made using a spreadsheet program.

The flow depth at 800 m downstream of the lake is 2.4/m. The flow depth will be 2.5 m at 470 m from the lake.

Table P6.2 – Direct Step Method

Table P6-2
Direct Step Method

Q = 250.6 m3/s B = 10 m
So = 0.01 n = 0.013

y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
m	m2	m	m/s				m	m	m	m
4	40	2.222	6.265	0.002287			6.000521			0
3.8	38	2.159	6.595	0.002634	0.002461	0.007539	6.016644	0.016123	2.14	2.14
3.6	36	2.093	6.961	0.003059	0.002846	0.007154	6.069779	0.053135	7.43	9.57
3.4	34	2.024	7.371	0.003587	0.003323	0.006677	6.168887	0.099108	14.84	24.41
3.2	32	1.951	7.831	0.004251	0.003919	0.006081	6.325814	0.156927	25.80	50.21
3	30	1.875	8.353	0.005101	0.004676	0.005324	6.556482	0.230668	43.32	93.54
2.8	28	1.795	8.950	0.006206	0.005653	0.004347	6.882696	0.326214	75.05	168.59
2.6	26	1.711	9.638	0.007675	0.006941	0.003059	7.334961	0.452265	147.83	316.41
2.5	25	1.667	10.024	0.008594	0.008134	0.001866	7.621334	0.286373	153.49	469.90
2.45	24.5	1.644	10.229	0.009111	0.008852	0.001148	7.782501	0.161167	140.41	610.31
2.4125	24.125	1.627	10.388	0.009527	0.009319	0.000681	7.912067	0.129566	190.22	800.53

6.3

Given

Trapezoidal channel

S_0 = 0.001

Q = 75 m^3/s

B_0 = 50m

s = 1.5

Control at downstream end that raises the water depth to 12 m.

n = 0.025

Determine : Amount by which the channel banks must be raised along its length.

Solution Compute the critical path.

$$\frac{Q}{\sqrt{g}} = A\sqrt{D}$$

$$\frac{75}{\sqrt{9.81}} = \frac{[(50+1.5y_0)y0]^{\frac{3}{2}}}{(50+1x1.5y_0)^{\frac{1}{2}}}$$

Solving for y_0 we get, y0 = 0.608 m

Compute the normal depth: $\frac{nQ}{\sqrt{S_0}} = AR^{\frac{2}{3}}$

$$\frac{(0.025)(75)}{\sqrt{0.001}} = \frac{[(50+1.5y_n)y_n]^{\frac{5}{3}}}{(50+2\sqrt{1+1.5^2}y_n)^{\frac{2}{3}}}$$

The normal depth, y_n = 10106 m

$y_0<y_n$ => Mild channel. The new backwater curve is M1.

Table P6.3 shows the computations of the new profile. The last column (Dh) is the minimum distance by which the banks must be raised if the flow was uniform before the construction of the central structure.

A freeboard must be provided for safety reasons. The water depth will be normal at 11.8 km upstream from the central structure.

Table P6-3
Direct Step Method

Q = 75 m3/s B = 50 m
So = 0.001 n = 0.025 z = 1.5

y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2	Dh
m	m2	m	m/s				m	m	m	m	m
12	816	8.749	0.092	2.929E-07			12			0	10.89
11	731.5	8.159	0.103	4.001E-07	3.465E-07	0.0009997	11.001	-0.999895	-1000.24	-1000.24	9.89
10	650	7.553	0.115	5.615E-07	4.808E-07	0.0009995	10.001	-0.999857	-1000.34	-2000.58	8.89
9	571.5	6.931	0.131	8.145E-07	6.88E-07	0.0009993	9.0009	-0.999801	-1000.49	-3001.07	7.89
8	496	6.291	0.151	1.231E-06	1.023E-06	0.000999	8.0012	-0.999712	-1000.74	-4001.80	6.89
7	423.5	5.629	0.177	1.958E-06	1.594E-06	0.0009984	7.0016	-0.999567	-1001.16	-5002.97	5.89
6	354	4.942	0.212	3.333E-06	2.645E-06	0.0009974	6.0023	-0.999311	-1001.96	-6004.93	4.89
5	287.5	4.226	0.261	6.225E-06	4.779E-06	0.0009952	5.0035	-0.998819	-1003.62	-7008.54	3.89
4	224	3.477	0.335	1.33E-05	9.763E-06	0.0009902	4.0057	-0.997755	-1007.59	-8016.14	2.89
3	163.5	2.688	0.459	3.518E-05	2.424E-05	0.0009758	3.0107	-0.994989	-1019.71	-9035.85	1.89
2	106	1.853	0.708	0.0001375	8.634E-05	0.0009137	2.0255	-0.985209	-1078.31	-10114.16	0.89
1.5	78.375	1.414	0.957	0.0003605	0.000249	0.000751	1.5467	-0.478843	-637.59	-10751.74	0.39
1.11	57.3482	1.062	1.308	0.0009866	0.0006735	0.0003265	1.1972	-0.3495	-1070.58	-11822.32	0

6.4

Given:

5 km long canal with free over fall at the downstream end.

Y_0 = 4m (at the fall)

n = 0.013

Entrance loss $= 0.2\dfrac{V^2}{2g}$

B_0 = 8.0 m

s = 1.5

S0 = 0.0001

Determine : The minimum water level held in the lake for these conditions.

Solution : Compute the channel discharge using the critical depth relation

$$A\sqrt{D} = \frac{Q}{\sqrt{g}}$$

Where A = (8+4x1.5)4 = 56

$$D = \frac{(8+4x1.5)4}{8+2x4x1.5} = 2.8$$

And Q $= 56\ \sqrt{2.8}\sqrt{9.81}$

Q = 293.49 m^3/s

Table P6.4 shows the computations of the flow profile using Standard Step Method.

The water depth downstream of the entrance section is 6.34m. Using the velocity-head of this section the local losses are :

$$0.2\ \frac{V^2}{2g} = (0.2)(0.356) = 0.071$$

The minimum water level in the lake is :

H_{lake} = 6.34 + 0.07 = 6.41 m

Table 6-4

Standard Step Method

x m	y m	A m2	P m	V m/s	V2/2g m	z m	H m	Sf1	Sf ave	Dx m	hf m
0	4.000	56.00	22.42	5.241	1.400	0.00	5.400	0.001370			
-100	4.513	66.65	24.27	4.403	0.988	0.01	5.511	0.000852	0.001110	-100	0.1111
-500	5.021	77.98	26.10	3.763	0.722	0.05	5.793	0.000557	0.000704	-400	0.2818
-1000	5.340	85.49	27.25	3.433	0.600	0.10	6.041	0.000434	0.000495	-500	0.2476
-2000	5.730	95.23	28.68	3.082	0.484	0.20	6.419	0.000324	0.000379	-1000	0.3791
-3000	5.990	101.73	29.60	2.885	0.424	0.30	6.714	0.000271	0.000298	-1000	0.2984
-4000	6.184	106.83	30.29	2.747	0.385	0.40	6.968	0.000238	0.000255	-1000	0.2545
-5000	6.340	110.96	30.85	2.645	0.358	0.50	7.195	0.000215	0.000226	-1000	0.2262

6.5

Given

Natural stream with the following cross section:

n = 0.035
Q = 80 m^3/s
Flow depth at a bridge downstream = m
S_0 = 0.0002

Determine : Flow depth 3.0 km upstream of the bridge.
Solution:

First, determine the type of flow profile by comparing the normal depth, critical depth and the water elevation at the bridge.
For the normal depth, we solve for y_n from

$$\frac{nQ}{\sqrt{S_0}} = \frac{A^{\frac{S}{3}}}{P^{\frac{2}{3}}}$$

where

$$\frac{nQ}{\sqrt{S_0}} = \frac{(0.035)(80)}{\sqrt{0.0002}} = 198$$

Then

$$\frac{[(10+2y_n)y_n]^{1.667}}{(10+2\sqrt{5}y_n)^{0.6667}} = 198 \quad assu\min g \quad y_n < 5_m$$

The solution is y_n = 4.833 m

For the critical depth we solve,

$$A\sqrt{D} = \frac{Q}{\sqrt{g}}$$

Or

$$\frac{[10+2y_0)y_0]^{\frac{3}{2}}}{[10+2x2y_0]^{\frac{1}{2}}} = 25.54 \qquad \text{assuming } y_0 < 5$$

Or, y_0 = 1.662 m
Given that $y_{bridge} > y_u > y_0$, the channel is MILD and the curve is M1 type.

Table P6.5 shows the computations of the backwater flow profile using Standard Step method.
At 3km upstream from the bridge, the water depth is y = 7.46 m

Table 6-5

Standard Step Method

x	y	A	P	V	V2/2g	z	H	Sf1	Sf ave	Dx	Hf
m	m	m2	m	m/s	m	m	m			m	m
0	8.00	244.00	55.84	0.320	0.00548	0.00	8.0055	1.8E-05			
-100	7.98	243.07	55.79	0.330	0.00552	0.02	8.0073	1.9E-05	1.9E-05	-100	0.00198
-300	7.95	241.22	55.69	0.330	0.00561	0.06	8.0111	1.9E-05	1.9E-05	-200	0.00378
-500	7.89	238.46	55.54	0.335	0.00574	0.12	8.0169	2E-05	1.9E-05	-300	0.00580
-1000	7.82	234.81	55.33	0.341	0.00590	0.20	8.0250	2.1E-05	2E-05	-400	0.00810
-1500	7.73	230.28	55.08	0.347	0.00620	0.30	8.0360	2.2E-05	2.1E-05	-500	0.01070
-2000	7.64	225.80	54.83	0.354	0.00640	0.40	8.0470	2.3E-05	2.2E-05	-500	0.01130
-2500	7.55	221.37	54.58	0.361	0.00660	0.50	8.0590	2.5E-05	2.4E-05	-500	0.01200
-3000	7.46	216.99	54.33	0.369	0.00692	0.60	8.0720	2.6E-05	2.5E-05	-500	0.01270

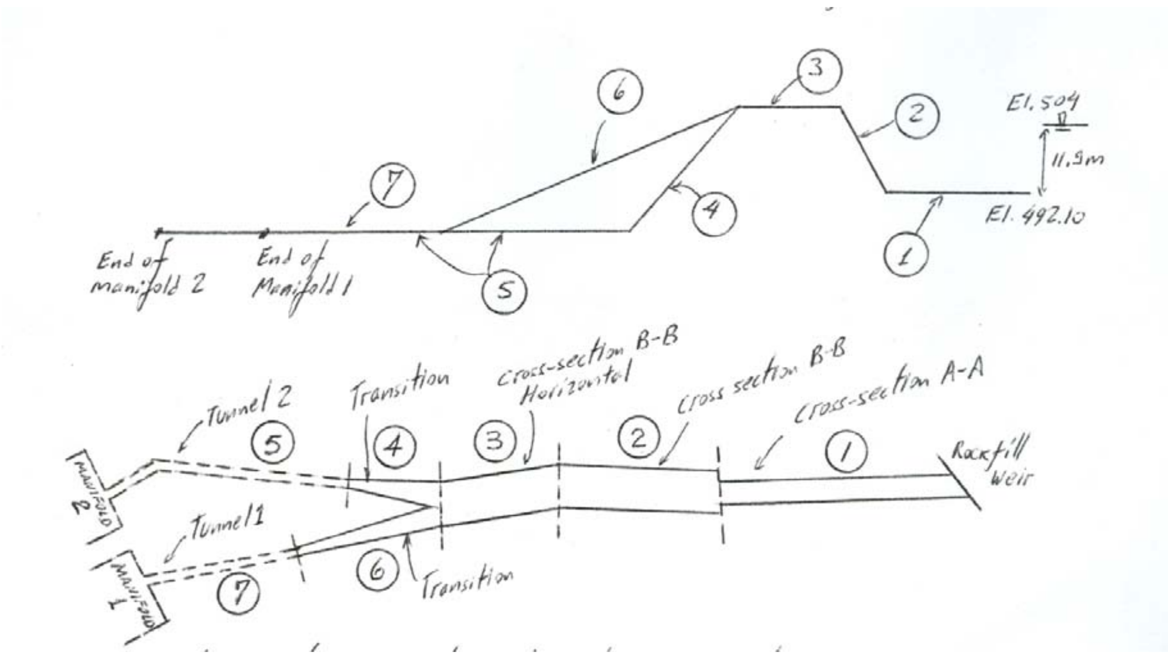
6.14

Given : The tailrace system of a hydropower plant shown in the next figure, with

Q = 1688m3/5

Downstream water elevation = 504.00 m.

Determine : The water level in each manifold.



The system was divided into 7 sections. Table 1 shows the channel properties for each section. The following assumptions are made to solve this problem.

1. All channels and tunnels are concrete-lined with n = 0.013
2. The transitions are assumed rectangles in cross-section.
3. The horse-shoe section is "standard with D = 18 m (see figure below)
4. The flow through each turbine is the same : Q = 1688/2
   or Q = 844 m^3/s

Standard horse-shoe section.

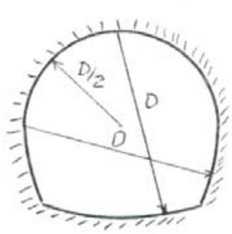
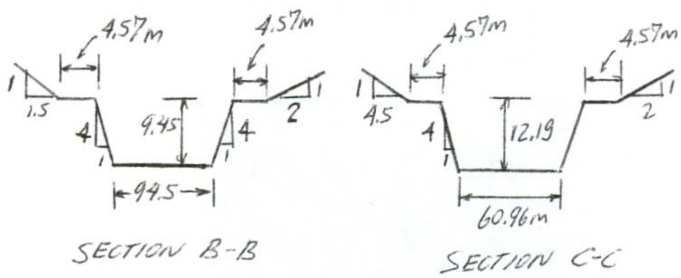


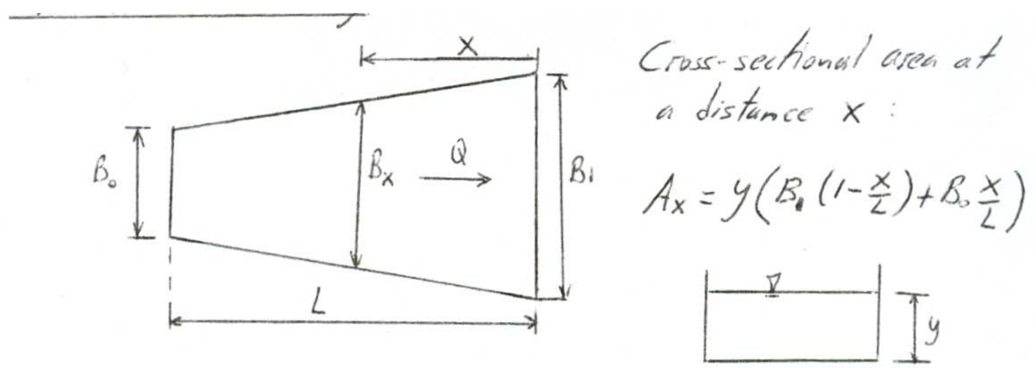
Table 1: Channel Properties

Channel N^2 and section	Upstream Elev. (m)	Downstream Elev. (m)	L (m)	S_0	Q (m^3/s)
1 C-C	492.56	492.10	502.92	0.000915	1688
2 B-B	496.52	492.56	137.16	0.028871	1688
3 B-B	496.52	496.52	97.54	0.0	1688
4 Transition 2	488.50	496.52	122.0	-0.06245	844

5 Horse-shoe 2	489.50	188.90	573.0	.0001047	844
6 Transition 1	488.90	496.52	198.0	-0.03848	844
7 Horse-shoe 1	489.50	488.90	405.0	0.001431	844



TRANSITIONS GEOMETRY



The backwater profile is started at compound section A-A with a water depth of 11.9m of the weir. The computations are carried out upstream using Standard Step Method as shown in Table 2. This corresponds to channel N2 1 in Table 1.

The energy and continuity equations are used at the junction of Channel 1 and channel 2 to obtain the elevation at the beginning of channel 2, as follows:

CHANNEL FUNCTION ANALYSIS.

Energy $E_1 = E_2$ + Losses

$$E_A + y_A + \frac{V_A^2}{2g} = E_B + y_B + (1+k)\frac{V_B^2}{2g} \quad \text{---------} \ (1)$$

Continuity : QA = QB

$V_A A_A = V_B A_B = Q$ --------- (2)

Assuming negligible losses at the junction (k=0) and combining Eq. 1 and Eq. 2, we get,

$$Y_A + \frac{Q_A^{\;2}}{2g}\frac{1}{A_A^{\;2}} = Z_B + y_B + \frac{V_B^{\;2}}{2g} = H_B \text{ ----- (3)}$$

H_2 is known from the last line in Table 2:

H_B = 504.275 m

If the bottom surface at the junction is continuous we have,

E_A = E_B = 492.56 m

Then Y_A can be obtained from Eq. 3 as

Y_A = 11.607 m

and A_A = 1158.37 m2

V_A = 1.457 m/s

Table 3 shows the computation of the water profile in channel 2 beginning with Y_A = 11.607m. Table 4 continues the computations for channel 3 (horizontal channel). The standard step method was used in both cases.

An energy analysis similar to the one made for the junction was made for the branch where channel 3 joins channels 4 and 6. The details are as follows:-

Energy Equation (neglecting losses):

$$Z_B + Y_B + \frac{V_B^{\;2}}{2g} = H_A \text{ ----------- (4)}$$

$$Z_C + Y_C + \frac{Vc^2}{2g} = H_A \text{ ----------- (5)}$$

Continuity Equation :

$Q_A = Q_B + Q_C$ ---------- (6)

and

$Q_B = Q_C = Q_A/2$ ---------- (7)

From Eq. 4 :

$$Y_B + \frac{Q_B^{\ 2}}{2gA_B^{\ 2}} = H_A\text{-}Z_B \text{ ------------ (8)}$$

Using Q_B = 844 m^3/s and HA = 504.288 m and Z_B = 496.52 m from the last section of table 4, to solve for Y_B in eq. 8, we get :

$$Y_B = 7.706 \text{ m}$$

Similarly we have Y_A = 7.706 m. There are the downstream depth for channels 4 and 6.

Assuming that the width of the downstream section in the rectangular transitions is equal to the top width of the main channel of section B-B we have, for y=9.45m, the width is 99.23m. The width of the upstream end of the transition is equal to the diameters of the standard horse-shoe section, this is 18 m. The channel is increased linearly between these two values. Table 1 shows the length of each transition.

The equation for gradually varied flow in a channel with variable cross-section derived in problem 5-9 was used here. The Improved Euler method with the modifications required for a rectangular transition was used in the computations. Table 5 and 6 show these results.

Finally, the Direct Step Method with the help of a table of the geometric properties of a standard horse-shoe section was used to compute the water surface profile in the tunnels. Table 7 and 8 show the computations. Interpretation between the last two lines of these table gives:

From Table 7, at manifold 1, y = 16.75 m (Elev. 506.25m)

From Table 8, at manifold 2, y = 16.71 m (Elev. 506.21m)

Table 2

Standard Step Method for Channel #1 (Section A-A)

x	y	A	P	V	V2/2g	z	H	Sf	Save	Dx	hf
(m)	(m)	(m2)	(m)	(m/s)	(m)	(m)	(m)			(m)	(m)
0.00	11.900	760.830	85.492	2.219	0.251	492.100	504.251	4.5E-05			
-50.00	11.855	757.783	85.399	2.228	0.253	492.146	504.253	4.6E-05	4.55E-05	-50.00	0.00227
-100.00	11.809	754.740	85.305	2.237	0.255	492.192	504.255	4.62E-05	4.61E-05	-50.00	0.00230
-200.00	11.718	748.659	85.117	2.255	0.259	492.283	504.260	4.74E-05	4.68E-05	-100.00	0.00468
-300.00	11.627	742.583	84.930	2.273	0.263	492.375	504.265	4.85E-05	4.79E-05	-100.00	0.00479
-400.00	11.536	736.512	84.742	2.292	0.268	492.466	504.270	4.97E-05	4.91E-05	-100.00	0.00491
-502.92	11.443	730.269	84.549	2.311	0.272	492.560	504.275	5.1E-05	5.04E-05	-102.92	0.00518

Table 3

Standard Step Method for Channel #2 (Section B-B)

x	y	A	P	V	V2/2g	z	H	Sf	Save	Dx	hf
(m)	(m)	(m2)	(m)	(m/s)	(m)	(m)	(m)			(m)	(m)
0.00	11.607	1157.240	131.833	1.459	0.108	492.560	504.275	1.99E-05			
-25.00	10.868	1072.250	128.850	1.574	0.126	493.280	504.276	2.48E-05	2.23E-05	-25.00	0.00056
-50.00	10.125	989.270	125.850	1.706	0.148	494.000	504.276	3.1E-05	2.81E-05	-25.00	0.00070
-75.00	9.376	908.020	113.830	1.859	0.176	494.730	504.277	3.67E-05	3.41E-05	-25.00	0.00085
-100.00	8.622	833.410	112.270	2.025	0.209	495.447	504.278	4.79E-05	4.23E-05	-215.00	0.00106
-137.16	7.481	721.000	109.920	2.341	0.279	496.520	504.280	7.55E-05	6.17E-05	-37.16	0.00229

Table 4

Standard Step Method for Channel #3 (Section B-B, Horizontal)

x	y	A	P	V	V2/2g	z	H	Sf	Save	Dx	hf
(m)	(m)	(m2)	(m)	(m/s)	(m)	(m)	(m)			(m)	(m)
0.00	7.481	720.950	109.920	2.341	0.279	496.520	504.280	7.55E-05			
-25.00	7.483	721.146	109.930	2.341	0.279	496.520	504.282	7.54E-05	7.55E-05	-25.00	0.00189
-50.00	7.485	721.347	109.931	2.340	0.279	496.520	504.284	7.54E-05	7.54E-05	-25.00	0.00189
-75.00	7.487	721.547	109.935	2.339	0.279	496.520	504.286	7.53E-05	7.53E-05	-25.00	0.00188
-97.54	7.489	721.727	109.939	2.339	0.279	496.520	504.288	7.53E-05	7.53E-05	-22.54	0.00170

Table 5
IMPROVED EULER METHOD FOR RECTANGULAR TRANSITIONS
TRANSITION No 1
B0= 99.2 M
S = .0
SO = -.0385
N = .013
Q = 844.00 M3/S
YD = 7.706 M
BUP= 18.0 M

X m	B m	A m2	Y m
.0	99.230	764.666	7.706
-10.0	95.127	770.822	8.103
-20.0	91.025	773.702	8.500
-30.0	86.922	773.323	8.897
-40.0	82.820	769.699	9.294
-50.0	78.717	762.841	9.691
-60.0	74.615	752.759	10.089
-70.0	70.512	739.462	10.487
-80.0	66.410	722.957	10.886
-90.0	62.307	703.252	11.287
-100.0	58.205	680.356	11.689
-110.0	54.102	654.275	12.093
-130.0	45.897	592.882	12.918
-140.0	41.795	557.293	13.334
-150.0	37.692	518.579	13.758
-160.0	33.590	476.773	14.194
-180.0	25.385	385.070	15.169
-198.0	18.000	293.445	16.303

Table 6
IMPROVED EULER METHOD FOR RECTANGULAR TRANSITIONS
TRANSITION No 2
B0= 99.2 M
S = .0
SO = -.0625
N = .013
Q = 844.00 M3/S
YD = 7.706 M
BUP= 18.0 M

X m	B m	A m2	Y m
.0	99.230	764.666	7.706
-10.0	92.572	773.060	8.351
-20.0	85.914	772.848	8.996
-30.0	79.255	764.093	9.641
-40.0	72.597	746.841	10.287
-50.0	65.939	721.131	10.936
-60.0	59.281	687.001	11.589
-70.0	52.623	644.488	12.247
-80.0	45.964	593.642	12.915
-90.0	39.306	534.539	13.599
-100.0	32.648	467.316	14.314
-110.0	25.990	392.265	15.093
-122.0	18.000	293.762	16.320

Table 8
Direct Step Method
STANDARD HORSE SHOE SECTION FOR TUNNEL #2
Q = 844 m3/s D = 18 m
So = 0.00105 n = 0.013

y/D	A/D^2	R/D	y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
			m	m2	m	m/s				m	Q	m	m
0.9	0.7884	0.3005	16.2	255.4416	5.409	3.304	0.000194			16.75642	0		0
0.91	0.7943	0.2988	16.38	257.3532	5.3784	3.280	0.000193	0.000194	0.000853	16.92818	0.171765	201.27	201.27
0.92	0.7999	0.2969	16.56	259.1676	5.3442	3.257	0.000192	0.000192	0.000855	17.10054	0.172351	201.67	402.94
0.93	0.8052	0.2947	16.74	260.8848	5.3046	3.235	0.000191	0.000192	0.000855	17.27344	0.172908	202.12	605.06

Table 7
Direct Step Method
STANDARD HORSE SHOE SECTION FOR TUNNEL #1
Q = 844 m3/s D = 18 m
So = 0.00148 n = 0.013

y/D	A/D^2	R/D	y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
			m	m2	m	m/s				m	m	m	m
0.9	0.7884	0.3005	16.2	255.4416	5.409	3.304	0.000194			16.75642	0		0
0.91	0.7943	0.2988	16.38	257.3532	5.3784	3.280	0.000193	0.000194	0.001287	16.92818	0.171765	133.42	133.42
0.92	0.7999	0.2969	16.56	259.1676	5.3442	3.257	0.000192	0.000192	0.001289	17.10054	0.172351	133.75	267.17
0.93	0.8052	0.2947	16.74	260.8848	5.3046	3.235	0.000191	0.000192	0.001289	17.27344	0.172908	134.09	401.26
0.94	0.8101	0.2922	16.92	262.4724	5.2596	3.216	0.000191	0.000191	0.00129	17.44701	0.173566	134.56	535.82

6.17

Given :

The channel of problem 3.18

Rectangular outlet

B_0 = 10 ft.

L = 500 ft.

S_0 = 0.001

Q = 324.6 cfs

Invert Elev. At the entrance = 122 ft.

Concrete channel (n = 0.013)

Plot the water surface profile

Solution:

The bottom elevation at the river entrance is :

122-500 x 0.001 = 121.5 ft.

The water depth at this section is :

131-121.5 = 9.5 ft.

The normal depth is obtained from

$$\frac{(10y_n)^{\frac{5}{3}}}{(10+2y_n)^{\frac{2}{3}}} = \frac{(0.013)(324.6)}{1.49\sqrt{0.001}} = 89.56$$

Y_n = 4.896 ft.

The critical depth is $Y_0 = \sqrt[3]{\frac{Q^2}{g}} = \sqrt[3]{\frac{(\frac{324.6}{10})^2}{32.2}}$

Y_0 = 3.199 ft.

Therefore

$Y_{river} > Y_n > Y_0$ => MILD CHANNEL, M1 PROFILE

Table P6.17 shows the result obtained by using Standard Step Method..

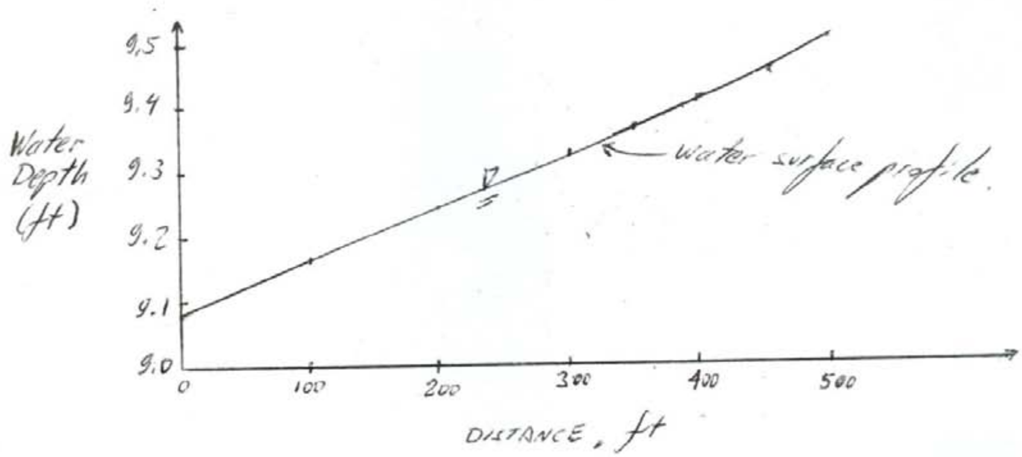


Table P6-17
Standard Step Method

x	y	A	P	V	V2/2g	z	H	Sf1	Sf ave	Dx	hf
ft	ft	ft2	ft	ft/s	ft	ft	ft			ft	ft
0	9.5	95	29	3.4168	0.1813	0	9.6813	0.000183		0	
-50	9.458	94.575	28.915	3.4322	0.1829	0.05	9.6904	0.000185	0.000184	-50	0.009188
-100	9.415	94.152	28.83	3.4476	0.1846	0.1	9.7	0.000187	0.000186	-50	0.00929
-150	9.373	93.729	28.746	3.4632	0.1862	0.15	9.7092	0.000189	0.000188	-50	0.00939
-200	9.331	93.307	28.661	3.4788	0.1879	0.2	9.7186	0.000191	0.00019	-50	0.0095
-300	9.247	92.466	28.493	3.5105	0.1913	0.3	9.738	0.000195	0.000193	-100	0.01932
-400	9.163	91.629	28.326	3.5426	0.1949	0.4	9.7577	0.0002	0.000198	-100	0.019755
-500	9.079	90.795	28.159	3.575	0.1985	0.5	9.7779	0.000204	0.000202	-100	0.020205

6.18

Given

Y_{bridge} = 12 ft.

Trapezoidal channel

B_0 = 20 ft.

S = 20

S_0 = 0.0003

Q = 800cfs

n = 0.025

Determine how far the effect of dogging due to the debris accumulated at the bridge entered.

Solution :

Consider that the flow was uniform before the accumulation of debris. Then compute the normal depth as :

$$AR^{2/3} = \frac{nQ}{1.49 S_0^{\frac{1}{2}}}$$

Or

$$\frac{[20 + 2y_n)y_n]^{\frac{5}{3}}}{[20 + 2\sqrt{5}y_n]_{\frac{2}{3}}} = 774.97$$

Then Y_n = 7.59 m

Now determine if the flow is subcritical or supercritical by computing the critical depth and comparing it with the normal depth.

$$A\sqrt{D} = \frac{Q}{\sqrt{g}}$$

Or

$$\frac{[20+2y_0)y_0]^{\frac{5}{3}}}{[20+2\sqrt{5}y_0]_{\frac{2}{3}}} = 140.98$$

Y0 = 32.28 m.

Then Y_0 < Y_n => sub critical flow.

Also y_{bridge} > yn > y0 => M1 profile, starting at a depth of 12 m at the bridge and approximating the normal depth asymptotically.

Table P6.18 shows the computations obtained by using Direct Step Method.

The effect of clogging extends 36775 ft (6.96 mi) upstream of the bridge.

Table P6.18 (**Direct Step Method**).

Table P6-18
Direct Step Method

Q = 800 cfs B = 20 ft C = 1.49
So = 0.0003 n = 0.025 s = 2 g = 32.2 m/s2

y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
ft	ft2	ft	ft/s				ft	ft	ft	ft
12	528	7.168	1.515	4.68E-05			12.03565			0
11.5	494.5	6.923	1.618	5.58E-05	5.13E-05	0.000249	11.54064	-0.49501	-1990.44	-1990.44
11	462	6.677	1.732	6.71E-05	6.15E-05	0.000239	11.04656	-0.49408	-2071.57	-4062.01
10.5	430.5	6.429	1.858	8.13E-05	7.42E-05	0.000226	10.55362	-0.49294	-2183.37	-6245.38
10	400	6.180	2.000	9.93E-05	9.03E-05	0.00021	10.06211	-0.49151	-2343.94	-8589.33
9.5	370.5	5.929	2.159	0.000122	0.000111	0.000189	9.572397	-0.48972	-2588.34	-11177.67
9	342	5.676	2.339	0.000152	0.000137	0.000163	9.084965	-0.48743	-2994.44	-14172.11
8.5	314.5	5.421	2.544	0.000191	0.000172	0.000128	8.600474	-0.48449	-3776.51	-17948.62
8	288	5.163	2.778	0.000243	0.000217	8.27E-05	8.119814	-0.48066	-5815.50	-23764.12
7.6	267.52	4.955	2.990	0.000298	0.000271	2.93E-05	7.738861	-0.38095	-13010.67	-36774.79

6.19

Given:

10 ft. square Box CULVERT

$L_{culvert}$ = 150 ft.

S_{culver} = 0.01

Depth upstream box entrance : 15 ft = M_0

The accumulation of debris at a channel crossing 0.5 mi downstream of the culvert raises the water level 5 ft. at the crossing. The channel characteristics are:

Trapezoidal

B_0 = 10 ft.

S = 1.5

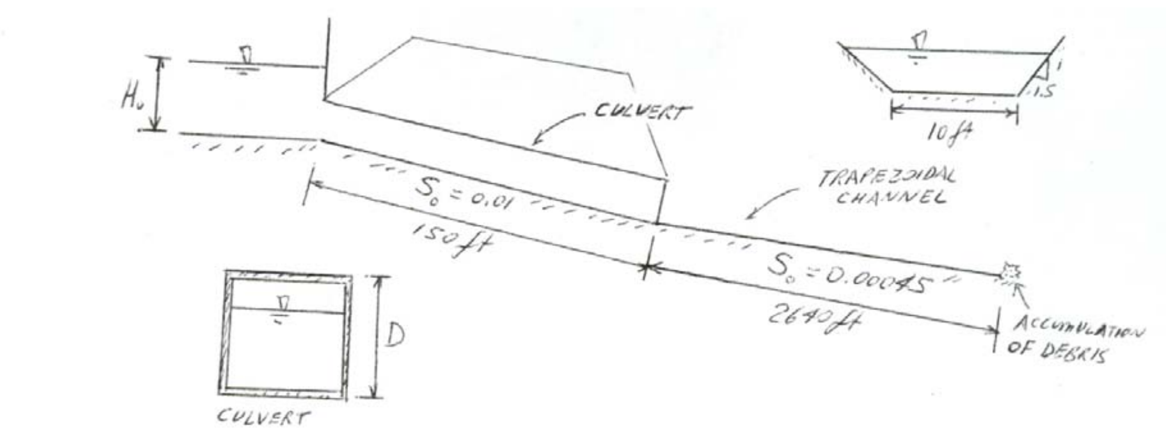
Uniform flow prior to the accumulation

$$S_{CHANNEL} = \frac{1.2ft}{2640ft} = 0.00045$$

where 0.5mi = 2640 ft.

Compute and plot the water-surface profile in the channel and inside the culvert.

The following sketch shows the channel and culvert profile.



Solution:

The culvert discharge is given by the orifice equation, as discussed in Chapter 10, Sec. 10.4.

In this case H_0 > 1.2 D, where D is the height of the culvert. Using eq. 10.14, we get, Q = $CB_0D\ \sqrt{2g(H_0 - CD)}$

where B_0 is the culvert width and C is a coefficient of contraction. For a squared-edge entrance C = 0.6,

Then,

$$Q = (0.6)(10)(10)\ \sqrt{2x32.2(15-(0.6)(10)}$$

Q = 1444.5 ft3/5

The critical depth for the culvert is computed from

$$A\ \sqrt{D_h} = \frac{Q}{\sqrt{g}}$$

Where D_h = hydraulic depth. Then,

$Y_0^{3/2}$ = 254.56

Finally

Y_0 = 8.65 ft.

Compute the normal depth in the culvert to determine the type of profile

$$\frac{nQ}{1.49S_0^{\frac{1}{2}}} = \frac{(0.013)(1444.5)}{(1.49)(0.01)^{\frac{1}{2}}} = 126.03.$$

Solve for the normal depth from

$$AR^{\frac{2}{3}} = \frac{(10y_n)^{\frac{5}{3}}}{(2y_n+10)^{\frac{2}{3}}} = 126.03$$

The answer is Yn= 6.348 ft.

We have yn<y<y0 and the curve is S2 type. The flow inside the culver4t approaches the normal depth asymptotically.

Now, we compute the normal and critical depth in the trapezoidal channel.

For normal depth solve,

$$\frac{[(10+1.5y_n)y_n]^{\frac{5}{3}}}{(10+2\sqrt{3.25}y_n)^{\frac{2}{3}}} = \frac{0.013x1444.5}{(1.49)(0.00045)^{\frac{1}{2}}} = 594.11$$

$Y_n = 6.657$ ft.

For critical depth solve:

$$\frac{[(10+1.5y_0)y_0]^{1.5}}{(10+2x1.5y_0)^{\frac{1}{2}}} = \frac{1444.5}{\sqrt{32.2}} = 254.56$$

$Y_0 = 6.329$ ft.

$Y_n \quad > \quad y_0 \quad$ => MILD CHANNEL.

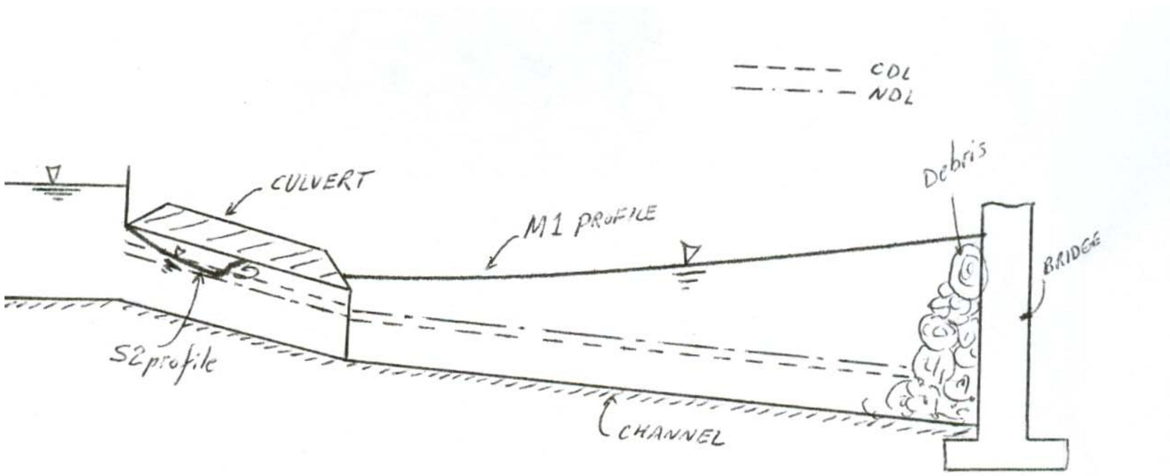
FLOW PROFILE:

The water profile in the channel is computed beginning with the water elevation at the bridge (y_b=y_n+5 ft.), yb = 11.66 ft. The computations in the channel are carried out upstream from the bridge up to a distance of 2640 ft. The computations in the culvert start upstream, at the central section near the culvert inlet. They are carried out up to a distance of 150 ft. downstream, where the culvert meets the channel. It is assumed that the flow depth is critical near the culvert inlet.

The Standard Step Method was used in these computations.
Table 9-19a and 9-19b show the results.

SKETCH OF WATER SURFACE PROFILE

Figure 1 is a sketch of the water surface profile. After the accumulation of debris at the bridge the outlet of the culvert is submerged. The culvert flow is partially full and a hydraulic jump forms inside. (Read section 10.4 for more details on culverts).



CDL
NDL
CULVERT
M1 PROFILE
Debris
BRIDGE
S2profile
CHANNEL

Table 6-19a
Standard Step Method for channel

x	y	A	P	V	V2/2g	z	H	Sf1	Sf ave	Dx	hf
ft	ft	ft2	ft	ft/s	ft	ft	ft			ft	ft
0	11.66	320.53	52.04	4.51	0.875	0.000	12.535	0.00014			
-100	11.62	318.67	51.89	4.53	0.886	0.045	12.549	0.00014	0.00014	-100	0.01381
-500	11.45	311.32	51.29	4.64	0.928	0.225	12.601	0.00015	0.00014	-400	0.05746
-1000	11.25	302.33	50.56	4.78	0.984	0.450	12.684	0.00016	0.00015	-500	0.07708
-2000	10.85	285.14	49.12	5.07	1.106	0.900	12.857	0.00019	0.00017	-1000	0.17380
-2640	10.60	274.70	48.23	5.26	1.192	1.188	12.984	2.10E-04	0.00020	-640	0.12623

Table 6-19b
Standard Step Method for culvert

x	y	A	P	V	V2/2g	z	H	Sf1	Sf ave	Dx	hf
ft	ft	ft2	ft	ft/s	ft	ft	ft			ft	ft
0	8.65	86.50	27.30	16.70	4.330	2.688	15.668	0.00456			
20	7.94	79.44	25.89	18.18	5.134	2.488	15.566	0.00565	0.00510	20	0.10210
40	7.71	77.09	25.42	18.74	5.451	2.288	15.449	0.00609	0.00587	20	0.11736
60	7.55	75.47	25.09	19.14	5.689	2.088	15.324	0.00643	0.00626	20	0.12516
100	7.32	73.18	24.64	19.74	6.050	1.688	15.056	0.00695	0.00669	40	0.26751
150	7.12	71.24	24.25	20.30	6.385	1.188	14.696	0.00744	0.00719	50	0.35975

6.20

Given

Rectangular Channel

B = 10 m

Concrete Lined (n = 0.013)

S_0 = 0.01

Constant level lake upstream, H = 6m.

If the flow depth at the channel entrance is critical, determine the location where the flow depth is 3.9, 3.7, 3.5, 3.3 and 3.0 m

Solution:

From example 5.3, we know that :

Y_0 = 4m (at entrance)

Q = 250.6 m3/5

Y_n = 2.37m

The Direct Step Method is particularly appropriate for this type of problem. Table 6.20 shows the computations.

The locations required are :

Flow depth (m)	**Location from the entrance (m)**
3.9	0.51
3.7	5.09
3.5	15.89
3.3	35.64
3.0	93.31

Table P6-20
Direct Step Method

Q = 250.6 m3/s B = 10 m
So = 0.01 n = 0.013

y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
m	m2	m	m/s				m	m	m	m
4	40	2.222	6.265	0.002287			6.000521			0
3.9	39	2.191	6.426	0.002452	0.00237	0.00763	6.004427	0.003906	0.51	0.51
3.7	37	2.126	6.773	0.002835	0.002644	0.007356	6.038082	0.033654	4.57	5.09
3.5	35	2.059	7.160	0.003308	0.003072	0.006928	6.112926	0.074844	10.80	15.89
3.3	33	1.988	7.594	0.003899	0.003604	0.006396	6.239241	0.126316	19.75	35.64
3	30	1.875	8.353	0.005101	0.0045	0.0055	6.556482	0.317241	57.68	93.31

6.21

Given

Rectangular Channel

B_0 = 10m

Q = 50 m^3/s

Concrete lined (deteriorated) : n = 0.015

S_0 = 0.0006

Free outfall downstream.

Assume y = y_0 at $4y_0$ upstream of the fall.

Compute Water depth 2 km upstream of the fall and the water surface profile.

Solution:

Compute critical and normal depth to determine if the channel is mild or steep.

Critical depth $$\frac{Q}{\sqrt{g}} = B_0 y_0^{\frac{3}{2}}$$

$$\frac{50}{\sqrt{9.81}} = 10 y_0^{\frac{3}{2}}$$

Y_0 = 1.366 m => 4 y_0 = 5.46 m

Normal depth

$$AR^{\frac{2}{3}} = \frac{nQ}{\sqrt{S_0}}$$

$$\frac{(y_n B_0)^{\frac{5}{3}}}{(2y_n + B_o)^{\frac{2}{3}}} = \frac{(0.015)(50)}{\sqrt{0.0006}} \qquad = 30.619$$

Solving for y_n, we get y_n = 2.274 m.

Y_n > y_0 => MILD CHANNEL

Table 6.21 shows the computations by using the Standard Step Method.

The water depth 2 km upstream of the fall is 2.23 m.

Table P6-21

Standard Step Method

x	y	A	P	V	V2/2g	z	H	Sf1	Sf ave	Dx	hf
m	m	m2	m	m/s	m	m	m			m	m
-5.46	1.366	13.66	12.732	3.66	0.6829	0	2.04887	0.00274		0	
-100	1.777	17.7709	13.5542	2.8136	0.4035	0.05672	2.23729	0.00124	0.00199	-94.5	0.18842
-300	1.942	19.4225	13.8845	2.5743	0.3378	0.17672	2.45675	0.00095	0.0011	-200	0.21946
-500	2.029	20.2975	14.0595	2.4633	0.3093	0.29672	2.63576	0.00084	0.0009	-200	0.17901
-700	2.087	20.8734	14.1747	2.3954	0.2925	0.41672	2.79652	0.00077	0.0008	-200	0.16076
-1200	2.174	21.7386	14.3477	2.3	0.2696	0.71672	3.16022	0.00068	0.0073	-500	0.3637
-1600	2.21	22.0979	14.4196	2.2627	0.2609	0.95672	3.42745	0.00065	0.00067	-400	0.26723
-2000	2.232	22.3202	14.464	2.2401	0.2558	1.19672	3.68451	0.00063	0.00064	-400	0.25706

6.22

Given

Trapezoidal Channel

B_0 = 10 m

S = 1.5

Q = 80 m3/5

S_0 = 0.002

n = 01.015

ΔY_{dam} = 10 m

Compute : Flow depth at 250, 500 and 750 m upstream from the dam.

Solution

Critical depth computation : $A\sqrt{D} = \dfrac{Q}{\sqrt{g}}$

Or

$$\frac{80}{\sqrt{9.81}} = \frac{[(10+1.5y_0)y_0]^{1.5}}{(10+2x1.5y_0)^{\frac{1}{2}}}$$

Y_0 = 1.707 m

Normal Depth Computation :

$$AR^{\frac{2}{3}} = \frac{nQ}{\sqrt{S_0}}$$

$$\frac{[(10+1.5y_n)y_n]^{\frac{5}{3}}}{(10+2y_n\sqrt{1+1.52})^{\frac{2}{3}}} = \frac{0.015x80}{(0.002)^{\frac{1}{2}}} \qquad = 26.833$$

$Y_n = 1.742$ m

The channel bottom slope is practically the critical slope ($y_n \approx y_0$). Assuming that the initial flow in the channel is uniform, the dam will raise the water elevation above the critical depth forming a C1 profile upstream of the dam.

Table 6.22 shows the computations for the water profile using Standard Step Method. The initial water depth at the dam is $y_n = \Delta y_{dam} = 11.74$ m.

The required flow depth are :

Location (upstream of dam, m)	**Water depth (m)**
250	11.24
500	10.74
750	10.24

Table 6-22
Standard Step Method

x	y	A	P	V	V2/2g	z	H	Sf1	Sf ave	Dx	hf
m	m	m2	m	m/s	m	m	m			m	m
0	11.74	324.14	52.33	0.247	0.00310	0.00	11.743	1.20E-06		0	
-100	11.54	315.15	51.61	0.254	0.00328	0.20	11.743	1.30E-06	1.25E-06	100	0.00013
-250	11.24	301.90	50.53	0.265	0.00358	0.50	11.743	1.46E-06	1.34E-06	150	0.00021
-500	10.74	280.41	48.72	0.285	0.00415	1.00	11.744	1.78E-06	1.62E-06	250	0.00040
-750	10.24	259.66	46.92	0.308	0.00484	1.50	11.744	1.98E-06	4.30E-06	250	0.00049

6.23

Given :

$Y_n = 2m$

Rectangular Channel

$B_0 = 10m$

$S_0 = 0.001$

$n = 0.020$

$\Delta Y_{bridge} = 1m$

Determine the distance from the bridge at which y = 2.5 m

Solution:

Assuming that previous to the construction of the bridge the channel had uniform flow; then, at the bridge section the water depth is

$Y_{bridge} = 2+1 = 3m$

The channel flow is given by :

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

$$Q = \frac{1}{0.02} \frac{(10x2)^{\frac{5}{3}}}{(4+10)^{\frac{2}{3}}} (0.001)^{\frac{1}{2}}$$

$Q = 40.111\ m^3/s$

The critical depth is

$$Y_0^{3/2} = Bo \frac{Q}{\sqrt{g}}$$

$$Y_0 = [\frac{40.111}{\sqrt{9.81}X10}]^{\frac{2}{3}}$$

$Y_0 = 1.179$ m

$Y_0 < y_n$ => MILD CHANNEL

$Y_b > y_n$ => M1 profile

Table 6.23 shows the result using direct step method.

The flow depth is 2.5 m at 777 m upstream of the dam.

Table P6-23
Direct Step Method

Q = 40.111 m3/s B = 10 m
So = 0.001 n = 0.02

y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
m	m2	m	m/s				m	m	m	m
3	30	1.875	1.337	0.000309			3.091114			
2.9	29	1.835	1.383	0.000341	0.000325	0.000675	2.997506	-0.09361	-138.66	-138.66
2.8	28	1.795	1.433	0.000376	0.000358	0.000642	2.904595	-0.09291	-144.82	-283.48
2.7	27	1.753	1.486	0.000418	0.000397	0.000603	2.812487	-0.09211	-152.74	-436.21
2.6	26	1.711	1.543	0.000465	0.000441	0.000559	2.721306	-0.09118	-163.25	-599.47
2.5	25	1.667	1.604	0.000521	0.000493	0.000507	2.631204	-0.0901	-177.80	-777.27

Chapter 7
RAPIDLY VARIED FLOW

7.1

$$H = Z + y + \frac{v^2}{2g}$$

$$H_0 = H\text{-}z = y + \frac{Q^2}{2gA^2}$$

$$\frac{dH_0}{dx} = \frac{dy}{dx} - \frac{Q^2}{gA^3}\frac{dA}{dx}$$

$$S_0 = \frac{dy}{dx} - \frac{Q^2B}{gA^3}\frac{dA}{dx}$$

$$S_0 = \left(1 - \frac{Q^2}{gB^2y^3}\right)\frac{dy}{dx} \qquad \text{------------------ (1)}$$

$$H_0 = Y + \frac{Q^2}{2gA^2}$$

For critical section $\frac{dH_0}{dx} = 0$

$$\frac{dH_0}{dy} = 1 - \frac{2Q^2}{2gB^2y^3} = 0$$

$$Yc = \sqrt[3]{\frac{Q^2}{gB^2}} \qquad \text{--------------------- (2)}$$

Or

$$\frac{Q^2}{gB^2{y_c}^3} \quad = \text{Yc}$$

$$\frac{{V_1}^2}{2g} = \frac{Yc}{2}$$

$$\therefore \quad \text{Hmin} = \text{Yc} + \frac{{V_1}^2}{2g} = Yc + \frac{Yc}{2} = \frac{3}{2}Yc$$

From eqn (2)

$$H_{min} = \frac{3}{2}\sqrt[3]{\frac{Q^2}{gB^2}}$$

From Eqn. (1)

$$\int_0^x S_0 dx = \int_{Y0}^{y}(1 - \frac{Q^2}{gB^2y^3})dy$$

$$S_0x = y + \frac{Q^2}{2gB^2y^2} - y_c \quad \frac{Q^2}{2gB^2{y_0}^2}$$

$$S_0x = y + \frac{1}{2}\frac{{Y_c}^3}{Y^2} - Y_c - \frac{1}{2}\frac{{Y_c}^3}{{Y_c}^2}$$

$$S_0x = Y + \frac{1}{2}\frac{{Y_c}^3}{Y^2} - \frac{3}{2}Y_c$$

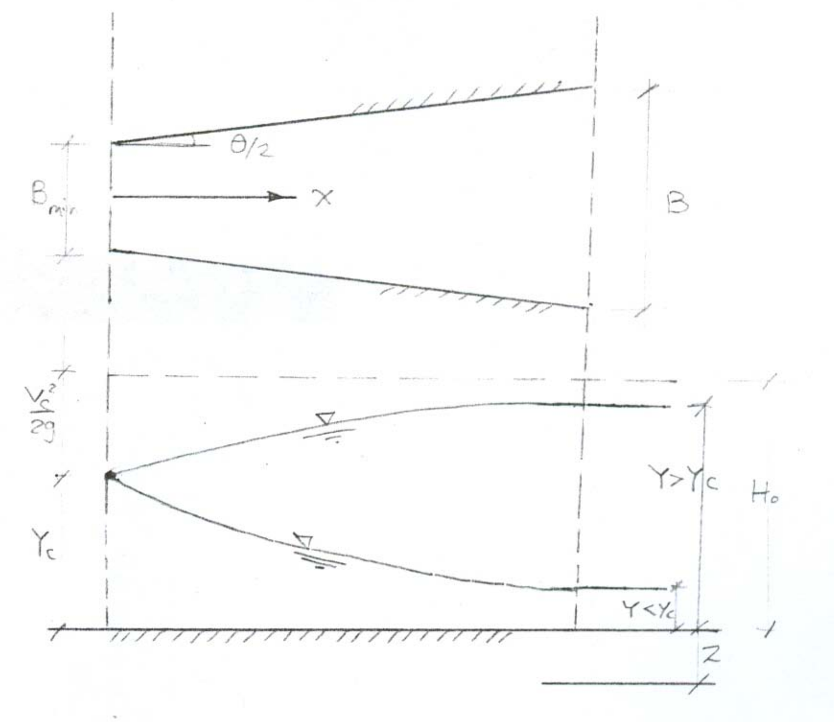
7.2

$\theta \quad = \tan^{-1} \dfrac{dB}{dx} = \dfrac{dB}{dx}$

$\theta dx \quad = dB \qquad$ -- (1)

$Q \quad = BY \sqrt{2g(H_0 - y)}$

$B \quad = \dfrac{Q}{Y\sqrt{2g(H_0 - y)}} \qquad$ -- (2)



$H0 \quad = Y + \dfrac{Q^2}{2gA^2}$

At the critical depth,

$$\frac{dH_0}{dy} = 1 - \frac{2Q^2}{2gB^2Y^3} = 0$$

$$Yc = \sqrt[3]{\frac{Q^2}{gB^2}} \qquad Yc = \frac{Q^2}{gB^2 y_c{}^2} = \frac{Vc^2}{g}$$

$$H_0 = Yc + \frac{Vc^2}{2g} = Yc + \frac{Yc}{2} = \frac{3}{2}Yc$$

$$H0 = \frac{3}{2}\sqrt[3]{\frac{Q^2}{gB^2}}$$

$$Bmin = \frac{Q}{\sqrt{g}} \frac{1}{(2H_0/g)^{\frac{3}{2}}}$$

From (1)

$$\int_0^x \theta\, dx = \int_{B\min}^{B} dB$$

$$\theta\, x = B - B_{min}$$

$$\theta\, x = \frac{Q}{Y\sqrt{2g(H_0 - y)}} - \frac{Q}{\sqrt{g}} \frac{1}{(2H_0/3)^{\frac{3}{2}}}$$

7.4

$Y_2 = Y_1$

$b_2 = b_1$

$E_1 = E_2$

$M_2 = M_3$

γ = b_3/b_1

S = Y_3/Y_1

E_1 = $Y_1 + V_1^2/2g$

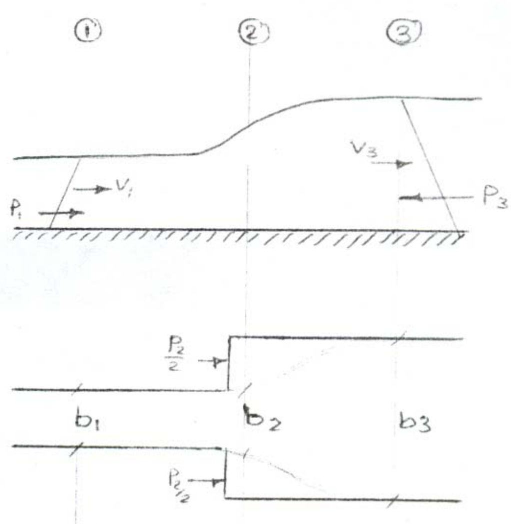
E_3 = $Y_3 + V_3^2/2g$

Q_1 = Q_2 = Q_3

A_1V_1 = $A_2V_2 = A_3V_3$

$b_1y_1v_1$ = $b_2y_2v_2 = b_3y_3v_3$

41



$$\Delta E = Y1 + \frac{V_1^2}{2g} - y_3 - \frac{v_3^2}{2g}$$

$$\Delta E = \frac{V_1^2}{2g}[1 + \frac{2gy_1}{v_1^2} - \frac{2gy_3}{v_1^2}\frac{V_3^2}{V_1^2}]$$

$$\Delta E = \frac{v_1^2}{2g}[1+\frac{2}{F_{r1}^2} - \frac{2S}{F_{r1}^2} - \frac{1}{r^2S^2}] \quad \text{---------- (1)}$$

Momentum

$$P_1 + P_2 - P_3 = \frac{rQ}{g}(v_3 - v_1)$$

$$P_1 = \frac{\gamma y_1^2 b_1}{2}$$

$$P_2 = \frac{\gamma(b_3 - b_1)y_2^2}{2} \qquad P_2 = \frac{\gamma(b_3 - b_1)y_1^2}{2}$$

$$P_3 = \frac{\gamma b_3 y_1^2}{2}$$

$$\frac{\gamma Q(v_3 - v_1)}{g} = \frac{\gamma y_1^2 b_1}{2} + \frac{\gamma y_1^2 b_3}{2} - \frac{\gamma y_1^2 b_1}{2} - \frac{\gamma y_3^2 b_3}{2}$$

$$\frac{b_1 y_1 v_1}{g}(v_3 - v_1) = ½\, b_3\, [y_1^2 - y_3^2]$$

$$\frac{b_1 y_1 v_1}{g}\left(\frac{b_1}{b_3}\frac{y_1}{y_2}v_1 - v_1\right) = ½\, b_3\, [y_1^2 - y_3^2]$$

$$\frac{v_1^2}{g y_1}\left(\frac{1}{\gamma s} - 1\right) = \frac{1}{2}\frac{b_3}{b_1}\left(\frac{y_1^2}{y_1^2} - \frac{y_3^2}{y_1^2}\right)$$

$$F\gamma_1^2\left(\frac{1}{\gamma^2} - 1\right) = \frac{1}{2}\gamma(1 - S_2)$$

$$\frac{F\gamma_1^2}{\gamma s} - F\gamma_1^2 = \frac{\gamma}{2} - \frac{\gamma S_2}{2}$$

$$F\gamma_1^2 + \frac{\gamma}{2} = \frac{F\gamma_1^2}{\gamma S} + \frac{\gamma S_2}{2}$$

----------- (2)

Eqn. 2 can be solved for S

$$S \qquad = 1 + \frac{F\gamma_1{}^2(\gamma - 1)}{\gamma^2}$$

$$S2 \qquad = 1 + 2\frac{F\gamma_1{}^2(\gamma - 1)}{\gamma^2} + \frac{F\gamma_1{}^4(\gamma - 1)}{\gamma^4}$$

Eqn. (1)

$$\Delta E \qquad = \frac{V_1{}^2}{2g}[1 + \frac{2}{F\gamma_1{}^2} - \frac{2S}{F\gamma_1{}^2} - \frac{1}{\gamma^2 S^2}$$

$$\Delta E \qquad = \frac{V_1{}^2}{2g}[1 + \frac{2}{F\gamma_1{}^2} - \frac{1}{\gamma^2 + 2F\gamma_1 2(\gamma - 1)} - \frac{2}{F\gamma^2} - \frac{2(\gamma - 1)}{\gamma^2}$$

$$= \frac{V_1{}^2}{2g}[1 + \cancel{\frac{2}{F\gamma_1{}^2}} - \frac{1}{\gamma^2 + 2F\gamma_1 2(\gamma - 1)}\,\frac{\gamma^2 - 2F\gamma_1 2(\gamma - 1)}{\gamma^2 - 2F\gamma_1 2(\gamma - 1)} - \cancel{\frac{2}{F\gamma^2}} - \frac{2}{\gamma} + \frac{2}{\gamma^2}]$$

$$= \frac{V_1{}^2}{2g}[1 - \frac{\gamma^2 - 2F\gamma_1{}^2(\gamma - 1)}{\gamma^4 - 2F\gamma_1{}^4(\gamma - 1)^2} - \frac{2}{\gamma} + \frac{2}{\gamma^2}]$$

$$= \frac{v_1{}^2}{2g}[\left(1 - \frac{2}{\gamma} + \frac{1}{\gamma^2}\right) - \frac{1}{\gamma^2} + \frac{2F\gamma^2(\gamma - 1)}{\gamma^4} + \frac{1}{\gamma^2}]$$

$$= \frac{v_1{}^2}{2g}[(1 - \frac{1}{r})^2 + 2\frac{Fr^2(r - 1)}{\gamma^4}]$$

$$= \frac{v_1^{\,2}}{2g}\left[(1-\frac{b_1}{b_3})^{\,2}+2\frac{Fr^{\,2}(\frac{b_2}{b_1}-1)}{\left(\frac{b_3}{b_1}\right)^4}\right]$$

$$\Delta E \quad = \frac{v_1^{\,2}}{2g}\left[(1-\frac{b_1}{b_3})^{\,2}+2Fr^{\,2}\,\frac{b_1^{\,3}}{b_3^{\,4}}(b_3-b_1)\right] \quad ---- \quad (3)$$

$$\Delta E \quad = \frac{v_1^{\,2}}{2g}(1-\frac{b_1}{b_3})^2$$

As Fr < 0.5 and $\frac{b_3}{b_1}$ > 1.5, the last term in eqn. (3) vanish also $y_1 = y_2 = y_3$

$$\Delta E \quad = \frac{v_1^{\,2}}{2g}\left[1-\frac{2b_1}{b_3}+\frac{b_1^{\,2}}{b_3^{\,2}}\right] \qquad = \frac{v_1^{\,2}}{2g}\left[1-\frac{2V_3}{V_1}+\frac{V_3^{\,2}}{V_1^{\,2}}\right]$$

$$= \frac{1}{2g}[v_1^{\,2}-2v_3v_1+v_3^{\,2}]$$

$$\Delta E \quad = \frac{(V_1-V_3)^2}{2g}$$

7.5

$$Yr \quad = \frac{1}{2}[-1+\sqrt{1+8Fr_1^{\,2}}\,]$$

$$= -\frac{1}{2}+\frac{1}{2}\left[8Fr_1^{\,2}\left(1+\frac{1}{8Fr_1^{\,2}}\right)\right]^{1/2}$$

$$= -½ + ½\,(8Fr_1^2)^{½}\left(1+\frac{1}{2}\frac{1}{8Fr_1^{\,2}}+........\right)$$

$$= -½ + \sqrt{2}\,Fr_1$$

Yr $= \sqrt{2}\,Fr_1 - ½$ for $Fr_1 > 2$

7.6

h_i $= E1 - E2 = \left(Y_1+\frac{V_1^{\,2}}{2g}\right)-\left(Y_2+\frac{V_2^{\,2}}{2g}\right)$

$$= (y_1 - y_2) + \frac{q^2}{2g}\left(\frac{1}{y_1^{\,2}}-\frac{1}{y_2^{\,2}}\right)$$

$$= (y_1 - y_2) + \frac{q^2}{2gy_1^{\,2}}\left(1-\frac{y_1^{\,2}}{y_2^{\,2}}\right)$$

$$= (y_1 - y_2) +[1- \frac{q^2}{2gy_1^{\,2}}\left(\frac{y_2+y_1}{y_2^{\,2}}\right)$$

h_i $= (y_1 - y_2) +[1- \frac{Fr_1^{\,2}}{2}\frac{y_1(y_2+y_1)}{y_2^{\,2}}]$

Fr_1^2 $= ½\,\frac{y_2}{y_1}[\frac{y_2}{y_1}+1]$

hi $= (y_1 - y_2)\left[1-\frac{1}{4}\frac{y_2}{y_1}\left(\frac{y_2}{y_1}+1\right)\frac{y_1}{y_2^{\,2}}(y_2+y_1)\right]$

$$h_i = (y_1 - y_2)\left[1 - \frac{1}{4}\frac{(y_1+y_2)^2}{y_1 y_2}\right]$$

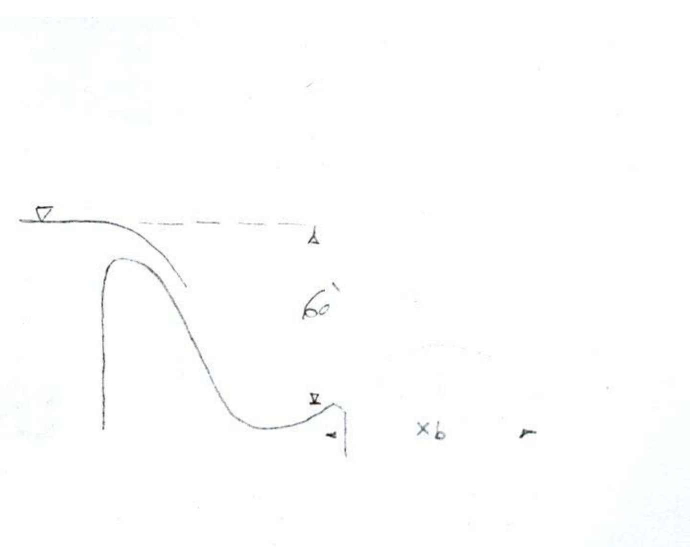
$$= (y_1 - y_2)\,\frac{4y_1y_2 - y_1^{\,2} - y_2^{\,2} - 2y_1y_2}{4y_1y_2}$$

$$= -\,(y_1 - y_2)\,\frac{y_1^{\,2} + y_2^{\,2} - 2y_1y_2}{4y_1y_2}$$

$$= \frac{(y_2 - y_1)(y_2 - y_1)^2}{4y_1y_2}$$

$$h_i = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

7.7



$$\frac{v_1{}^2}{2g} + y1 = \frac{v_2{}^2}{2g} + y_2 + hi$$

$$60 = \frac{v_2{}^2}{2g} + 6$$

$$\frac{v_2{}^2}{2g} = 54$$

$$\frac{X_b}{h_0} = 2 \sin 2\alpha$$

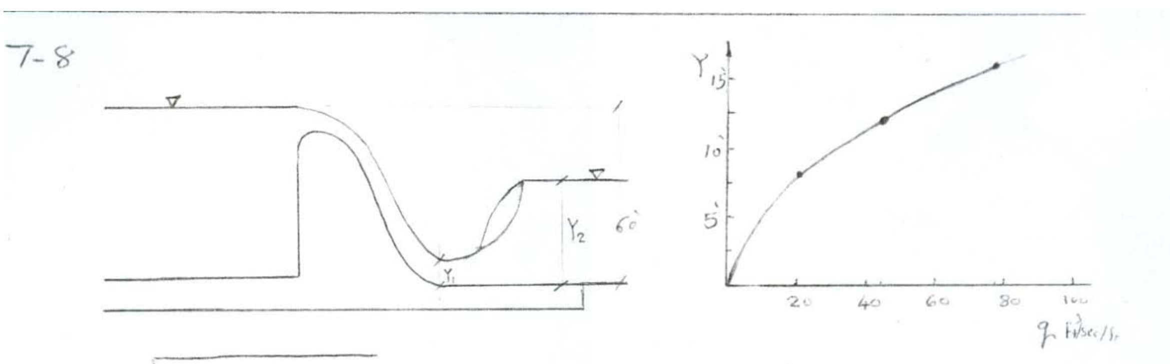
$$X_b = 54\,x\,2\,x \sin 2(20)$$

Xb = 104 sin 40

For design

Xb = 0.8 x 104 x sin 40 = 53.5

7.8



$V_1 = \sqrt{2\,x\,32.2(60-6)} = 58.97\,ft/\text{sec}.$

$$Y_1 = \frac{Q}{58.97B}$$

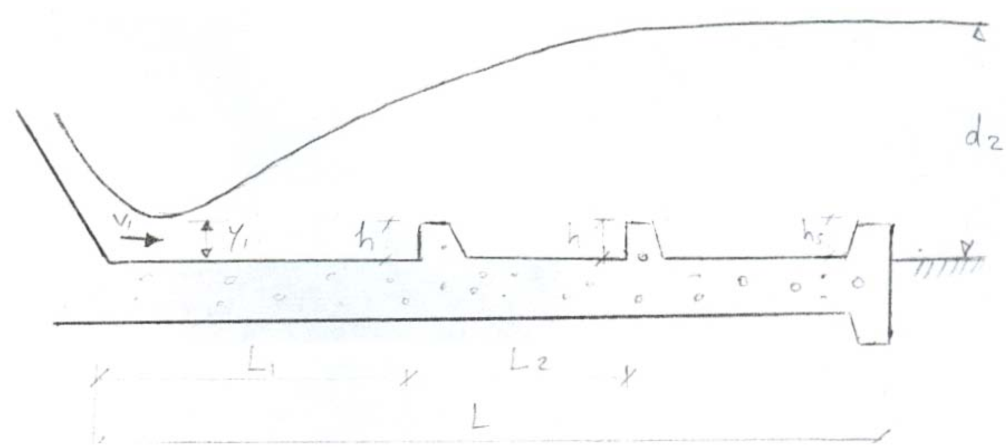
$$Fr_1^2 = \frac{V_1^2}{gy_1} = \frac{58.97^2 x 58.97B}{32.2Q} = 6368.69\frac{B}{Q}$$

$$\frac{Y_2}{y_1} = \frac{1}{2}[-1+\sqrt{1+8Fr_1^2}]$$

$$\frac{Y_2\, 58.97B}{y_1} = \frac{1}{2}[-1+\sqrt{1+\frac{8x6368.89B}{Q}}]$$

$$Y2 = \frac{q}{117.94}\left[-1+\sqrt{1+\frac{50957.095}{q}}\right]$$

Stilling basin design



Assume,

Fr_1 = 11.79, y_2 = 12.39 y_1 = 0.766

$L_1 \quad = [1.5 + \frac{1}{11}(Fr_1 - 4.6)]\ y_2$

$= [1.5 + \frac{1}{11}\ (11.79 - 4.6)]\ x\ 12.39 \qquad = 26.68$

$h \quad = [1 + 0.13\ (Fr_1 - 4.6)]\ y_1$

$= [1 + 0.13\ (11.79 - 4.6)]\ 0.766$

$= 1.48 \qquad \approx \qquad 1.5$

$L_2 \quad = 2.5\ h$

$= 2.5\ x\ 1.5 \qquad = \qquad 3.75$

$h_s \quad = h/2 \quad = 1.5/2 = \qquad 0.75$

$L \quad = 26.68 + 3.75 = \qquad 30.4$

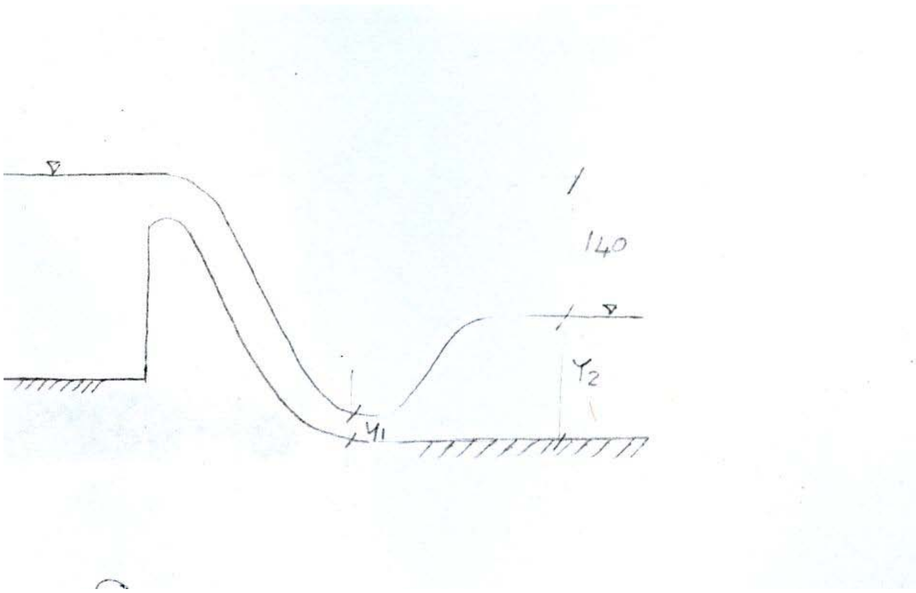
$L \quad \geq 4y2$

$\geq 4\ x\ 12.39 \qquad \geq 49.56$

$L \quad = 50$

$d_2 \quad = 0.85\ x\ 12.39 = 10.5$

7.9



$Q = 360{,}000$ Cfs

$V_1 = \sqrt{2(32.2)(140 + y_2)}$

$V_1 = 8.02\ \sqrt{140 + y_2}$

$Q = B_1 .\ v_1 .\ y_1$

$$Y_1 = \frac{Q}{50\,x\,68.02\sqrt{140+y_2}} = \frac{Q}{2407.49\sqrt{140+y_2}}$$

$$Fr_1^{\ 2} = \frac{V_1^{\ 2}}{g y_1} = \frac{64.32(140+y_2)}{32.2.\dfrac{Q}{2407.49\sqrt{140+y_2}}}$$

$$= \quad 4809.03\ \frac{(140+y_2)^{\frac{3}{2}}}{Q}$$

$$\frac{Y_2}{Y_1} \quad = \quad \frac{1}{2}[-1+\sqrt{1+8F_1^{\,2}}]$$

$$Y_2 \ \frac{2407.49\sqrt{140+y_2}}{Q} \quad = \ \frac{1}{2}\left[-1+\sqrt{1+8x4809.03\frac{(140+y_2)^{\frac{3}{2}}}{Q}}\right]$$

$$Y_2 \ \sqrt{140+y_2} \qquad = \frac{Q}{4814.98}\left[-1+\sqrt{1+\frac{38472.24}{Q}(140+y_2)^{\frac{3}{2}}}\right] \quad \text{----- (1)}$$

For Q = 260,000

$$Y_2\sqrt{140+y_2} \qquad = -74.77+74.77\sqrt{1+0.107(140+y_2)^{\frac{3}{2}}}$$

Trial and error,

Y_2 = 90.5

$$Y1 \quad = \ \frac{360{,}000}{2407.49\sqrt{140+90.5}} \qquad = 9.85$$

Design of Flip bucket :

Assume no losses $\quad \therefore h_0 \ = \dfrac{V_1^{\,2}}{2g} \quad = \dfrac{121.76}{2x32.2} \ = 230$

Yb = 0 $\qquad \alpha$ = 30

$$\frac{x_b}{h_0} \quad = 2\sin 2\alpha$$

Xb = 230 x 2sin 60 = 398

For design,
Xb = 0.8 x 398 = 318.7 → 320

Bucket redius = 15m x $\dfrac{1}{0.3048}$ = 49.2 → 50

Bucket top above the bucket invert by = 10/100 x 50 = 5

Design of Roller bucket:-

Fig. 7.20

$$\frac{p}{Hd} = 2 \qquad \text{Cd} = 0.485$$

Fig. 7.21

$$\frac{He}{Hd} = 1 \qquad \text{Kd} = 0.02$$

Q = 360,000 Cfs

Le = Ln-2(N Kp + Ka) He

Le = 6 x 50 – 2(5 x 0.02) He

Le = 300 – 0.2 He

$$360{,}000 = 0.485\ \text{Le}\ \sqrt{2x32.2}He^{1.5}$$

92484.9 = (300-0.2 He) He1.5

$$He_3 = \frac{92494.9^2}{(300-0.2He)^2} \qquad \text{Trial and error} \qquad He = 46.61$$

If we take Y_2 = 90.5 and y_1 = 9.85, from the first part,

$$V_1 = \sqrt{2x32.2(140+90.5-9.85)} = 119.21 \text{ ft/sec.}$$

$$Y_1 = \frac{360{,}000}{119.21x6x50} = 10$$

$$Fr_1 = \frac{119.21}{\sqrt{32.2x10}} = 6.62 \quad \rightarrow \quad \text{Fig. 7.28} \quad \frac{R}{y_1 + \frac{V_1^2}{2g}} = 0.25$$

$$R = 0.25\ (10 + \frac{119.21^2}{2x32.2} = 57.67 \approx 58$$

Fig. 7.26 a wing Fr = 6.62 and $\dfrac{R}{y_1 + \dfrac{V_1^2}{2g}} = 0.25$

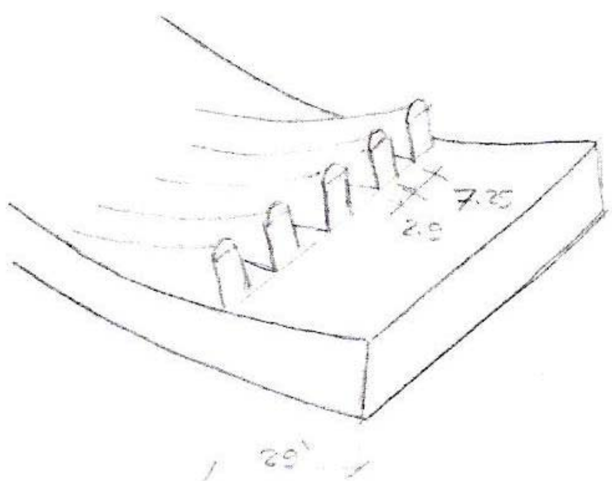
$\dfrac{T_{\min}}{y_1} = 10$ T_{min} = 10 x10 = 100

From Fig. 7.26 b

$\dfrac{T_{\max}}{y_1} = 13$ T_{max} = 10 x13 = 130

For best performance T_{min} ≈ tail water depth

$\dfrac{Ts}{y_1} = 9$ Ts = 9 x 10 = 90



R = 58’

0.05 R = 2.9

0.125 R = 7.25

0.5 R = 59

Stilling basin : notation on figure 7.24

Fr_1 = 6.62 y_1 = 10 y_2 = 90.5

L_1 = [1.5 + 1/11 (Fr1 – 4.6)] y_2

= [1.5 + 1/11(6.62 -2.6)] 90.5 = 152.4

h = [1 + 0.13(Fr1 – 4.6)] y_1

= [1 + 0.13 (6.62-4.6) 10 = 12.6

L_2 = 2.5 h = 2.5 x 12.6 = 31.6

hs = h/2 = 12.6/2 = 6.3

L = L_1 + L_2 = 152.4 + 31.6 = 184

L $\geq 4y_2$ $\geq$ 4 x 90.5 = 362

L = 362

d_2 = 0.85 x 90.5 = 76.95

i : Roller bucket

ii : Flip bucket

iii : Roller bucket

7.10

Q = 260,000 Cfs Hs = 46.61

$\frac{He}{Hd} = \frac{55.93}{46.61}$ = 1.2 → Fig. 7.21 type 2 Kp = -0.005

$\frac{He}{Hd}$ = 1.2 → Fig. 7.20 c/cd = 1.015

C = 0.485 x 1.015 = 0.492

Le = Ln – 2 (NKp + Ka)Hc

Le $= 50 \times 6 - 2\,[5 \times (-0.005) + 0]\,55.93 = 302.796$

Q $= 0.492 \times 302.796\,\sqrt{2x32.2}\,59.93^{1.5}$

Q $= 500{,}063.73$ Cfs at H = 55.93

$\frac{He}{Hd} = \frac{37.23}{46.61}$ = 0.8 Fig. 7.21 type 2 Kp = 0.025

$\frac{He}{Hd}$ = 0.8 Fig. 7.20 c/cd = 0.965

C $= 0.483 \times 0.965 = 0.468$

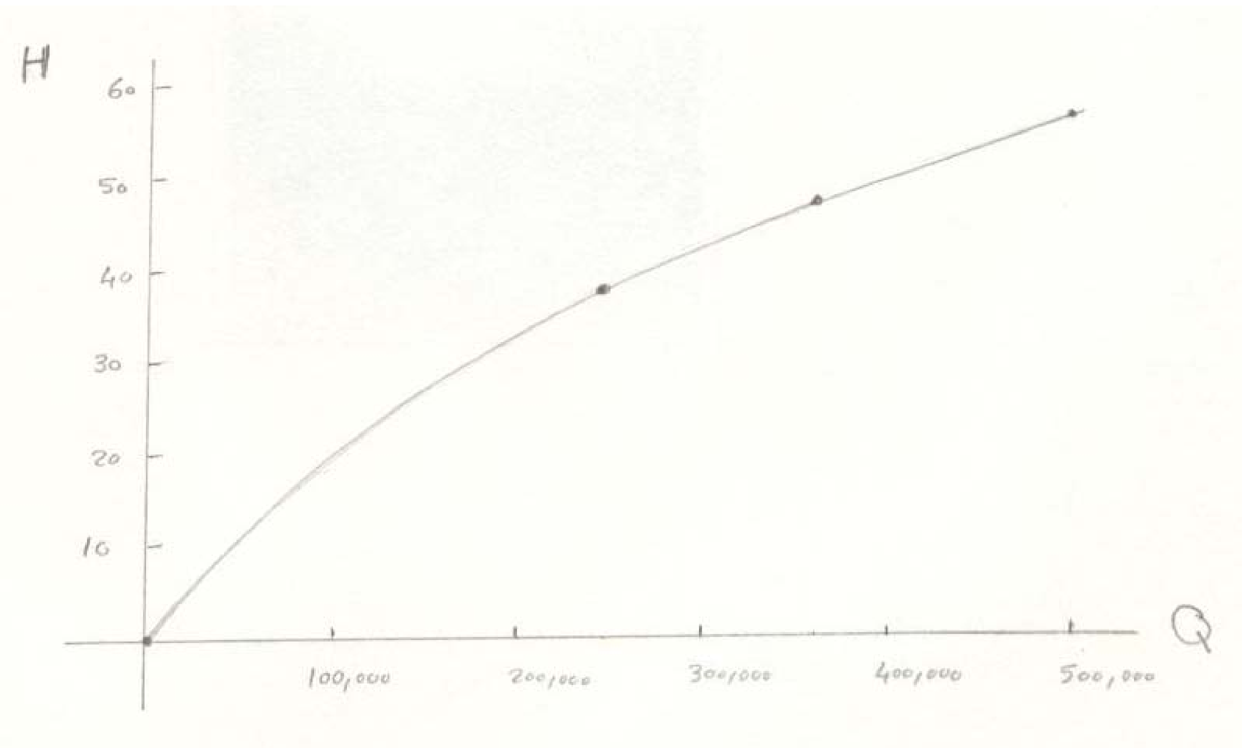
Le $= Ln - 2\,(NKp + Ka)Hc$

$= 50 \times 6 - 2\,[5\,(0.25)+0]\,37.23 = 290.69$

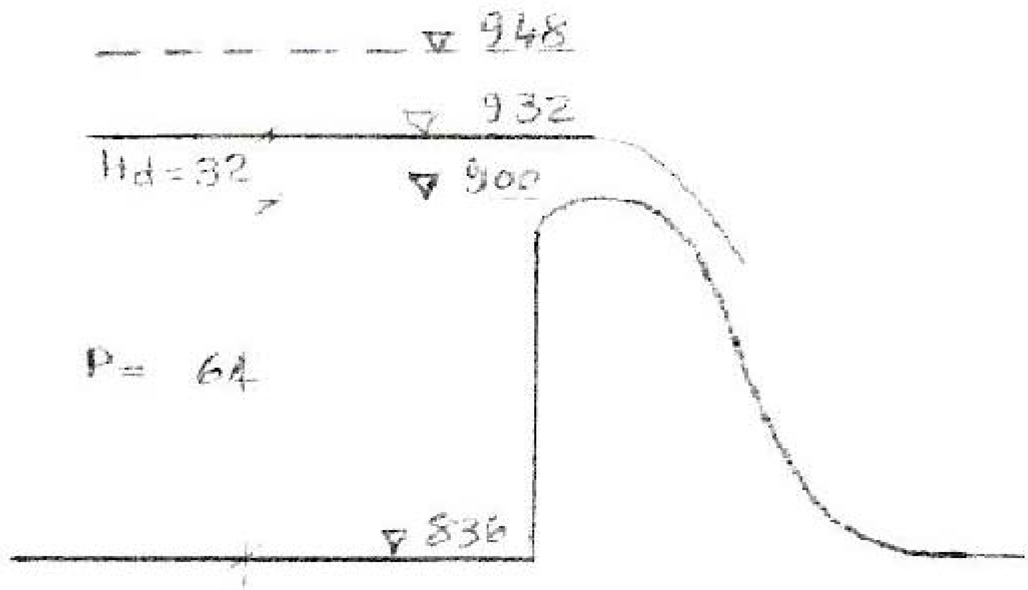
Q $= 0.468 \times 290.69\,\sqrt{2x32.2}\,37.23^{1.5}$

$= 248{,}003.84$ Cfs

Q $= 248{,}003.84$ Cfs at H = 37.23



7.11



Le $= Ln - 2(NKp + Ka)Hc$

Fig. 7-21 $Kp = 0.0125$

Le $= 4 \times 50 - 2(3 \times 0.0125)32$

Le $= 197.6$

$\frac{p}{Hd} = \frac{64}{32} = 2$ Fig. 7.20.8 cd = 0.49

Q $= C\, Le \sqrt{2g}\, He^{1.5}$

$= 0.49 \times 197.6 \sqrt{2x32.2} \times 32^{1.5}$

$= 140653.59$ Cfs at H = 32

He $= 48$

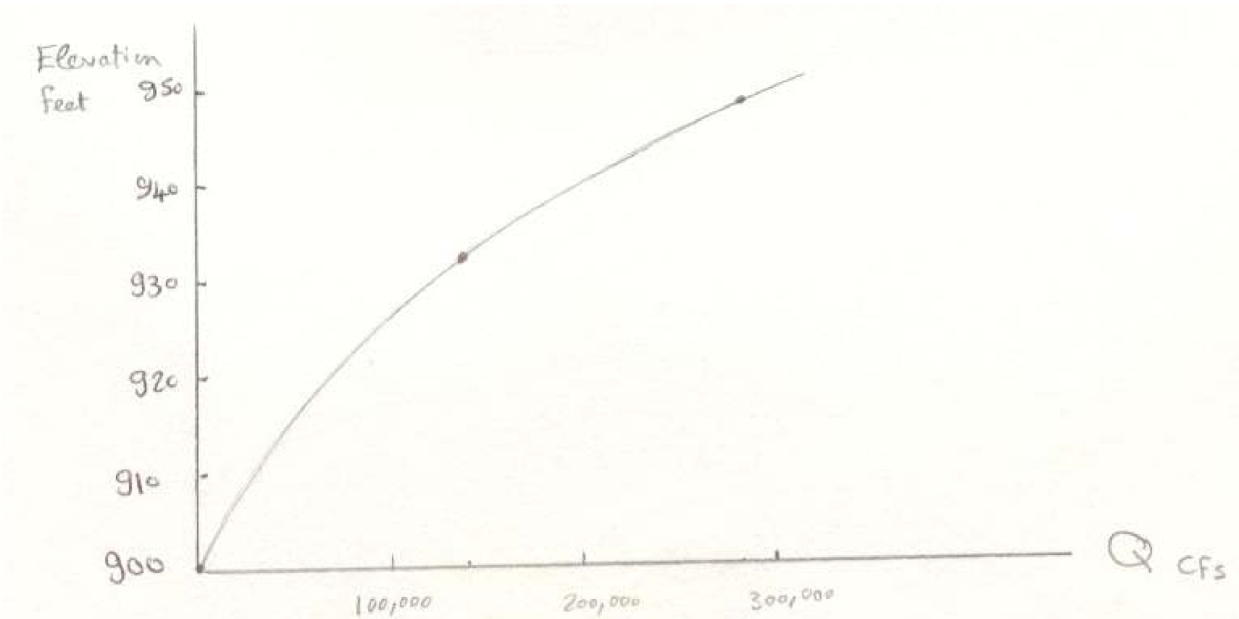
$\frac{Hc}{Hd} = \frac{48}{32} = 1.5$ Fig. 7.21 Kp $= -0.025$

Le $= 4 \times 50 - 2[3 \times (-0.025)]48 = 207.2$

Fig. 7.20 b $\frac{H}{Hd} = 1.5 \rightarrow c/c_1 = 1.05$ C = 1.05 x 0.49 = 0.5145

Q $= 0.5145 \text{ x } 207.2 \sqrt{2x32.2} \text{ x } 48^{1.5}$

Q = 284498.44 Cfs at H = 48



7.12

Q $= CLc\ H^{1.5}$

Q $= 3.8\ Lc\ H^{1.5}$

C $= 3.8 = Cd \sqrt{2g} = Cd\sqrt{2x32.2}$

For S I

C $= \dfrac{3.8}{\sqrt{2x32.2}}\sqrt{2x9.81} = 2.097$

Q $= C.\ Lc.\ H^{1.5}$

Q $= 2.057 \text{ x } 100 \text{ x } 10^{1.5} = 6631.295\ m^3/s$

7.13

(i) Flip bucket :

- so much spray un-desirable for road, bridge and electric equipments.
- Act enough submergence D.S
- Water level fluctuation
- current and eddies around the plunge pool

(ii) Roller bucket

(iii) Stilling basin :

- control the jump location
- Low apron level, may required a lots up cavitations and

concrete

- Chute blocks and baffle blocks and end stills are used in control the jump.

7.14

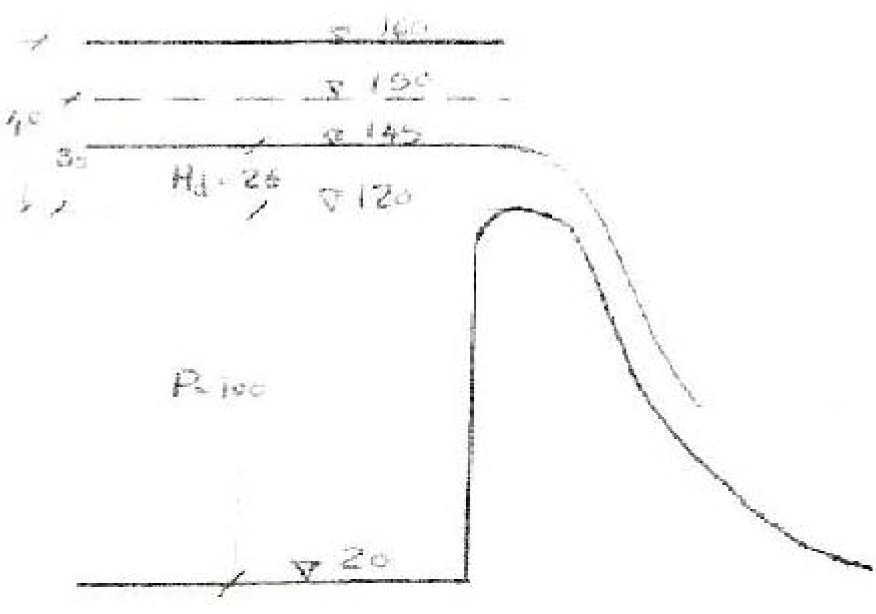


Fig. 7-20 $\dfrac{p}{Hd} = \dfrac{100}{25} = 4 \rightarrow$ Cd = 0.492

Fig. 7.21	Kp	= 0.0125	
	Le	= Ln – (N Kp + Ka) He	
	Le	= 60 x 5 – 2(4 x 0.0125 +0) 25	= 297.5
	Q	= 0.492 x 297.5 $\sqrt{2x32.2}$ x 25 $^{1.5}$	
		= 146826.67 Cfs	

$\frac{H}{Hd} = \frac{30}{25}$ = 1.2   Fig. 7.20 b   C/Cd = 1.01

C = 1.01 x 0.492 = 0.497

Fig. 7.21	Kp	= 0	
	Le	= 60 x 5 – 2 (4 x 0 + 0)25	= 300
	Q	= 0.497 x 300 $\sqrt{2x32.2}$ x 30$^{1.5}$	
		= 196608.54 Cfs	

$\frac{H}{Hd} = \frac{40}{25}$ = 1.6   Fig. 7.20 b   C/Cd = 1.07

C = 1.07 x 0.492 = 0.5264

Fig. 7.21	Kp	= -0.025	
	Le	= 5 x 60 – 2(4 x (-0.025)+ 0) 40	= 308
	Q	= 0.5264 x 308 x $\sqrt{2x32.2}$ x 40$^{1.5}$	
		= 329154.25 Cfs	

E_L	= 150	Q	= 196608.54 Cfs
E_L	= 160	Q	= 329154.25 Cfs

7.15

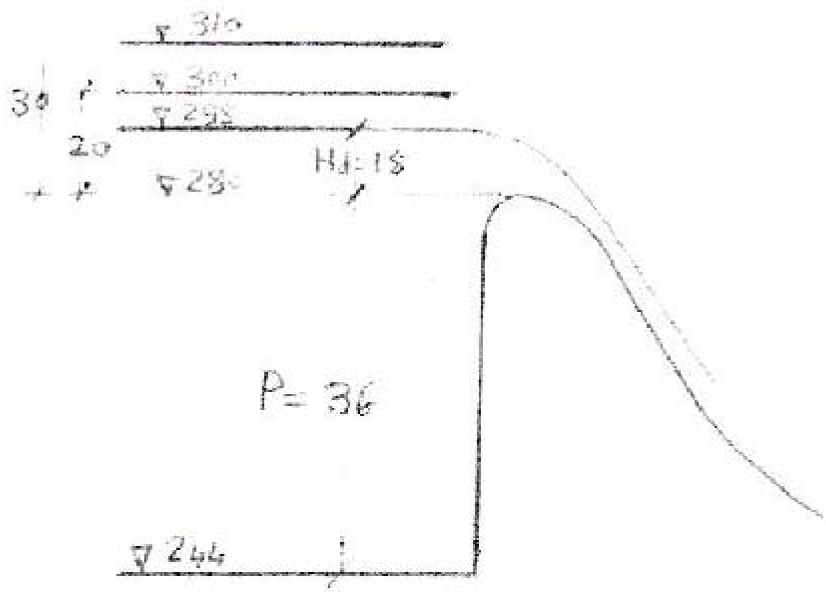


Fig. 7.20 Q

$\frac{p}{Hd} = \frac{36}{18} = 2$ Cd = 0.49 Le = 20

Q $= 0.49 \times 20 \times \sqrt{2x9.81}\, 18^{1.5} = 3315$ m^3/sec.

Elev. 300 $\frac{H}{Hd} = \frac{20}{18} = 1.11$ Fig. 7.20 b C/Cd = 1.05

C $= 1.05 \times 0.49 = 0.5145$

Q $= 0.5145 \times 20 \sqrt{2x9.81}\, 20^{1.5} = 4076.7$ m^3/sec.

Elev. 310 $\frac{H}{Hd} = \frac{30}{18} = 1.67$ Fig. 7.20 b C/Cd = 1.09

C $= 1.09 \times 0.49 = 0.5341$

Q $= 0.5341 \times 20\ \sqrt{2x9.81}\ \times 30^{1.5}$ $= 7774.7\ m^3/s$

7.16 Assume Hd = 30

Hd $= 0.75\ (652 - H_{crest}) = 30$ $\therefore\ H_{crest} = 612$

60000 $= Cd\ L\ \sqrt{2x32.2}\ \times 30^{1.5}$

$\frac{p}{Hd} = 2$ Cd = 49 Vs bed level = 552

60000 $= 0.49 \times L \times \sqrt{2x32.2}\ 30^{1.5}$

L = 92.9

Elev. 620 $\frac{H}{Hd} = \frac{8}{30} = 0.267$ Fig. 7.20 b C/Cd = 0.86

C = 0.86 x 0.49 = 0.42

Q $= 0.42 \times 92.9\ \sqrt{2x32.2}\ \times 8^{1.5} = 7085.05$ Cfs

Elev. 648 $\frac{H}{Hd} = \frac{36}{30} = 1.2$ Fig. 7.20 b C/Cd = 1.02

C = 1.02 x 0.49 = 0.5

Q $= 0.5 \times 92.9\ \sqrt{2x32.2}\ \times 36^{1.5} = 80516$ Cfs

7.17

$$\frac{Q}{\sqrt{g}} = A\sqrt{D}$$

$$\frac{800}{\sqrt{32.2}} \qquad = (B_0 + 5y)\,y\sqrt{\frac{(B_0+5y)y}{(B_0+25y)}}$$

$$140.98 \qquad = (15+y)y\sqrt{\frac{(15+y)y}{15+2y}}$$

Yc = 4.05

$$AR^{\frac{2}{3}} \qquad = \frac{nQ}{\sqrt{S_2}} \qquad = \frac{0.028\,x\,800}{\sqrt{0.05}}$$

$$\frac{\left[(15+y)y\right]^{\frac{5}{3}}}{\left[15+2\sqrt{2}y\right]^{\frac{2}{3}}} = 100.176$$

Yn_1 = 3.11

$$AR^{\frac{2}{3}} \qquad = \frac{nQ}{\sqrt{S_2}} \qquad = \frac{0.028\,x\,800}{\sqrt{0.0003}}$$

$$\frac{\left[(15+y)y\right]^{\frac{5}{3}}}{\left[15+2\sqrt{2}y\right]^{\frac{2}{3}}} = 1293.265$$

Yn_2 = 12.8

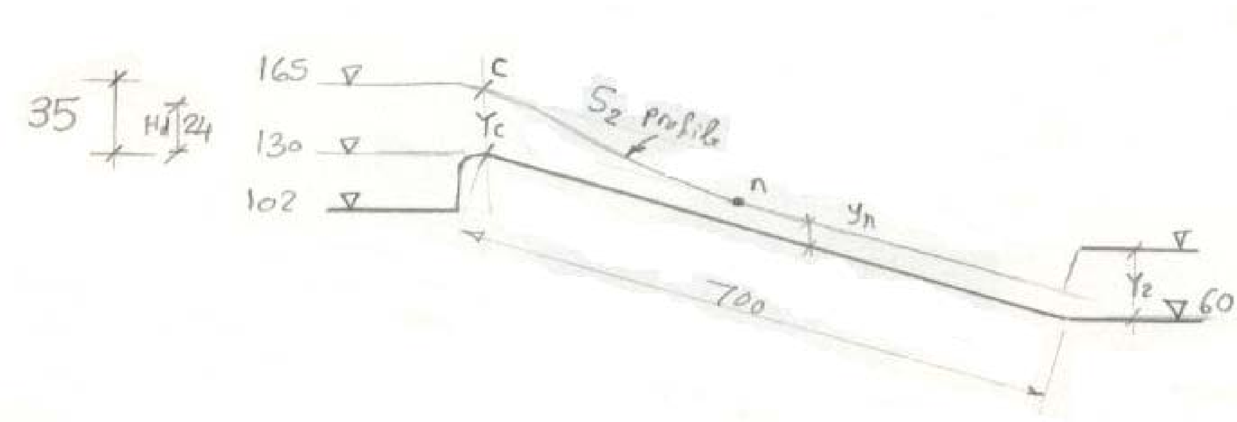
V1 = $\frac{800}{3.11(15+3.11)}$ = 14.204 ft/sec.

Fr_1 = $\frac{V}{\sqrt{gy}} = \frac{14.204}{\sqrt{32.2\,x\,3.11}} = 1.419$

Y2 = $\frac{3.11}{2}\left[\sqrt{1+8x1.419^2}-1\right]$ = 4.877

The jump will move upstream till the exact position where Y_1, Y_2 will be satisfied.

7.18



(i) L = 100'

$$\frac{p}{Hd} = \frac{28}{24} = 1.167 \qquad Cd = 0.483 \qquad \text{using Fig. 7.20 a}$$

At Elev. 165

$$\frac{H}{Hd} = \frac{35}{24} = 1.46 \qquad \text{Fig. 7.20 b} \qquad C/Cd = 1.05$$

C = Cd x 1.05 = 0.483 x 1.05 = 0.507

Q = 0.507 x 100 x $\sqrt{2x32.2}$ x $35^{1.5}$ = 84246.7 ft^3/sec.

(ii) assumen = 0.013 for concrete

To find critical depth

$$\frac{Q}{\sqrt{g}} = A\sqrt{D}$$

$$\frac{84246.7}{\sqrt{32.2}} = bY^{1.5} = 100\,y_0^{1.5}$$

Yc = 28.04

Sc $= \dfrac{n2Q2}{A2R\frac{4}{3}}$ $= \left[\dfrac{0.013\,x\,84246.7}{100\,x\,28.04}\right]^2 \dfrac{1}{\left(\dfrac{100\,x\,28.04}{100+2\,x\,28.04}\right)^{\frac{4}{3}}}$

= 0.00324

But So = $\dfrac{70}{700}$ = 0.1 $\therefore$ $S_0 \geq S_c$ Steep channel

To find the normal depth

$AR^{\frac{2}{3}} = \dfrac{nQ}{\sqrt{S_0}}$

$100\,yn\left(\dfrac{100y_n}{100+2y_n}\right)^{\frac{2}{3}} = \dfrac{0.013\,x\,84246.7}{\sqrt{0.1}}$

$Yn\left(\dfrac{100y_n}{100+2y_n}\right)^{\frac{2}{3}}$ = 34.6335

Trial and error yn = 8.96

From point C to point n, S2 profile. Then yn will be the lowest possible epth other than spilling. The type of the flow profile developed in the steep canal depends mainly on the tail water situation.

(iii) Y1 = 8.96 Fr1 = 5.5

Y2 $= \dfrac{Y_1}{2}\left[-1+\sqrt{1+8Fr^2}\right]$

$= \dfrac{8.96}{2}\left[-1+\sqrt{1+8\left(\dfrac{84246.7}{100\,x\,8.36}\right)^2 \dfrac{1}{32.2\,x\,8.96}}\right]$

Y2 = 65.81

Tail water level = 65.81 + 60 = 125.81

(iv) h2 = E1 – E2

$$= \left(y_1 + \frac{v_1^{\ 2}}{2g}\right) - \left(y_2 + \frac{v_1^{\ 2}}{2g}\right)$$

$$h2 = 8.96 + \left(\frac{84246.7}{100\,x\,8.96}\right)^2 \frac{1}{2\,x\,32.2} - \left[125.81 + \left(\frac{84246.7}{100\,x\,125.81}\right)^2 \frac{1}{2\,x\,32.2}\right]$$

= 19.73

7.19

Q = 50,000 Cfs

$$AR^{\frac{2}{3}} = \frac{nQ}{\sqrt{S_0}}$$

$$100\,y_n \frac{(100\,y_n)^{\frac{2}{3}}}{(100+2y_n)^{\frac{2}{3}}} = \frac{0.013\,x\,50000}{\sqrt{0.1}}$$

$$y_n \left(\frac{100\,y_n}{100+2y_n}\right)^{\frac{2}{3}} = 20555$$

Trial and error = 6.44

Y_1 = 6.44 $\quad v_1 = \frac{50000}{100\,x\,6.44}$ = 77.64 ft/sec. Fr_1 = 5.39

$$Y2 = \frac{Y1}{2}\left[-1+\sqrt{1+8Fr_1^{\ 2}}\right]$$

$$= \frac{6.44}{2}\ [-1+\sqrt{1+8(5.39)^2}\ ] = 45.989$$

But the tail water level = 72

Tw = 72 – 60 = 12

There is not enough submergence – use flop bucket

Flip bucket design:

$$h0 = \frac{v_1^2}{2g} = \frac{77.64^2}{2x32.2} = 93.6 \text{ ft.}$$

$Yb = 0 \quad \alpha = 30^0$

$$\frac{Xb}{h_0} = 2 \sin 2\alpha$$

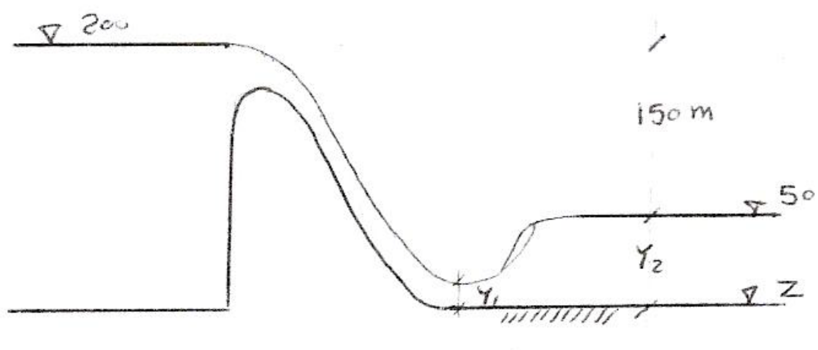
$Xb = 93.6 \text{ x } 2 \sin 60 = 162.12$

For design, Xb= 0.8 x 162.12 = 129.7

Bucket radius = 15 m = Sc

Bucket lip above the bucket invert by = (10/100) x 50 = 5

7.20



Invert elev = z

$Y_2 = 50 - z$

$V_1 = \sqrt{2xg(200-z)}$

$Q = Bv_1y_1$

$2700 = 50 \text{ y1 } \sqrt{2x9.81(200-z)}$

$$Y_1 = \frac{2700}{50\sqrt{19.62(200-z)}} = \frac{12.19}{\sqrt{200-z}}$$

$$Fr_1^2 = \frac{v_1^2}{gy_1} = \frac{19.62(200-z)(200-z)^{½}}{9.81x12.19}$$

$$= 0.1641\ (200\text{-}z)^{1.5}$$

$$\frac{y_2}{y_1} = \frac{1}{2}(-1+\sqrt{1+8Fr_1^2})$$

$$\frac{50-z}{12.19\big/\sqrt{200-z}} = \frac{1}{2}\left[-1+\sqrt{1+8x0.1641(200-z)_{1.5}}\right]$$

$$(50\text{-}z)\sqrt{200-z} = -6.095+6.095\sqrt{1+1.3128(200-z)^{1.5}}$$

Trial and error z = 25.06 m

Y_2 = 24.94 m

If one-row of battle block and en sill is used the invert deviation may be raised and when it saves excavation and stabilizes the jump.

Chapter 8
COMPUTATION OF RAPIDLY VARIED FLOW

8.1

The computer program can be written in two ways:

a. By using a false transient approach wherein the two-dimensional unsteady flow equations are solved.
b. By solving the steady flow equations.

Note that in both the approaches, the governing equations are hyperbolic partial differential equations. However, the equations are two dimensional in the first approach and are one-dimensional in the second approach. Herein, results are represented for the first approach. In this, concepts of coordinate transformation and artificial viscosity are used. Results by MacCormack scheme and by Lax scheme are presented in Fig. 8.1.

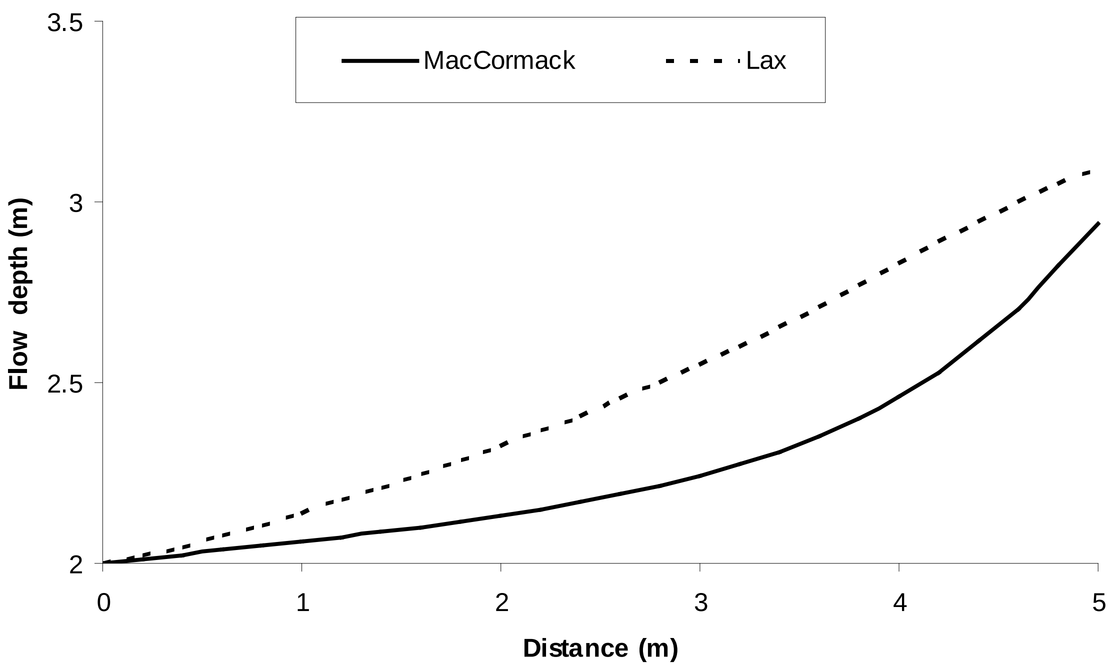


Figure 8.1: Surface profile in a channel contraction

8.2
Flow depths at all the points are computed by using the computer program developed in 8.1. The maximum flow depth at any section is considered to be the shok wave. Thus, the height and location of the shock wave is determined. Results for the height of the shock wave are presented in Fig. 8.2. Both MacCormack scheme and the Lax scheme is used to determine the shock wave.

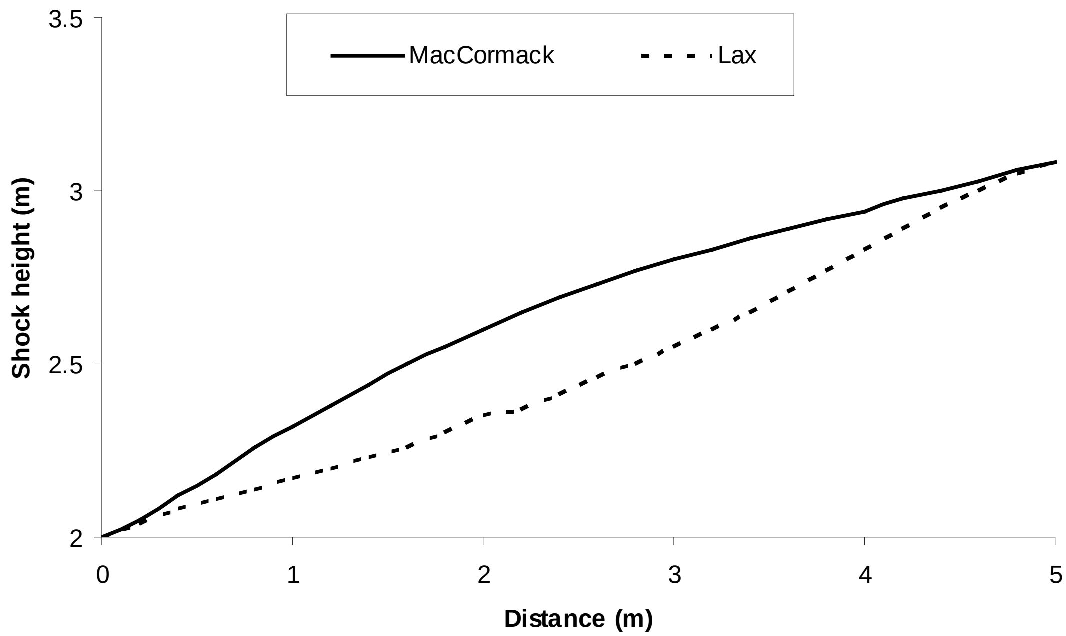


Figure 8.2: Height of the shockwave in a channel contraction

8.3
MacCormack scheme is suitable for both sub- and super critical flows. Therefore, the computer program given in the appendix can be used to compute the flow. Note that this problem is same as the computation of a hydraulic jump.

Chapter 9
CHANNEL DESIGN

9.1

Given Q $= 50\ m^3/S$

$S_0 = 0.0002$

Channel material : Rock

Design the canal.

Solution:

For rock in good conditions n = 0.035 and side slopes could be almost vertical (S = 0.25)

From Monning's equation, we get,

$$AR^{\frac{2}{3}}\ \frac{nQ}{S_0^{\frac{1}{2}}} = \frac{(0.035)(50)}{(0.0002)^{\frac{1}{2}}} = 123.74$$

For S = 0.25, we can take $B_0 = 2y$, then :

$A = (2y + 0.25y)\ y \quad = 9/4\ y^2$

$P = 2y + ½\sqrt{1.25}y \quad = 2.259\ y$

$R = \frac{9}{4}\frac{y^2}{2.559y} \quad = 0.879\ y$

And

$$AR^{\frac{2}{3}} = (2.25y^2)(0.879y)^{\frac{2}{3}}$$

$123.74 = 2.0646\ y^{2.67}$

Or

y = 4.63 m.

Then $B_0 = 2y = 9.26$ m

SELECT CHANNEL BOTTOM WIDTH

Use $B_0 = 10.0$ m, then for $AR^{2/3} = 123.74$, the value of y obtained by solving:

$$\frac{[(10+1.25y)y]^{\frac{5}{3}}}{(10+2.062y)^{\frac{2}{3}}} = 123.74$$

The answer is $y = 5.37$ m.

ADD APPROPRIATE FREEBOARD.

Freeboard equation : $F_b = \sqrt{0.8x5.37} = 2.0$ m

From Table 9.1, $F_b = 0.75$ m

Use $F_b = 0.75$ m

DETERMINE CHANNEL DEPTH

The total depth is :

$Y_T = 5.37 + 0.75 = 6.12$m

CHECK MINIMUM ALLOWANCE VELOCITY

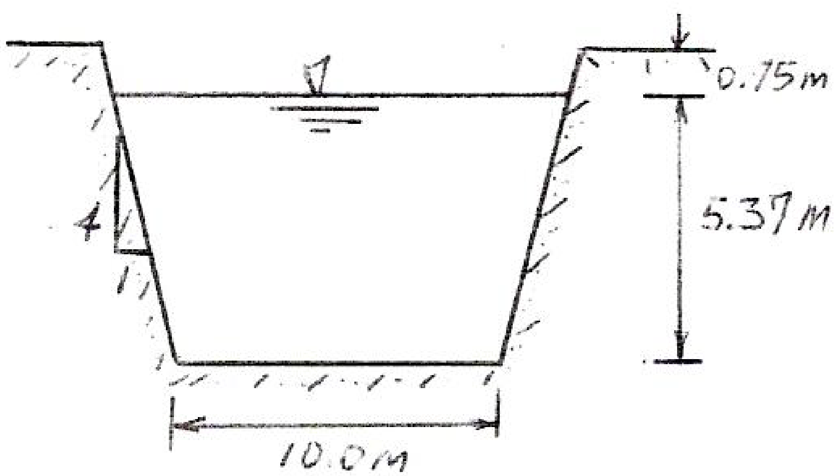
Flow Area = 60.9m2

Flow Velocity : Q/A = 50/60.9 = 0.82 m/s > V_{min}. Ok

(Vmin. = 06.m/s)

SKETCH CHANNEL CROSS-SECTION

SKETCH CHANNEL CROSS-SECTION



9.2

Design :

Irrigation Channel

100 km^2 farmland

0.1 $m^3/s/km^2$ (water demand)

$S_0 = 1/2000$

Soil is stuff clay.

Solution:

1. Permissible –Velocity Method:

We choose a trapezoidal channel with side slopes of 1:1 (from Table 9.2). For stiff clay n = 0.025. The discharge is Q = 100 km^2 x 0.1 m^3/s/km2)

$= 10\ m^3/s$

From Table 9.3 the permissible velocity is : Vp = 1.8 m/s.

A = Q/V = 10/1.8 = 5.56 m2

$$R = [\frac{nQ}{AS_0^{\frac{1}{2}}}]^{1.5}$$

$$R = [\frac{(0.025)(10)}{(0.0005)^{\frac{1}{2}}(5.56)}]^{1.5}$$

R = 2.85 m (Hydraulic Radius)

Water perimeter : P = A/R = 1.95 m

Then, compute the water depth :

$A = (B_0 + y)y = 5.56$

$P = B_0 + 2\ldots..y = 1.95$

Or $B_0 = 1.95\text{-}2\sqrt{2}y$

Then

$A = (1.95\text{-}2\sqrt{2}y\text{+}y)y = 5.56$

or $1.828\ y^2 – 1.95y + 5.56 = 0$

This equation has not real roots ! The reason for the inconsistency is that the permissible velocity is very high for this flat channel. The problem could be solved by reducing the channel flow velocity.

Let's try: $V_p = 0.9$ m/s

$A = Q/v_p = 10/0.9 = 11.11$ m2

$$R = \left[\frac{0.025x10}{\sqrt{0.0005x11.11}}\right]^{1.5} = 1.01 \text{ m}$$

$P = 11.01$ m

Solve for 'y' now :

$A = (B_0 + y)y = 11.11$

$P = B_0 + 2\sqrt{2}\, y = 11.01$

=> $B_0 = 11.01 - 2\sqrt{2}\, y$

$A = (11.01 - 2\sqrt{2}\, y + y)y = 11.11$

or $1.828y^2 – 11.01\, y + 11.11 = 0$

Solving for 'y', we get.

$Y = 1.282$ m

Therefore, $B_0 = 7.38$ m.

The other root of the quadratic equation is real, but less than zero. The result is physically impossible.

The freeboard could be : $F_b = \sqrt{0.8x1.28} = 1.01$ m

or 0.75 m, depending on the criterion followed.

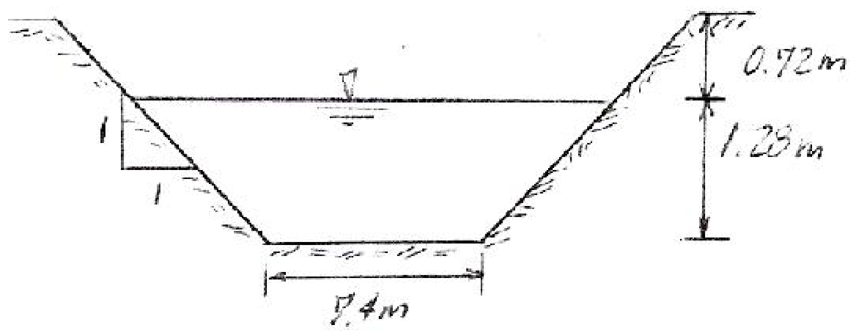
Using 0.75 (Table 9.1), the cross-section depth is

$D = 1.287 + 0.75 = 2.03$ m

Choose $d = 2.0$ m

CHANNEL SKETCH.

CHANNEL SKETCH :



9.3

Given:

Runoff Area = 200 km^2

Flow = 0.5 $m^3/s/m^2$

Material size = 2 mm.

S_0 = 0.00002

Design the channel by using :

(i) Permissible velocity method,

(ii) Tractive Force method and (iii) Regime Theory.

1. **Permissible Velocity Method**:

For fine sand (loose sandy soil) we have

Side slope = 2:1 (Trapezoidal Channel)

Manning's n = 0.020

Permissible Velocity = 0.57 m/s.

Then A = Q/V = 100/0.57 = 175.439 m^2

$$R = [\frac{nQ}{AS_0^{\frac{1}{2}}}]^{1.5} = [\frac{0.02x100}{175.439x\sqrt{0.00002}}]^{1.5}$$

$R = 4.0699$ m

$P = A/R = \dfrac{175.439}{4.0699} = 43.106$ m

$P = B_0 + 2\sqrt{1+5^2} = 43.106$

or

$B_0 = 43.106 - 2\sqrt{5}\,y$

Also $A = (B_0 + 5y)y = 175.439$

Substituting B_0 in the expression for the area, we get,

$(43.106 - 2\sqrt{5}\,y + 2y)y = 175.439$

or

$-2.472\,y^2 + 43.106\,y - 175.439 = 0$

The solutions for 'y' are :

$Y_0 = 6.472$ m, $y^2 = 10.966$ m

The first root (y1) gives $B_0 = 14.16$ m, while the second gives $B_0 < 1$ which is not possible.
Add 0.78 m of freeboard to have a total height of 7.25 m and use $B_0 = 14$m.

CHANNEL DIMENSIONS USING PREMISSIBLE VELOCITY METHOD.

2. **TRACTIVE FORCE METHOD**.

This is a fine method (<5 mm), therefore, the effect of angle of repose is negligible.

The permissible shoes stress is $J_{crit} = 0.06$ lb/ft^2 or 2.872 M/m^2 (From Figure 9.4 assuming clew water).

The unit tractive force on the side is 0.76 @ S0y

=> $J_t = 0.76 \times 999 \times 9.81 \times 0.00002 \times y$

Equating 'J_t' to 'J_{crit}', we get, y = 19.28 m.

This method predicts a very high value of 'y'. The reason is that the channel slope is extremely small. This results in a very small coefficient of tractive force (0.76&S_0) and, therefore, a very high flow depth. Actually, Manning's equation predicts a negative value of the bottom width, which is totally unrealistic.

3. **REGIME THEORY**

From table 9.4, the silt factor can be taken as low as 1.44. The higher the value of silt factor, the wider the channel is, then we look for a better proportionate channel.

$f_s = 1.44$

$P = 4.75\ \sqrt{Q} = 4.75 \times 10 = 47.5$ m

$R = 0.47\ (Q/f_s)^{1/3} = 1.932$ m

$A = PR = 91.76\ m^2$.

Also

$A = (B_0 + 2y)y = 91.76$

$P = B_0 + 2\sqrt{(1+4)}\ y = 47.5$

Eliminating B_0 from these two equations, we get,

$-2.472\ y^2 + 47.5\ y - 91.76 = 0$

The roots are : $y_1 = 2.18$ m and $y_2 = 17.04$ m

For $y = y_1$, $B_0 = 37.76$m. The other root gives $B_0 < 0$

Then, use $y = 2.18$, $B_0 = 37$ m and 0.82 m of freeboard. However, the channel is very wide and shallow. This is not a good option. The permissible velocity method gave a better proportionate channel.

9.4

Design a storm sewer,

Area = 4 km^2

Runoff = 0.15 $m^3/s/km^2$

S0 = 1/2000

Try using a concrete pipe : n = 0.013

Discharge Q = 0.15 x 4 = 0.6 $m^{3/5}$

Slope S_0 = 0.0005

$$\frac{nQ}{\sqrt{S_0}} = \frac{(0.013)(0.6)}{\sqrt{0.0005}} = 0.35$$

Then, $\frac{AR^{2/3}}{D^{2/3}} = \frac{0.35}{D2.667} = 0.16$

The valve 0.16 was taken using Figure 4.5. The value yn/D0 was used approximately as 0.5 to get $\frac{AR^{2/3}}{D^{2/3}}$ =0.16. This ensures that the final cross-section is close to the most efficient hydraulic section.

Then D = 1.34m, use D = 1.37m (54 inches pipe). Then θ = 3.11 rad and the flow depth is y = 0.67m.

Also : A = 0.72 m^2

V = Q/A = 0.6/0.72 = 0.83m

$V > V_{min}$. Ok.

Use a circular concrete pipe with D = 1.37 m

9.5

Design an irrigation channel.

Q = 1100 ft^3/s

S0 = 2 ft/mile = 2/5280 = 0.00038

Soil is clay.

Solution:

Tractive Force Method.

(a) For clay we select a trapezoidal channel with s=1 (Table 9.2) and n = 0.024.

For fairly compact clay and a voids ratio of 0.8, the critical shear stress is (Fig. 9.5)

Jc = 0.17 lb/ft^2

(b) Determine the flow depth.

Jc = 0.17 = 0.76 HyS_0

$$Y = \frac{0.17}{(0.76)(62.4)(0.00038)} = 9.43 \text{ ft.}$$

(c) Compute B_0 : For clay use n = 0.024, then

$$\frac{[(B_0 + y)y]^{\frac{5}{3}}}{[B_0 + 2\sqrt{2}y]^{\frac{2}{3}}} = \frac{(0.024)(1100)}{1.44x\sqrt{0.00038}} = 908.92$$

For y = 9.43 ft, the solution for B_0 is B_0 = 19.95ft.

Use B_0 = 20 ft (Try solving this problem by the permissible velocity method!).

Use a freeboard of 2.57 ft. to have a total channel depth of :

d = 9.43 + 2.57 = 12 ft.

9.6

Design a flood control channel for

Q = 500ft^3/s

S_0 = 0.003

Channel paved with bricks (n = 0.013)

This is a non-erodible channel which could be designed using the criterion of the most efficient hydraulic section. A trapezoidal section is selected.

The most efficient trapezoidal section is half of a hexagon for which

A = √3 y2

P = 2√3 y

R = ½ y

Section Factor : $AR2/3 = \frac{(0.013)(500)}{1.49(0.003)^{\frac{1}{2}}} = 79.65$

Then $\sqrt{3}\ y^2 \left(\frac{y}{2}\right)^{2/3} = 79.65$

$1.091y^{2.667} = 79.65$

Finally $y = 5$ft.

From here, we get,

$A = 43.3\ ft^2$

$A = (B_0 + \frac{\sqrt{3}}{3} y)y = 43.3\ ft^2$

$B_0 = \sqrt{3}\ y - \frac{\sqrt{3}}{3} y = 2\frac{\sqrt{3}}{3} y$

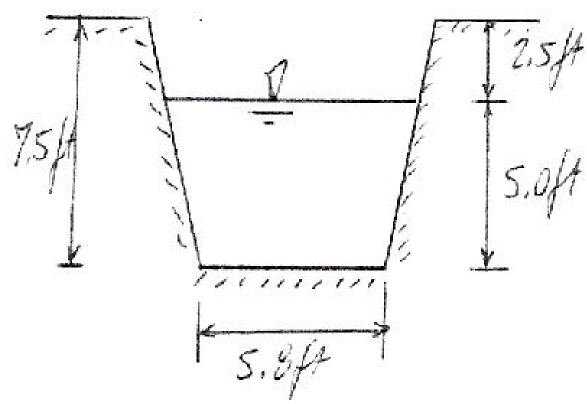
Finally $B_0 = 5.77$ ft.

Flow Velocity = Q/A = 500/43.3 = 11.55 ft/s.

This velocity is acceptable for a paved channel.

Add 2.5 ft. of freeboard.

Channel Sketch.



9.7

Given

2 km long tunnel with horseshoe section (standard).

Material : sound rock

Inlet bottom level : 100 m

Exit bottom level : 98.5 m

$Q = 100\ m^3/s$ (design flow)

Free flow in the tunnel

Downstream water level 102 m

(a) Plot the water surface profile for the design flow.

(b) Plot the water surface profile for $Q = 150\ m^3/s$ and downstream elevation = 105 m.

Solution:

(a) First select the tunnel dimensions. Use

n = 0.035 (rock)

$$S0 = \frac{100 - 98.5}{2000} = 0.00075$$

Section Factor: $$AR^{2/3} = \frac{(0.035)(100)}{(0.00075)^{½}} = 127.80$$

A DIAMETER FOR THE HORSE-SHOE SECTION HAS TO BE ASSUMED HERE. A TABLE CONTAINING THE PROPERTIES OF A STANDARD HORSE-SHOE SECTION MAY ALSO BE NEEDED.

For a horse-shoe section at full capacity

$$\frac{A}{D^2} = 0.8293 \text{ and } R/D = 0.2538$$

where D is the diameter of a standard horse-shoe section.

Then $AR^{2/3} = (0.8293\ D^2)\ (0.2538D)^{2/3} = 127.80$

Therefore, the minimum diameter should be D = 9.32 m.

Try D = 15 m and compute the normal depth.

By using a table of Standard horse-shoe section, we get:

y (m)	**A (m^2)**	**R (m)**	**$AR^{2/3}$ ($m^{8/3}$)**
5	60.37	2.971	124.78 Low !
5.1	62.55	3.03	131.10 High !

The normal depth is 5.05 m (approximated from the values in the previous table)

For uniform conditions, the flow velocity is V = $\frac{100}{61.46}$

V = 1.63 m/s, The Frovde number is :

$$\frac{V}{\sqrt{gD_h}} = \frac{1.62}{\sqrt{9.81x14.6}} = 0.136 < 1$$

Then, the flow is sub critical.

The critical depth is computed next:

Section factor : $\frac{Q}{\sqrt{g}} = \frac{A^{3/2}}{T^{1/2}}$

where $\frac{Q}{\sqrt{g}}$ = 100/√9.81 = 31.93

Also:

y (m)	A (m^2)	T (m)	$A^{3/2}T^{-1/2}$	Remarks
3.0	32.78	13.62	50.85	high
1.95	18.88	12.87	22.87	low
2.55	26.73	8.83	46.49	high
2.25	22.77	13.10	30.01	low
2.40	24.75	13.21	33.87	high

From the last two lines we know that y_0 = 2.3 m. This confirms that y_0 < yn and the channel is MILD.

The downstream depth is 102-98.5 = 3.5 m < y_n.

Therefore, the profile is M2 type.

Table 9.7 shows the computations of the water surface profile for Q = 100 $m^{3/5}$.

(b) Flow profile for Q = 150 $m^{3/5}$ and downstream depth of 105-98.5 = 6.5 m.

Section Factor : $AR^{2/3}$ = 191.70

Normal depth computation:

y (m)	A (m^2)	R (m)	$AR^{2/3}$ ($m^{8/3}$)	Remarks
6.0	75.83	3.738	170.72	low

6.3	80.28	3.483	184.45	high
6.45	82.51	3.534	191.4	Ok

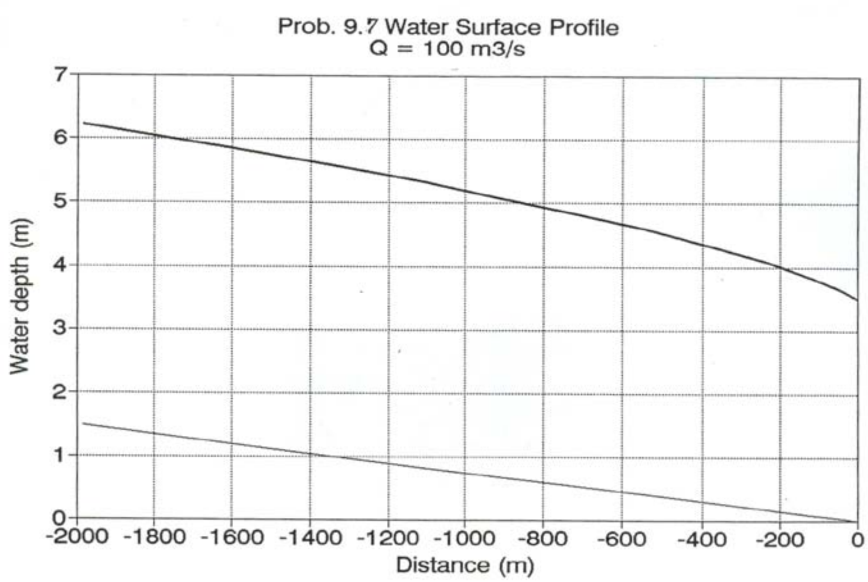
From the table we get y_n = 6.45 m. In this case the normal depth is equal to the downstream water depth; therefore, the flow in the tunnel is uniform at y_n = 6.5m.

The possibility of a smaller horse-shoe section should be considered for this problem. In this case, a new diameter (between 15m and 10 m) should be assumed and the computations repeated again.

Table 9-7
Direct Step Method

Q = 100 m3/s
So = 0.0008 n = 0.035

y	A	R	V	Sf	Sfave	So-Sfave	E	E2-E1	x2-x1	x2
m	m2	m	m/s				m	m	m	m
3.50	38.99	2.262	2.565	0.002714			3.83527			0
3.60	41.06	2.340	2.435	0.002339	0.002526	-0.00178	3.902317	0.067048	-37.74	-37.74
3.75	43.18	2.417	2.316	0.002026	0.002183	-0.00143	4.02336	0.121043	-84.50	-122.24
3.90	45.29	2.493	2.208	0.001767	0.001896	-0.00115	4.148483	0.125122	-109.14	-231.38
4.20	49.55	2.637	2.018	0.00137	0.001568	-0.00082	4.407593	0.259111	-316.70	-548.07
4.50	53.84	2.775	1.857	0.001084	0.001227	-0.00048	4.675829	0.268236	-562.75	-1110.82
4.65	56.00	2.843	1.786	0.00097	0.001027	-0.00028	4.812527	0.136698	-493.79	-1604.62
4.73	57.09	2.875	1.752	0.000919	0.000945	-0.00019	4.88638	0.073853	-379.37	-1983.98



9.8

Design a grass-lined channel for Q = 100 ft^3/s, $S_0 = 0.03$

Solution : Use Permissible Velocity Method.

For Bermuda grass and sandy silt the

permissible velocity is V – 1.8 m/s = 5.9 ft/s (from Table 9.3)

Use n = 0.030 (Manning's n is grossed channels varies depending on the depth of flow, the shape and slope of the channel and the grass growth conditions. A more detail analysis is recommended for any particular case and circumstances).

Lateral slope : use 2 : 1 (Tropezoidal cross-section)

Then : $A = \frac{Q}{V} = \frac{100}{5.9} = 16.95 ft^2$

Section Factor : $AR^{2/3} = \frac{0.03x100}{1.49x\sqrt{0.03}} = 11.62$

Also R = 0.57 ft and P = 29.84 ft.

Then : $(B_0 + y)y = 16.95$ => $B_0 = 29.84\text{-}2\sqrt{5}\, y$

Substituting B_0 in the section factor formula and solving for y, we get,

$3.47 y^2 - 29.84 y + 16.95 = 0$

From here : y = 0.61ft. Therefore, $B_0 = 27.11$ ft.

The channel is very wide compared to the water depth. In order to have better proportional dimensions, we can try s=6. In this case, we have,

$P = B_0 + 2\sqrt{1+36}\, y = 29.84$

Then, $B_0 = 29.84\text{-}2\sqrt{37}\, y$

and $(29.84 y - 11.165 y^2) = 16.95$

Finally we get, $y_1 = 0.82$ ft. and $B_0 = 19.87$ ft.

$$Y_2 = 1.85 \text{ ft. and } B_0 = 7.29 \text{ ft.}$$

We have two options for the channel. The top width in both cases will be similar. Take the second choice as our option:

$y = 1.85$ ft.

$B_0 = 7.29$ ft.

The freeboard given by the US Bureau of Reclamation formula is

$$F_b = \sqrt{0.8x1.85} = 1.2 \text{ ft.}$$

Chapter 10
SPECIAL TOPICS

10.1 Given : Rectangular channel connecting lakes B & C

B_0 = 10 m

L = 1000m

S_0 = 0.0002

n = 0.013

Channel bottom at Lake B entrance = 94 m.

(i) Plot the delivery curve if the water level in Lake B is of constant elevation El = 100 m and the water level in Lack C is variable.

Solution:

Use the computer program given in Appendix D4 for simultaneous solution of the energy equations for the channel reaches.

The input variable are :

NCHAN	= 1	sΦ	= 0.0002
MAXI	= 25	CHL	= 1000
G	= 9.81m/s^2	CMAN	= 0.013
TOL	= 0.0001	BOT	= 10
TOP	= 6.0 m	2Φ	= 0.0
Y_B	= 6.0 m	NR	= 20
CHELEV	= 94	ALPHA= 1	

QΦ is VARIED from 20 m^3/s to 251 m^3/s to get the delivery curve.

For every discharge, compute the critical depth and check that $y_0 \leq y_d$ where y_d = downstream depth (given by the depth at section 2.1 in the program output) and y_0 = critical depth.

The critical depth is obtained from

$$y_0 = \sqrt[3]{\frac{Q^2}{B^2 g}} = \sqrt[3]{\frac{Q^2}{981}}$$

The discharge corresponding to uniform flow conditions in this channel is

$Q_n = 127.41\ m^3/s$

However, the channel is <u>short</u> ; therefore, the discharge increases to a maximum of approximately 252 m^3/s. These conditions can be verified with the computer program.

Table 10.1a shows the results. The delivery curve is obtained by plotting Q vs y_d from this table.

(ii) Plot the channel discharge for different upstream lake levels (Lake B) if the water level in Lake C is El = 98 m.

The maximum discharge is obtained when the downstream depth is critical, this is

$$Q_{MAX} = \sqrt{Y_d{}^3 Y}\ B_0$$

Or

$$Q_{max} = \sqrt{[98-(94-1000x0.0002)]^3 x9.81}\,x10$$

$$Q_{max} = 269m^{3/5}$$

The normal discharge is given by

$$Q_n = \left(\frac{4.2x10}{0.013}\right)\left(\frac{4.2x10}{4.2x2+10}\right)^{\frac{2}{3}}\sqrt{0.0002}$$

$$Q_n = 79.21\ m^3/s$$

Other points for the discharge curve can be obtained by using the Standard Step method program given in appendix D2. The input data for this program is

Bϕ	= 10 m	ZD	= 93.8
S	= 0.0	G	= 9.81 m/s2

$S\phi$ = 0.0002 ALPHA= 1.0

y_D = 4.2 m

x(I) → use N Values between 0 and -1000 m

Q → Variable

The upstream depth for every flow corresponds to the valve of y at station 1000.

Table 10-1b shows the Q and Y_{up} Valves.

(iii) Plot a diagram between the water levels in both lakes and the channel discharge if the water levels in both lakes are variable.

Take for example Q=200M^3/s. Use the Standard Step method program given in Appendix D2. For the same discharge input different Valves of downstream depth, y_d, and determine the upstream depth, y_{up}. Table 10-1c shows the results.

Repeat the same process with different discharge and plot y_d vs y_{up} for each flow.

Table 10-1a
Delivery Curve

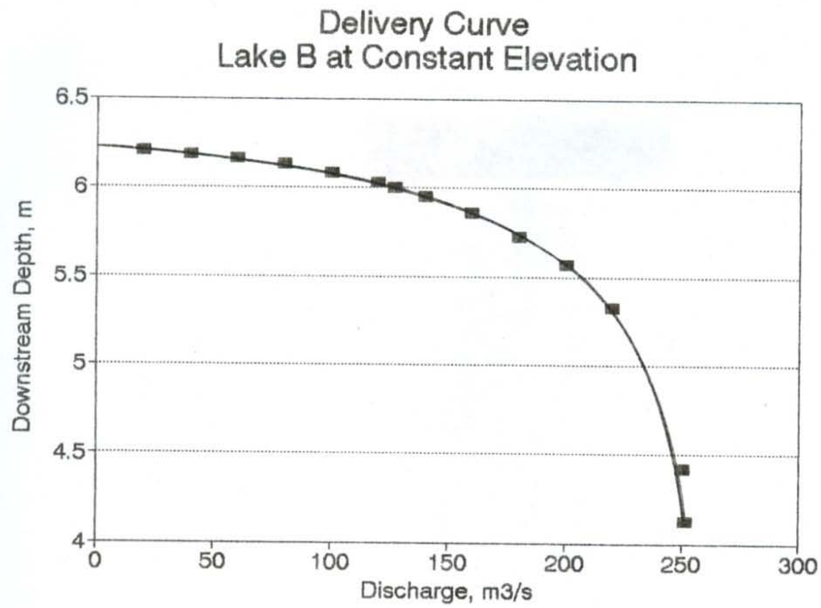
Q	yc	yd	Curve Type
m3/s	m	m	
20.0	0.74	6.20	M1
40.0	1.18	6.18	M1
60.0	1.54	6.16	M1
80.0	1.87	6.13	M1
100.0	2.17	6.08	M1
120.0	2.45	6.03	M1
127.4	2.55	6.00	Uniform
140.0	2.71	5.95	M2
160.0	2.97	5.86	M2
180.0	3.21	5.73	M2
200.0	3.44	5.57	M2
220.0	3.67	5.33	M2
250.0	3.99	4.42	M2
251.8	4.013	4.123	M2

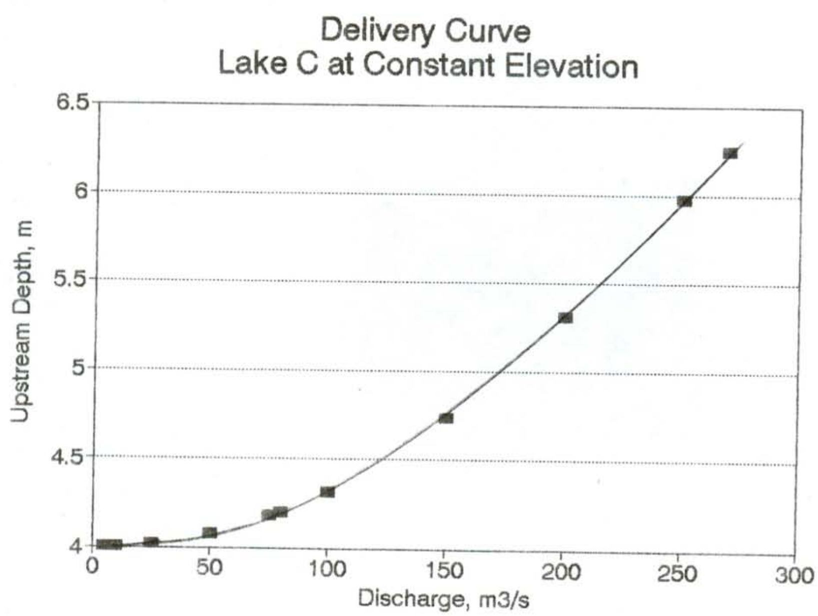
Table 10-1b
Lake B Level Curve

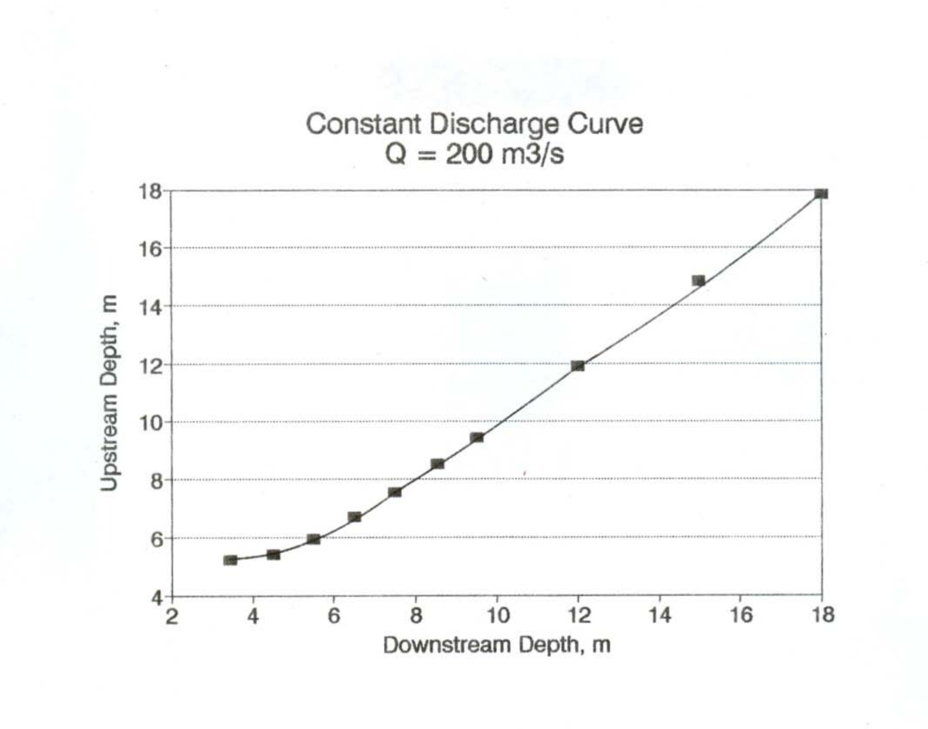
Q	yup
m3/s	m
269.0	6.250
250.0	5.980
200.0	5.310
150.0	4.740
100.0	4.320
80.0	4.200
75.0	4.179
50.0	4.079
25.0	4.020
10.0	4.003
5.0	4.001

Table 10-1c
Constant Discharge Curve
Q = 200 m3/s

yd	yu
m	m
3.45	5.21
4.50	5.41
5.50	5.95
6.50	6.71
7.50	7.58
8.55	8.55
9.50	9.45
12.00	11.89
15.00	14.85
18.00	17.83


Delivery Curve
Lake B at Constant Elevation
Downstream Depth, m
6.5
6
5.5
5
4.5
4
0
50
100
150
200
250
300
Discharge, m3/s


Delivery Curve
Lake C at Constant Elevation
Upstream Depth, m
6.5
6
5.5
5
4.5
4
0
50
100
150
200
250
300
Discharge, m3/s



10.2

For a concrete spillway with $\theta = 30^0$ and H_o = 8m, determine the development of the boundary layer thickness along the spillway length.

Solution:

For a concrete spillway take k_5=0.5mm and n=0.013.

(k_5=equivalent sand-roughness, n=Manning's coefficient).

The boundary layer development can be computed using equation 10-5.

$$\frac{\partial}{X_s} = 0.021\left(\frac{X_s}{h_s}\right)^{0.11}\left(\frac{K_s}{X_s}\right)^{0.10}$$

For our spillway h_s/x_s = cos30 = 0.866

Or

$$\delta = 0.0213\,(0.0005)^{0.1} X_5^{1.1}$$

$$\delta = 0.010\, X_5^{1.1}$$

Table 10.2 shows the boundary layer development Note that X_s is measured from the water surface over the spillway crest. The distance along the Spillway length is denoted by x.

TABLE 10.2 Boundary Layer Growth

X_5(m)	X(m)	δ (cm)	X_5(m)	X(m)	δ (cm)
6.93	0.0	8.4	12	5.07	15.3
8.00	1.07	9.8	15	8.07	19.6
10	3.07	12.5			

10.3

Given: Same spillway as problem 10.2

Determine: $\bar{c}$, y_{99}, $\bar{f}$ e, ye .

Solution:

The average air concentration depends upon the angle θ ; then, for $\theta = 30^0$:

$$\bar{C} = 0.75(\sin\theta)^{0.75} \qquad \text{(Eq.10.6)}$$

$$\boxed{\bar{C} = 0.446 \quad \text{or} \quad 44.6\ \%}$$

To estimate the uniform water depth at 99 % air concentration (y99) we need to computed the water depth corresponding to pure water. This is

$$Y_w = \left(\frac{nq}{\sqrt{S_0}}\right)^{3/5} \qquad \text{(wide rectangular channel)}$$

Where $q = \theta/L = C_d H_e^{1.5}$, H_e = total energy head on the crest. Neglecting the velocity head and assuming $H_o = H_d$ a typical valve for C_d is 4.0 (When H_d is in feet). Then:

$$q = (8x3.28)^{1.5}x0.4 = 53.76\ ft^3/ft$$

$$q = 5.0\ m^3/s\text{-}m$$

Then $y_w = \left(\frac{0.013x5}{\sqrt{\tan 30^2}}\right)^{0.6} = 0.23\ m$

From Eq.10.7 we get:

$$\text{ygg} = \text{yw}+1.35\text{yw}\left[\frac{\sin^3\theta y_w}{n^2 g^3}\right]^{0.25}$$

or, substituting the numbers : $\boxed{Y_{99}=0.43\text{m}}$

Given f_w = 0.015 we get 'fe' from Eq.10.8

as: fe $= \dfrac{f_w}{1+10c^{-4}}$

or

$\boxed{fe=0.0107}$

Finally the bulk flow depth is given by Eq.10.12

$$Ye = \left(\frac{f_e q^2}{8gS_0}\right)^{\frac{1}{3}}$$

$\boxed{ye=0.18\text{m}}$

10.4

Given : A box culvert 2m wide by 4m high

$S_0=0.005$

n=0.013

Downstream end un-submerged

Compute the rating curve

Solution:

Assuming inlet control we compute the culvert discharge using the weir and orifice equations.

For un-submerged entrance (H<1.2D)

$$Q = \frac{2}{3}CBH\sqrt{\frac{2}{3}gh}$$

Use B = 2m, C = 0.9 and g = 9.81 m/s^2 to get

$$Q = 3.07H^{3/2} \quad \text{if } H < 4.8\text{m}$$

For submerged entrance (H > 1.2D)

Q = CBD $CBD\sqrt{2g(H-CD)}$

Use C = 0.6 to get Q = 21.26 $\sqrt{H-2.4}$, If H > 4.8m

Also the critical and normal depth are given by

$$Yc = \sqrt[3]{\frac{Q^2}{Bg}} = \sqrt[3]{Q^2/19.62}$$

And $\dfrac{(By)^{\frac{5}{3}}}{(2y+B)^{\frac{2}{3}}}$ = 0.184 Q (use S_0 = 0.005)

Table 10.4 shows the results : head, H, discharge critical and normal depths.

For flow between 0.5m^3/5 and 43m3/5 the control is at the inlet (supercritical flow).

Therefore in this range of flows, the weir and orifice equations give the correct discharge.

Table 10.4

Rating Curve

H(m)	Q(m$^{3/5}$)	Yc(m)	Yn(m)
0.5	1.08	0.38	0.27
1.0	3.07	0.78	0.56
1.5	5.64	1.17	0.86
2.0	8.68	1.57	1.20
2.5	12.14	1.96	1.55
3.0	15.95	2.35	1.93
3.5	20.10	2.74	2.34
4.0	24.56	3.13	2.77
4.5	29.31	3.52	3.22
5.0	34.28	3.91	3.69
5.5	37.43	4.15	3.99
6.0	40.34	4.36	4.27
6.5	43.05	4.55	4.52
7.0	45.60	4.73	4.76

10.5

Given:

Box culvert of problem 10A with

$S_0 = 0.001$

L =100m

Tail water level remains below the culvert top at the outlet.

Compute : The rating curve.

Solution:

Determine of the control is at the inlet or not .Table 10-5a show the normal and critical depths for several discharge (these are the Q values computed in problem 10.4)

Table 10-5a

Q ($m^{3/s}$)	Yc (m)	Yn (m)
1.08	0.38	0.473
3.07	0.78	1.00
5.64	1.17	1.60
8.68	1.57	2.27
12.14	1.96	3.03
15.95	2.35	3.93
20.10	2.74	4.70
24.56	3.13	5.63
29.31	3.52	6.62

From Table 10-5a we conclude that the flow is sub-critical and the control is at the outlet. For outlet control the discharge and the water profile depends on the culvert length and the tail water level at the down streams end. It also depends on the head water level.

We compute the rating curve for the maximum flow in the culvert for a given tail water level. Recall that for a tail water level below the top of the culvert, the maximum flow in the culvert occurs when the tail water level is less or equal to the critical depth.

This is $Q_{max} = q_{max}B_0$

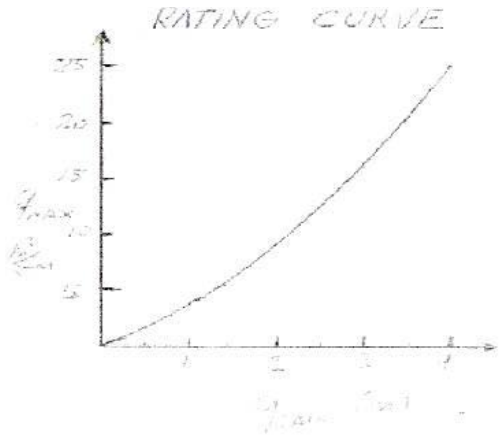
$$Q\text{max} = \sqrt{y_0{}^3 g} = \sqrt{y_{TAIL}{}^3 g}$$

Using these equations, the following rating curve for maximum flow as a function of the tail water level is obtained.

Table 10-5 b

Rating Curve

Y_{TAIL} (m)	q_{max} (m^3/sm)	Q_{max} (m^3/s)
0.5	1.11	2.22
1.0	3.13	6.26
1.5	5.75	11.5
2.0	8.86	17.72
2.5	12.38	24.76
3.0	16.27	32.54
3.5	20.51	41.02
4.0	25.06	50.12



10.6

Given: Culvert of prob.10-5

L=100m

The tail water level is 4.5m above the culvert invert at the outlet.

Compute and plot the water surface profile.

Solution:

The culvert of prob. 10-5 is 4m high; therefore , the outlet for the conditions given here is going to be submerged. Also the flow in the culvert is SUBSRITICAL (See Table 10-5a).

Under these conditions the control is at the outlet and culvert flow full or pressurized..

10.7

Given :Logarithmic Velocity Distribution.

Prove that the flow velocity at 0.368d is the depth-averaged flow velocity.

Solution:

Assume velocity distribution of the form

$$v = a \ln (by)$$

where y is the flow depth and ‘a’ and ‘b’ are constants.

Integrating ‘v’ across the flow depth, we have,

$\bar{v}$ = a [ln (bd) -1]

Then,

ln c = 1 => $\bar{v}$ = a [ln (bd)-ln e]

or $\bar{v}$ = a ln ($\frac{bd}{e}$)

But e = 2.718 => 1/e = 0.368,

Finally $\bar{v}$ = a ln (0.368db)

Then y = 0.368d corresponds to the depth at which the flow velocity is the depth averaged flow velocity.

10.8

Given :

Flow velocity at 0.2 d and 0.8 d

Logarithmic velocity distribution.

Show that the average

$$V_{av} = \frac{v_{0.2}d + V_{0.8}d}{2}$$

gives the depth-averaged flow velocity with an error of 2 %.

Recall that v = a ln (by)

Then $$V_{av} = \frac{v_{0.2}d + V_{0.8}d}{2}$$

$$V_{av} = \frac{a\ln(b0.2d) + a\ln(b0.8d)}{2}$$

$$V_{av} = \frac{a}{2}\ \ln(04bd)^2$$

V_{av} = a ln (0.4 bd)

The relative error is expressed as

$$\bar{\varepsilon} = \frac{\bar{\text{v}} - \text{Vav}}{\bar{\text{v}}} = \frac{a\ln(0.368db) - a\ln(0.4bd)}{a[\ln(bd) - 1]}$$

where $\bar{v}$ was taken from Prob. 10.7

$$\bar{\varepsilon} = \frac{0.084}{\ln(bd)-1}$$

Typically b ≈ 30/k (sec. Prob. 1.4) and 30d/k > 200 => ε ≈ 2% or less!

10.9

The water level in the upstream lake of the channel system shown in Fig. 10.10 remains constant at El. 108 m; the water level in the downstream lake may vary between El.98 and 108 m.

(i) Compute the rates of discharge on the channel for different downstream lake levels and plot the delivery curve.

A computer program similar to the one presented in Appendix …. for the computation of backwater flow profiles using simultaneous solution approach was used to solve this problem. The computation of the delivery curve only requires to specify the discharge and the program solves for the water depth.

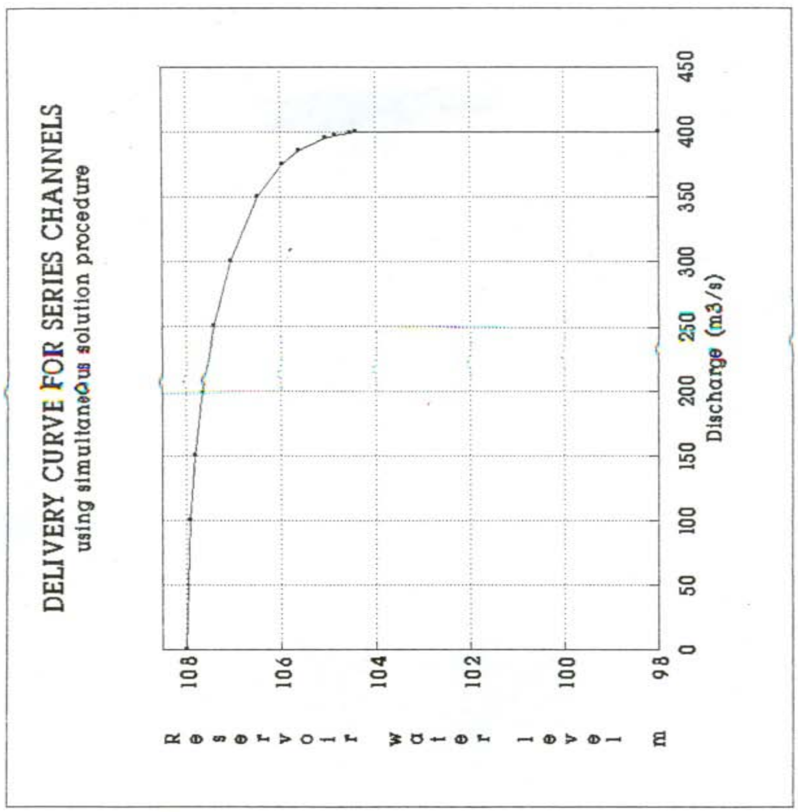
Table 10.9 presents relevant water depth for selected discharges.

Table 10.9

Normal, Critical and Downstream Depth at Channel 2

Q $m^{3/5}$	y_n m	y_0 m	y_d m
100	4.148	2.140	8.721
150	5.172	2.738	8.620
200	6.028	3.252	8.461
250	6.772	3.708	8.230
300	7.439	4.122	7.880
350	8.387	4.504	7.290
400	8.606	4.859	5.248

Figure 10.9 shows the delivery curve for the system. A maximum depth of 8.8 m at the downstream end of channel 2 corresponds to zero flow conditions. As the water level decreases the flow increases very rapidly, for high water level in the downstream reservoir. An increase in discharge from 0 to 200 m^3/s corresponds to only 0.30 m variation in the reservoir level. At this condition the channel is carrying approximately a half of the maximum flow for the system.



10.10

Given

Branch channel system consisting of the two channels of Prob. 10.9 and a branch channel (channel 3) this takes off from the junction of channels 1 and 2.

Channel 3 is, L_3 = 1000m
Same cross section as Ch. 2
n = 0.015
water level at downstream end = 105

(i) Compute and plot the delivery curves for channels 2 and 3 for the different downstream lake levels downstream of channel 2.

Solution:
A computer program for the solution of this problem using simultaneous solution procedure can be written following the explanations of Section 6.8. In this case, the solution will be easily obtained in a straight forward manner from the computer program. However, other procedure that uses the 'traditional' integration techniques (like Euler or Runge Kutta) to solve the gradually varied flow differential equation will be discussed here. In this case, the computations are more tedious because a trial and error procedure must be applied.

The first step is to determine the maximum discharge in channel 3. Theoretically, this corresponds to the valve for which the reservoir level is the critical depth. However, in this case the flow is restricted by the capacity of channel 1. After a trial and error process, trying to match the water elevation coming from reservoir 1 towards the junction with the constant elevation at reservoir 3, the maximum flow in channel 3 is estimated as 478 m^3/s, corresponding to a water elevation of 7.0 m at the junction. An elevation of 5.2 m at the junction represents zero flow in channel 3. So we look for water elevation at the junction between 7 and 5.2 m.

Now, by trial and error we find several combinations of flow and head that satisfy the continuity and the energy equations at the junction and, at the same time satisfy the boundary conditions of each channel. Table 10.10 shows the discharges head of the junction and depth at reservoir 2 that occur simultaneously.

Table 10.10

Discharge and water depth for the channel system.

Q_{ch1} m^3/s	Q_{ch3} m^3/s	Q_{ch2} m^3/s	y_{nor} m	y_0 m	y_{junc} m	y_d m	H m
500	448	52	2.94	1.46	6.76	7.34	106.54
520	393	127	4.73	2.48	6.43	6.88	106.08
530	355	175	5.62	3.00	6.19	6.41	105.61
535	320	215	6.26	3.39	6.02	5.87	105.07
537	308	229	6.47	3.52	5.93	5.50	104.70
538	298	240	6.63	3.62	5.88	5.13	104.33

Differences in velocity head at the junction were neglected in the computations. During the trial and error process a maximum difference of ±5 cm between the reservoir elevation of channel 2 (105 m) and the valve computed with the program was allowed.

As mentioned before, the maximum flow in channel 3 is approximately 478 m^3/s corresponding to a water elevation of 7.0 m at the junction. Elevations greater than 7.0 m correspond to flows less than 475 m^3/s in channel 1, causing a situation where the continuity equation is not satisfied.

Water elevations less than 7m at the junction correspond to flows greater than 478 m^3/s in channel 1. In these cases there is flow going from the junction towards reservoir 2 and 3.
The steps followed to obtain the values in table 10.10 are :

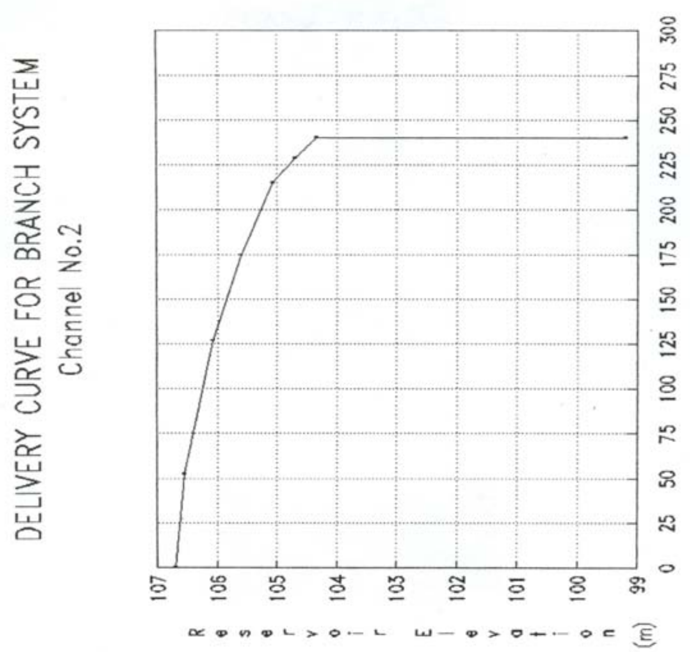
(i) Assume a discharge in channel 1 and compute the backwater profile up to the junction. We know that this flow must be greater than 478 m^3/s.

(ii) With the water depth at the junction obtained in Step 1, assume a discharge for channel 3 and use trial and error to match the end of the flow profile with the reservoir elevation.

(iii) Once step 2 is satisfied, apply the continuity equation at the junction to find the discharge in channel 2.

(iv) Knowing the discharge in channel 2 and the water depth at the junction, compute the backwater profile for channel 2 and obtain the reservoir elevation.

Repeat the same procedure for different discharges to get the delivery curve. Figure 10.10 shows the results for channel 2.



Chapter 11
UNSTEADY FLOW

11.1

Vw = absolute velocity. Assume rectangular channel.

Continuity eqn :

$(V_o+\Delta V-V_w)(Y_o+\Delta y) = (V_o-V_w)y_o$

$V_oy_o+y_o\Delta y-V_wy_o+V_o\Delta y+\Delta y\Delta v-V_w\Delta y= v_oy_o-v_wy_o$

$(v_o-v_w)\,\Delta y = -y_o\Delta v$

$(v_w-v_o)\,\Delta y = y_o\Delta v$

$$V_w-v_o = y_o\frac{\Delta v}{\Delta y}$$

$$C = y_o\frac{\Delta v}{\Delta y} \qquad \text{----------------(1)}$$

Momentum equation:

$$½[\frac{r}{2}(y_o+\Delta y)^2 - ½ry_o{}^2] = \frac{r}{g}(v_o\text{-}v_w)y_o[(v_o - v_w) - (v_o+\Delta v - v_w)]$$

$$\frac{1}{2}[y_o{}^2+\Delta y^2+2y_o\Delta y-y_o{}^2] = (v_o - v_w)\frac{y_o}{g}[v_0-v_w-v_o-\Delta v+v_w]$$

$$y_0\,\Delta\,y = v_0\text{-}v_w\,\frac{y_0}{g}(-\,\Delta\,v)$$

$$v_0\text{-}v_w = -g\frac{\Delta y}{\Delta v}$$

$$C = g\frac{\Delta y}{\Delta v} \qquad \text{---------------- (2)}$$

From (1): $\dfrac{\Delta y}{\Delta v} = \dfrac{y_0}{C}$

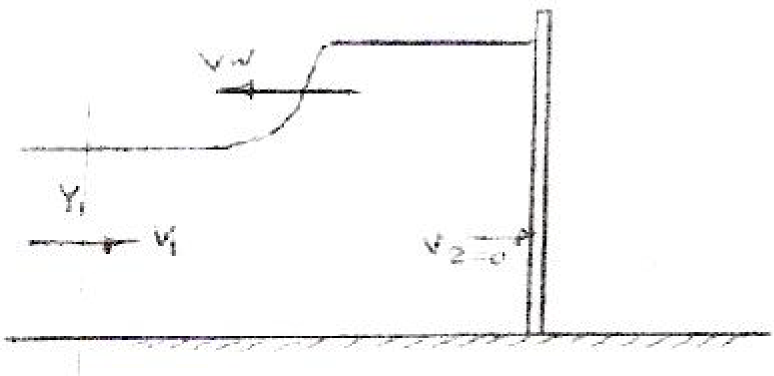
From (2): $C = g\dfrac{y_0}{C}$

$C2 = gy_0$

C $=$ $\sqrt{gy_0}$

Note that C is relative to flow velocity Vo.

11.2



V1 $= \dfrac{Q}{B_1 y_1} = \dfrac{7.5}{5x1.5} = 1m/\sec.$

(v1-v2)2 $= g\dfrac{y_1 - y_2}{2y_1y_2}(y_1^2 - y_2^2)$

$(1-0)^2 = \dfrac{9.81(1.5 - y_2)}{2x1.5y_2}(2.25-y_2^2)$

$0.3058 = \dfrac{(1.5 - y_2)}{y_2}(2.25-y_2^2)$

Use trial & error,

y2 $=$ 1.914 m

$v_w = \dfrac{v_2y_2 - v_1y_1}{y_2 - y_1} = \dfrac{0 - 1x1.5}{1.914 - 1.5} = 3.623m/\sec$

11.3

$\Delta E \quad = y_1 + \frac{V_1^{\ 2}}{2g} - y_2 - \frac{v_2^{\ 2}}{2g} \quad = \frac{V_1^{\ 2}}{2g}[(1-\frac{b_1}{b_2})^2 + 2Fr_1^{\ 2}\frac{b_1^{\ 3}}{b_2^{\ 3}}(b_2 - b_1)$

$= y_1 + \frac{(7.5/4y_1)^2}{2x9.81} - 1.5 - \frac{1}{2x9.81} \quad = \frac{(7.5/4y_1)^2}{2x9.81}[(1-\frac{4}{5})^2 + 2\frac{(7.5/4y_1)^2}{2x9.81}\frac{4^3}{5^4}(1)$

$= y_1 + \frac{0.1792}{y_1^{\ 2}} - 1.5 - 0.05097 \quad = \frac{0.1792}{y_1^{\ 2}}[0.04 + \frac{0.07339}{y_1^{\ 3}}]$

$y_1^6 - 1.55097y^5 - 0.172y^3 - 0.01315 \quad = 0$

$Y_1 \quad = \quad 1.474$ m

$Y_1` \quad = \quad (1.914-1.5)5/4+1.474 = 1.9915$ m

11.4

$y_1 + \frac{(7.5/7.5y_1)^2}{2x9.81} - 1.5 - \frac{1}{2x9.81} = \frac{(7.5/7.5y_1)^2}{2x9.81}[(1-\frac{7.5}{5})^2 + 2\frac{(7.5/7.5y_1)^2}{9.81x\,y_1}\frac{7.5^3}{5^4}(-2.5)$

$y_1 + \frac{1}{2x9.81xy_1^{\ 2}} - 1.5 - \frac{1}{2x9.81} = \frac{1}{2x9.8xy_1^{\ 2}}[0.25 + \frac{-0.344}{y_1^{\ 3}}]$

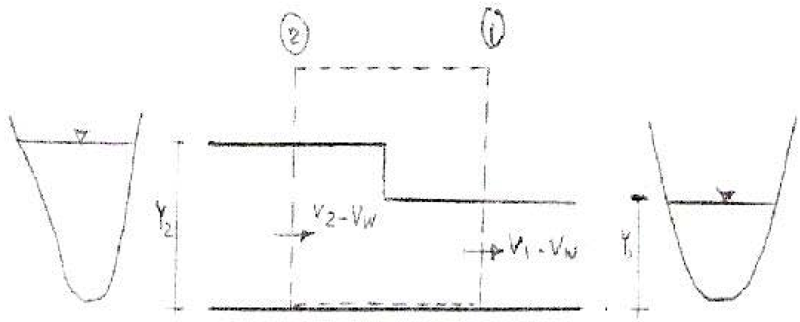
$y_1^6 - 1.55097y^5 + 0.0382y^3 + 0.01753 \quad = 0$

trial and error

y1 = 1.533 m

y1\` = (1.914-1.5)5/7.5+1.533 = 1.809 m.

11.5



$Q_1=Q_2$

$A_1(v_1-v_w) \quad = A_2(v_2-v_w)$ Eqn. 11.8

$$\frac{\rho}{g}Q[(v_1-v_w)-(v_2-v_w)] = \gamma\bar{y}_2A_2 - \gamma\,\bar{y}_1A_1$$

$$A_1(v_1-v_w)[v_1-v_w-\frac{A_1}{A_2}(v_1-v_w)] = g(\bar{y}_2A_2 - \bar{y}_1A_1)$$

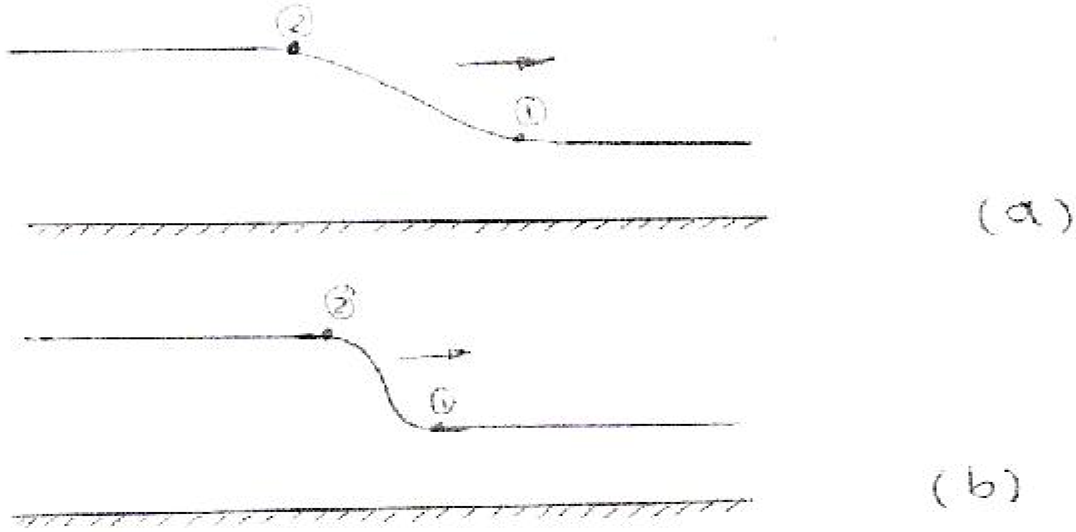
$$A_1\,(v_1-v_w)^2(1-\frac{A_1}{A_2}] \quad =$$

$$\frac{A_1}{A_2}(v_w-v_w)^2(A_2-A_1) \quad = g(\bar{y}_2A_2 - \bar{y}_1A_1)$$

$$(v_1-v_w)^2 \quad = \quad \frac{A_2g}{A_1(A_2-A_1)}[(A_2\overline{y_2} - A_1\overline{y_1})$$

$$C=v_w-v_1 \quad = \quad \sqrt{\frac{A_2g}{A_1(A_2-A_1)}[(A_2\overline{y_2} - A_1\overline{y_1})}$$ eqn. 11.17

11.6



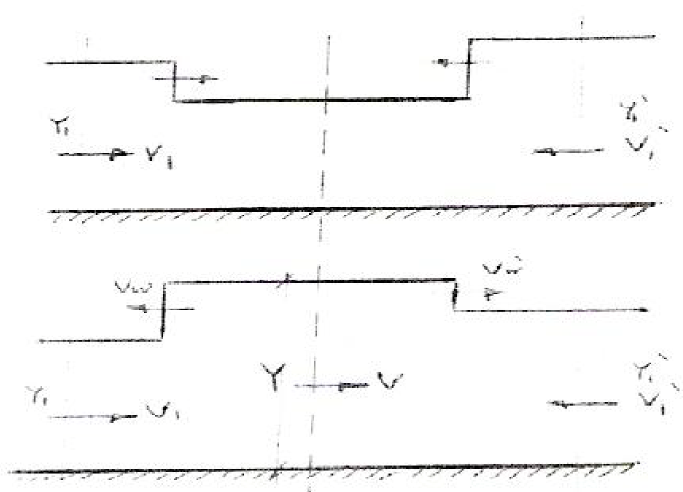
This is a positive wave

$$C = \sqrt{gy}$$

In fig. a : The depth at 1 is less than the depth at 2, the wave celerity c2 > c1. As the wave travel it tends to over take the front edge. Therefore, the wave tent becomes steeper until a bare form (figure b).

- a similar argument can be done for the negative wave front and it will flatten as it travel in the channel.

11.7



$(v_1-v_2)^2 = \quad (y_1-y_2)^2 \dfrac{(y_1+y_2)g}{2y_1y_2}$ ---- (1)

To the left side new singe

$V_2 = -vy_2 = y \qquad v_1=-v_1$

$(v_1-v)^2 \quad = \quad (y_1-y)^2 \dfrac{(y_1+y)g}{2y_1y}$ ---- (2)

To the right side new singe

$V_2=v \qquad v_1=v_1` \qquad y_2=y \qquad y_1=y_1`$

$(v_1`+v)^2 \quad = \quad (y_1`-y)^2 \dfrac{(y_1`+y)g}{2y_1`y}$ ---- (3)

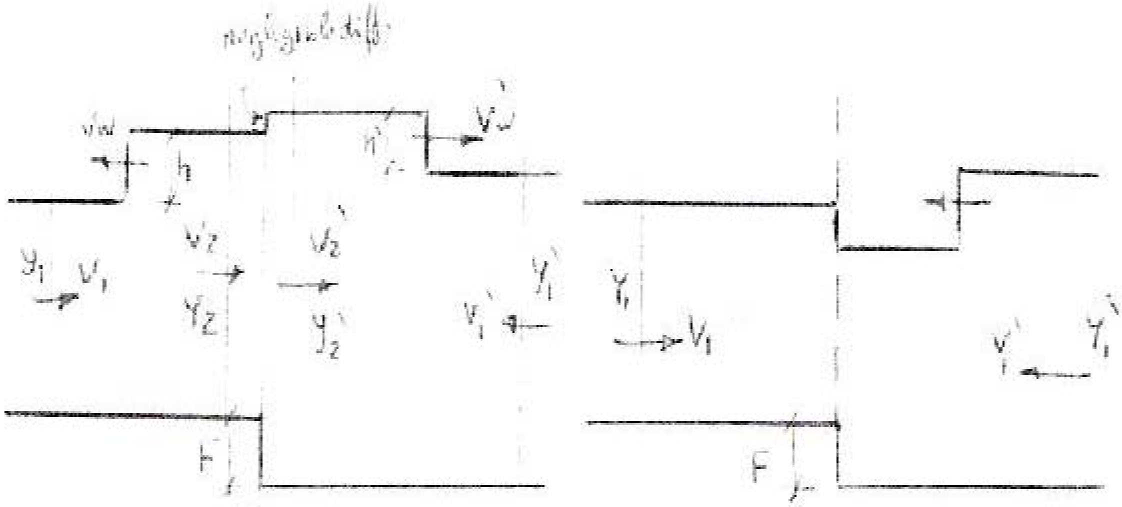
Solving eqn (2) and (3), Find v and y

$v_w \quad = \quad \dfrac{v_1y_1-v_2y_2}{y_1-y_2}$

Left side $\quad v_w \quad = \quad \dfrac{v_1y_1-vy}{y-y_1}$

Right side $\quad vw` \quad = \quad \dfrac{v_1`y_1`+vy}{y-y_1`}$

11.8



$(v_1-v_2)^2 = \quad (y_1-y_2)^2 \dfrac{(y_1+y_2)g}{2y_1y_2}$ ---- (1)

Applying eqn (1) to left side new singe

$(v_1-v_2)^2 = \quad (y_1-y_2)^2 \dfrac{(y_1+y_2)g}{2y_1y_2}$ ---- (2)

Applying eqn.(1) to right side new singe

$(v_1`+v_2`)^2 \quad = \quad (y_1`-y_2`)^2 \dfrac{(y_1`+y_2`)g}{2y_1`y_2`}$ ---- (3)

By geometric continuity

$y_2 + F \quad = y_2`$ -------- (4)

By hydraulic continuity

$V_2y_2 \quad = v_2`v_2`$

Solving 2,3,4,5 we get v_2, y_2, $v_2`$, $y_2`$

11.9

Stationary wave.

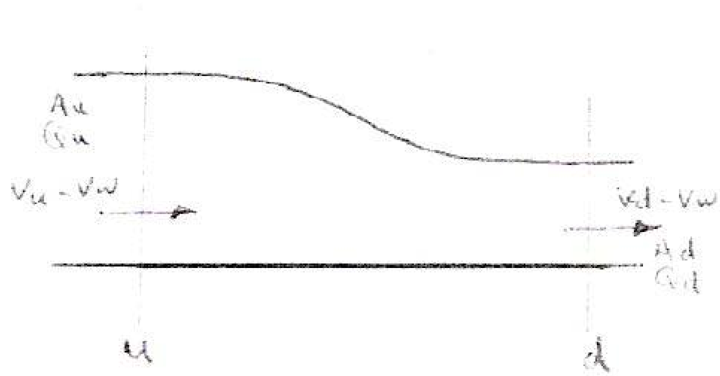
Au (vu-vw) = Ad (vd-vw)

Au (vw-vu) = Ad (vw-vd)

Auvw-Advw = Auvu – Advd

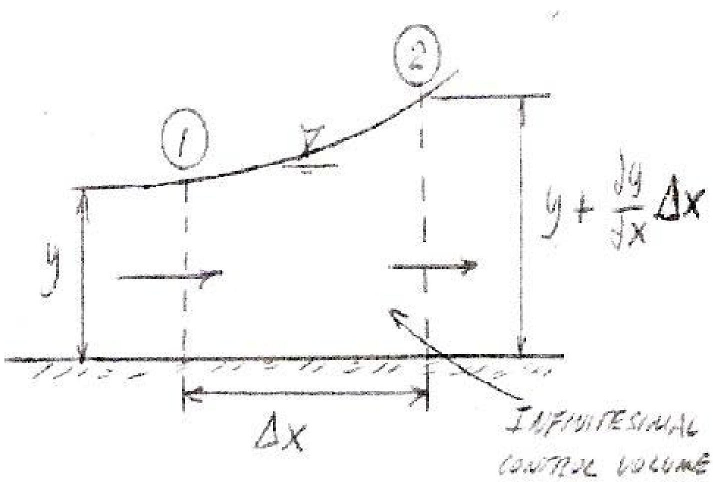
vw (Au – Ad) = Qu – Qd

$$vw = \frac{Qu - Qd}{Au - Ad}$$



Chapter 12
GOVERNING EQUATIONS FOR ONE-DIMENSIONAL FLOW

12.1- Derive the continuity equation for the one-dimensional unsteady flow using infinitesimal length of channel show in the figure.



The law of conservation of mass can be expressed as : "The time rate of increase of the mass inside the control volume must be equal to the net rate of mass inflow into the control volume ". From the figure we have the rate of mass in flow

as $\quad P_1Q_1=\frac{H}{g}V1\ A \quad$ Where H=specific weigh of water.

The rate of outflow is:

$$P_2Q_2 = \frac{H}{g}(A+\frac{\partial A}{\partial x}\Delta x)(V+\frac{\partial V}{\partial x}\Delta x)$$

Then the net rate of flow is :

$$\Delta m = P_2Q_2 - P_1Q_1$$

or

$$\Delta m = \frac{H}{g}AV - \frac{H}{g}(A+\frac{\partial A}{\partial x}\Delta x)(V+\frac{\partial V}{\partial x}\Delta x)$$

Neglecting second-order terms and simplifying we get :

$$\Delta m \quad = \frac{H}{g} V \frac{\partial A}{\partial x} \Delta x \; - \; \frac{H}{g} A \frac{\partial V}{\partial x} \Delta x \qquad (1)$$

The time rate of increase of mass inside the control volume is given by :

$$\frac{\partial m}{\partial t} = \frac{H}{g} \frac{\partial A}{\partial t} \Delta x \qquad (2)$$

According to the law of conservation of mass, Eq.1 is equal to Eq.2, therefore

$$\frac{\partial A}{\partial t} + V \frac{\partial A}{\partial x} + A \frac{\partial V}{\partial x} = 0 \qquad (3)$$

Recalling that Q=VA and $\frac{\partial Q}{\partial x} = \frac{\partial (AV)}{\partial X} = V \frac{\partial A}{\partial x} + A \frac{\partial V}{\partial x}$

We get the law of conversion of mass, Eq.3 as:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

This is the continuity equation in conservative form without lateral flow (Eq. 12.4).

12.2

Derivation of the momentum equation.

The law of conservation of momentum can be expressed as:

“The time rate of increase of momentum is equal to the net rate of momentum influx plus the summation of the forces acting inside the control volume “.

First we obtain an expression for the forces acting in the control volume shown in

Problem 12.1

$$\left.\begin{aligned} F_1 &= HA\bar{y} \\ F_2 &= HA\bar{y} + HA\frac{\partial y}{\partial x}\Delta x \end{aligned}\right\} \quad \text{Pressure forces}$$

$$F_3 = HAS_f\Delta x \qquad \text{Force due to friction}$$

S_f = slope of the energy grade line.

F_4 = W = $HAS_0\Delta x$ Gravity Force

S_0 = channel bottom slope

Adding the forces in the x-direction we get the resultant force:

$$F_R = -HA\,\frac{\partial y}{\partial x}\Delta x\ - HAS_f\ \Delta x + HAS_a\ \Delta x$$

The momentum influx is

$$\rho_1 Q_1 V_1 = \frac{H}{g}\ A_1 V_1^2 = \frac{H}{g}\ AV^2$$

The momentum influx:

$$\rho_2 Q_2 V_2 = \frac{H}{g}\ A_2 V_2^2 = \frac{H}{g}\ [AV^2 + \frac{\partial (AV^3)}{\partial x}\ \Delta x\ ]$$

The net change in momentum:

$$\rho Q_2 V_2 - PQ_1 V_1 = \frac{-H}{g}\ \frac{\partial}{\partial x}\ (AV^2)\,\Delta x$$

Then, the time rate of momentum is given by :

$$\frac{\partial (mv)}{\partial t} = \frac{\partial}{\partial t}\ (\frac{H}{g} AV\,\Delta x\,)$$

Finally, the conservation of momentum is

$$\frac{\partial}{\partial t}(\frac{H}{g} AV\,\Delta x\,) = -\ \frac{H}{g}\frac{\partial}{\partial x}\ (AV^2)\,\Delta x\ -\ HA\ \frac{\partial g}{\partial x}\ \Delta x + HAS_0\,\Delta x - HAS_F\,\Delta x$$

<u>Time rate of change of momentum</u> <u>Net rate of momentum influx</u> <u>Summation of forcer acting on the control volume</u>

Simplifying this equation we get:

$$\frac{\partial}{\partial t}(AV) + \frac{\partial}{\partial x}\ (AV^2)\ + gA\frac{\partial y}{\partial x} = gA\ (S_0 - S_f) \qquad (1)$$

Recalling that

$$\frac{\partial}{\partial x}(AV^2)\ = V^2\ \frac{\partial A}{\partial x} + A\ \frac{\partial (V^2)}{\partial x}$$

$$= V^2 \frac{\partial A}{\partial x} + 2A \frac{\partial V}{\partial x}$$

$$= V\,(V\frac{\partial A}{\partial x} + 2A \frac{\partial V}{\partial x}) \qquad (2)$$

and $$\frac{\partial (AV)}{\partial t} = V \frac{\partial A}{\partial t} + A \frac{\partial v}{\partial t} \qquad (3)$$

Substituting Eq.2 and Eq.3 in Eq.1 result in

$$V(\frac{\partial A}{\partial t} + V \frac{\partial A}{\partial x} + A \frac{\partial V}{\partial x}) + gA\frac{\partial y}{\partial x} + AV \frac{\partial V}{\partial x} + A\frac{\partial V}{\partial t} = gA(S_0 - S_F)$$

Dividing by A and noting that the first term in the left hand side is equal to zero (se prob.12.1,Eq.3), the find expression is:

This is the momentum equation or dynamic equation.

$$\boxed{g\frac{\partial y}{\partial x} + V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} = g(S_0 - S_F)}$$

12.3

Derive Eq.5.5 from Eq.12.17

Equation 5.5 is:

$$\frac{dy}{dx} = \frac{(S_0 - S_f)}{1 - \frac{BQ^2}{gA3}}$$

Equation 12.17 is: $$\frac{\partial V}{\partial t} + g \frac{\partial}{\partial x}(\frac{V^2}{2g} + y) = g\,(S_0 - S_f)$$

We know that Eq. 5.5 apply to STE ΔDy, gradually varied flow, then:

$$\frac{\partial V}{\partial t} = 0$$

Then, Eq. 12-17 becomes

$$g\frac{\partial}{\partial x}(\frac{V^2}{2g} + y) = g(S_0 - S_f) \qquad (1)$$

Recalling that: Q=VA then:

$$g\frac{(V^2/2g)}{\partial x} = g\ \frac{\partial(Q^2/2gA^2)}{\partial x}$$

$$= -\ \frac{Q^2}{g}\ \frac{B}{A^3}\ \frac{\partial y}{\partial x} \qquad (2)$$

Using the fact that $\qquad B=\frac{\partial A}{\partial y}$

Substituting Eq.2 into Eq.1 we get

$$-\frac{BQ^2}{A3}\ \frac{dy}{dx}+g\frac{dy}{dx}= g(S_0\text{-}S_f) \qquad (3)$$

Note that the partial derivatives are not needed in equation 3.

Rearranging Eq.3, we get:

$$\boxed{\frac{dy}{dx}= V\frac{(S_0\text{-}S_f)}{1-\frac{BQ^2}{gA3}}}$$

This is Eq.5-5 (Gradually varied flow equation).

12.4

If we use stage (the equation of water surface above a specified datum), Z instead of flow depth ,g, show that the continuity momentum equations for a prismatic channel become:

$$\frac{\partial Z}{\partial t}+V\frac{\partial Z}{\partial x}+\frac{A}{B}\ \frac{\partial v}{\partial x}+VS_0 = 0$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial Z}{\partial x}+gS_f = 0$$

Solution:

The relation between Z and y is

Y = Z-Z_0 Where Z_0 is the elevation of the channel bottom.

Then $\frac{\partial y}{\partial x}$ =V $\frac{\partial Z}{\partial x}$ - $\frac{\partial Z_0}{\partial x}$ = $\frac{\partial Z}{\partial x}$ +S_0

Where S_0 is the channel bottom slope.

Also $\frac{\partial y}{\partial t} = \frac{\partial z}{\partial t}$

Substituting $\frac{\partial y}{\partial x}$ and $\frac{\partial y}{\partial t}$ in terms of $\frac{\partial Z}{\partial x}$ and $\frac{\partial z}{\partial t}$ we get:

continuity Equation:

$$\frac{\partial y}{\partial t} + \frac{A}{B}\frac{\partial v}{\partial x} + \frac{\partial y}{\partial x} = 0 \qquad \longrightarrow$$

$$\frac{\partial z}{\partial t} + \frac{A}{B}\frac{\partial v}{\partial x} + V\frac{\partial z}{\partial t} + VS_0 = 0$$

Momentum Equation : (Eq. 12-16)

$$V\,[\,B\frac{\partial z}{\partial t} + A\frac{\partial v}{\partial x} + BV\frac{\partial z}{\partial t} + BVS_0\,] + A\,[\,\frac{\partial V}{\partial t} + V\frac{\partial v}{\partial x} + gS_0 + gS_f - gS_0] = 0$$

The first term in brockets is identically zero because of the continuity equation, Therefore

$$A\,[\,\frac{\partial V}{\partial t} + V\frac{\partial v}{\partial x} + g\frac{\partial z}{\partial x} + gS_f] = 0$$

Or

$$\frac{\partial V}{\partial t} + V\frac{\partial v}{\partial x} + g\frac{\partial z}{\partial x} + gS_f = 0$$

12.5

The momentum equation is

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(QV + gA\bar{Y}) = gA(S_0 - S_f)$$

If the flow is steady we have $\frac{\partial Q}{\partial t} = 0$

For uniform flow we have: $\frac{\partial}{\partial x}(Qv + gA\bar{Y}) = 0$

Then, the momentum equation becomes

$$gA\,(S_0 - S_f) = 0$$

Or $$S_0 = S_f.$$

12.6

If the wind stress on the flow surface is included, prove that the continuity and momentum equations becomes:

$$B\frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} - \frac{q}{l} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{Q}{A}\frac{\partial Q}{\partial x} + Q\frac{\partial}{\partial x}\left(\frac{Q}{A}\right) + gA\frac{\partial g}{\partial x} - gA(S_0 - S_f) - \frac{q}{l}u - Kw\,BV_W^2\cos\theta = 0$$

In which u=Velocity component of the latest flow in the positive x-direction, V_w= wind velocity and K_w=dimensionless wind stress coefficient.

The addition of wind stress on the surface flow does not modify the continuity therefore , is remains the same as Eq.12.5, this is

$$B\frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} - \frac{q}{l} = 0$$

The wind force can be expressed empirically as :

$$F_w = K_w V_w^2$$

Where K_w was defined as a dimensionless wind-stress coefficient.

If θ is the angle between the wind direction and the x-direction, the component of the wind force is the x-direction is:

$$F_w = K_w V_w^2 \cos\theta \Delta x$$

Adding this terms to the summation of forces in the momentum equation (problem 12.2) we get:

$$\frac{H}{g}\frac{\partial Q}{\partial t}\Delta x = -\frac{H}{g}\frac{\partial (QV)}{\partial x}\Delta x - HA\Delta x\frac{\partial y}{\partial x} + HAS_0\Delta x - HAS_f\Delta x - K_w V_w^2 \cos\theta + a\frac{q}{l}$$

Expending the partial derivative and simplifying we get:

$$\frac{\partial Q}{\partial t} + V\frac{\partial Q}{\partial x} + Q\frac{\partial v}{\partial x} + gA\frac{\partial y}{\partial x} - gA(S_0 - S_f) - K_w V_w^2 \cos\theta + a\frac{q}{l} = 0$$

Or

$$\frac{\partial Q}{\partial t} + \left(\frac{Q}{A}\right)\frac{\partial Q}{\partial x} + Q\frac{\partial}{\partial x}\left(\frac{Q}{A}\right) + gA\frac{\partial y}{\partial x} + gA(S_0 - S_f) - \frac{q}{l}a - K_w V_w^2 \cos\theta + = 0$$

Chapter 13
NUMERICAL METHODS

13.1

A computer program is developed for the transient by using the Method of Characteristics. The channel length is divided into 50 reaches (51 nodes). Other input values are as per the problem statement.

Results of surface profile due to sudden closure of the downstream gate at different time levels are presented in Fig. 1(a). Reflection of the wave is observed. Variation of flow depth with time at different locations in the reservoir is shown is Fig. 1 (b).

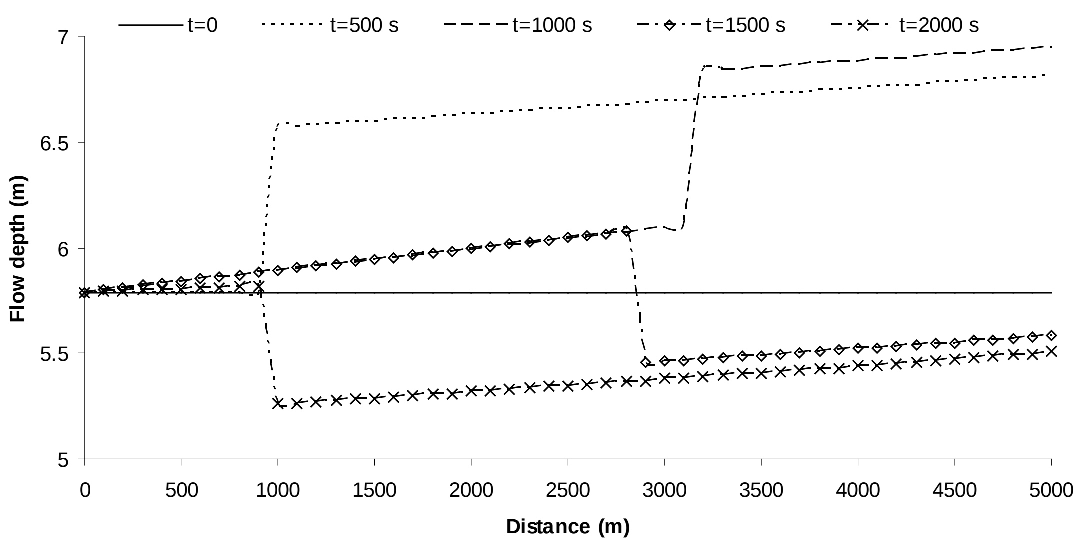


Fig. 1 (a) Surface profile at different times

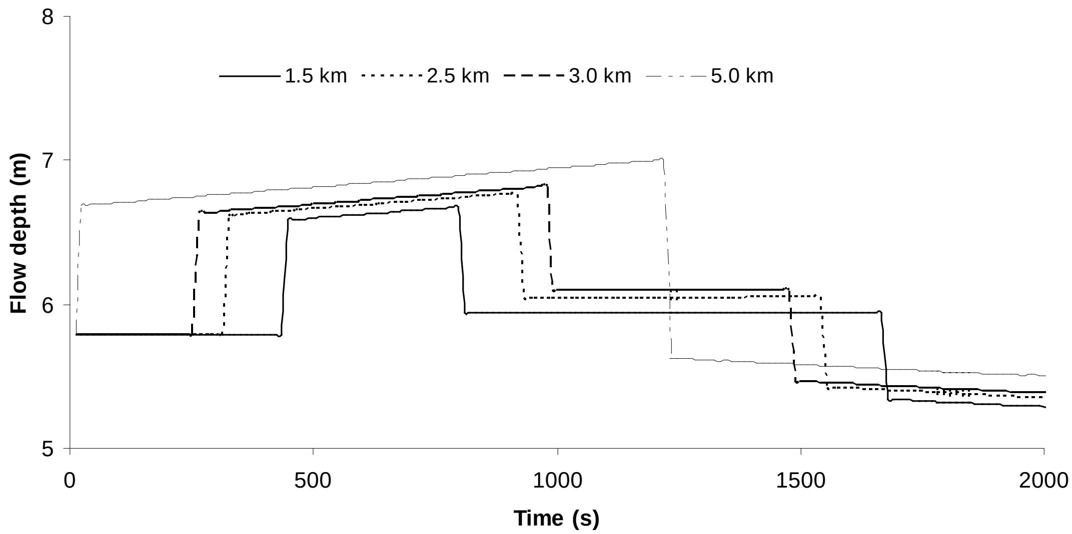


Fig. 1 (b) Variation of the flow depth with time at different locations.

13.2
The results are obtained by using different dt/dx. Result indicates diffusion errors in the wave shape as dt/dx becomes smaller.

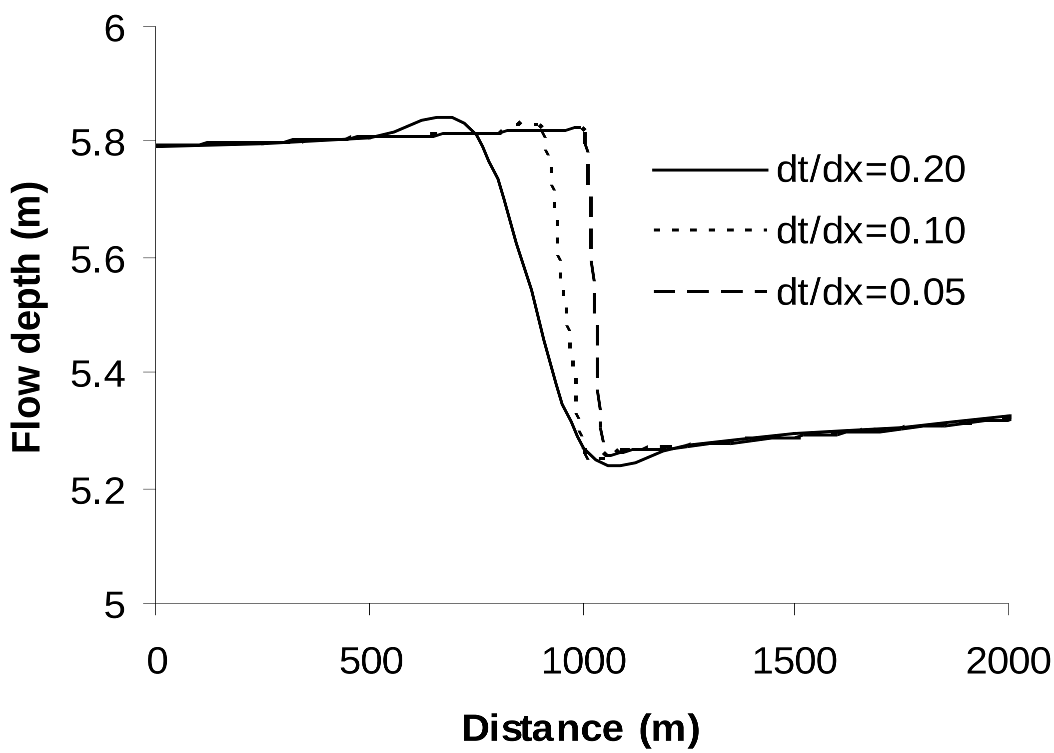


Fig. 13.2 Effect of dt/dx on the wave propagƒvation

Chapter 14
FINITE-DIFFERENCE METHOD

14.3

Use Von Newman analysis to show that the following scheme is unstable.

$$\frac{\partial f}{\partial x} \quad = \quad \frac{f_{i+1}{}^{k} - f_{i-1}{}^{k}}{2\Delta x}$$

$$\frac{\partial f}{\partial t} \quad = \quad \frac{f_i{}^{k+1} - f_i{}^{k}}{\Delta t}$$, where f refers to both ‘y’ and ‘v’ variables.

Solution

The linearized Saint-Venant equation (eq. 14.64 and 14.65) are

$$\frac{\partial V}{\partial t} + V_0 \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} = 0$$

$$\frac{\partial Y}{\partial t} + D_0 \frac{\partial V}{\partial x} + V_0 \frac{\partial y}{\partial x} = 0$$

Van newman assumed that the error has the form of a fourier series, this is

$$\varepsilon(x,t) = \sum X(t)\, e^{jmx} \quad \text{where } j=\sqrt{-1}$$

The finite difference approximation of the linearized equations using the given scheme is

$$V_i^{K+1} = V_i^{K} - \frac{V_0}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(V_{i+1}^{K} - V_{i-1}^{K}\right) - \frac{g}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(Y_{i+1}^{K} - Y_{i-1}^{K}\right)$$

$$Y_i^{K+1} = Y_i^{K} - \frac{D_0}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(V_{i+1}^{K} - V_{i-1}^{K}\right) - \frac{V_0}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(Y_{i+1}^{K} - Y_{i-1}^{K}\right)$$

The exact solution to these equation are:

$$Y_{exact} = Y_{comp} + \varepsilon$$

$V_{exact} = V_{comp} + \varepsilon$

Where V_{comp} is the approximation given by a real computer with limited accuracy. Since the exact solution, V_{exact} and Y_{exact} most satisfy the difference equation too we conclude that the same must be true for the error as long as the system is linear, therefore we have:

$$V_i^{K+1} = V_i^K - \frac{V_0}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(V_{i+1}^K - V_{i-1}^K\right) - \frac{g}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(W_{i+1}^K - W_{i-1}^K\right)$$

$$W_i^{K+1} = W_i^K - \frac{D_0}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(V_{i+1}^K - V_{i-1}^K\right) - \frac{Vo}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(W_{i+1}^K - W_{i-1}^K\right)$$

Where v and w represent the round of error introduced by the real-computer computations and given by

$$V_i^k = A^k e^{jmx}$$

$$W_i = B^k e^{jmx}$$

Since the system is linear, we can consider only one term in the error series; therefore, the error Equations becomes:

For υ

$$A(t+\Delta t)e^{jmx} = A(t)e^{jmx} - \frac{Vo}{2} r\,[A(t)\,e^{jm(x+\Delta x)} - A(t)\,e^{jm(x-\Delta x)}] - \frac{g}{2} r\,[B(t)\,e^{jm(x+\Delta x)} - B(t)\,e^{jm(x-\Delta x)}]$$

Where $r=\frac{\Delta t}{\Delta x}$, Calling $\eta=\frac{A(t+\Delta t)}{A(t)}$ the amplification factor we have :

$$\eta\, A(t)e^{jmx} = A(t)e^{jmx} - \frac{Vo}{2} r\,[A(t)e^{jmx}\,e^{jm\Delta x} + \frac{Vo}{2} r\,A(t)\,e^{jmx}\,e^{-jm\Delta x}$$

$$-\frac{g}{2} r\,[B(t)\,e^{jmx}\,e^{jm\Delta x} + \frac{g}{2} r\,B(t)\,e^{jmx}\,e^{-jm\Delta x}$$

Cancelling e^{jmx} and simplifying the equation we obtain:

$$[\eta-1+V_0 r\left(\frac{e^{jm\Delta x} - e^{-jm\Delta x}}{2}\right)]\,A + gr(\left(\frac{e^{jm\Delta x} - e^{-jm\Delta x}}{2}\right)B = 0$$

Defining $\int = m\Delta x$ and recalling that $\sin \int = \dfrac{e^{j\int} - e^{-j\int}}{2}$ we obtain:

$$[\eta - 1 + V_0 r \sin\!\int j]A + gr \sin\!\int B = 0$$

A similar procedure for the W_i^{K+1} equation gives:

$$[\eta - 1 + V_0 r \sin\!\int j]B + D_0 r \sin\!\int A = 0$$

In this derivation it was assumed that

$$\eta = \frac{A(t+\Delta t)}{A(t)} = \frac{B(t+\Delta t)}{B(t)}$$

To obtain a non-trivial solution for the amplitude (A and B) the following condition must be satisfied :

$$\begin{vmatrix} \eta - 1 + V_0 r \sin\delta j & gr\sin\delta j \\ D_0 r \sin\delta j & \} - 1 + V_0 r \sin\delta j \end{vmatrix} = 0$$

Or

$$(\eta - 1 + V_0 r \sin \delta j)^2 + D_0 g r^2 \mathrm{Sin}\delta = 0$$

$$\eta - 1 + V_0 r \sin \delta j = \pm j \sqrt{D_0 g} + \sin\delta$$

$$\text{Finally} \qquad \eta = 1 - (V_0 \pm \sqrt{D_0 g}\,) + \sin\delta j$$

In order to have the error bounded the condition must be $|\eta| < 1$

Where $|\eta|$ is the module of η

Or

$$(1 + (V_0 \pm \sqrt{D_0 g}\,)^2\, t^2 \sin^2\delta)^{½} < 1$$

This reduces to the impossible condition

$$(V_0 \pm \sqrt{D_0 g}\,)^2\, t^2 \sin^2\delta) \qquad < 0$$

Therefore the error cannot be bounded and the scheme is UNSTABLE .

14-6

The linearized Saint-Venant equations are:

$$\frac{\partial V}{\partial t}+V_0\frac{\partial V}{\partial x}+\frac{\partial Y}{\partial x} \quad = 0 \quad \text{--------------} \quad (1)$$

$$\frac{\partial Y}{\partial t}+D_0\frac{\partial V}{\partial x}+V_0\frac{\partial Y}{\partial x} = 0 \quad \text{--------------} \quad (2)$$

Predictor part of Mac Cormack Scheme applied to equation 1:

$$\frac{V_i^x - V_i^k}{\Delta t} + V_0\frac{V_i^K - V_{i-1}^K}{\Delta x} + g\frac{Y_i^x - Y_i^x}{\Delta x} = 0 \text{ -----------} \quad (3)$$

$$V_I^X \quad = \quad V_I^k - V0\left(\frac{\Delta t}{\Delta x}\right)\left(V_i^K - V_{i-1}^K\right) - g\left(\frac{\Delta t}{\Delta x}\right)\left(Y_i^K - Y_{i-1}^K\right) \quad \text{-----} \quad (4)$$

Similarly:

$$V_{i+1I}^X \quad = \quad V_{i+1}^k - V_0\left(\frac{\Delta t}{\Delta x}\right)\left(V_{i+1}^K - V_i^K\right) - g\left(\frac{\Delta t}{\Delta x}\right)\left(Y_{i+1}^K - Y_i^K\right) \qquad (5)$$

The corrector part for the same equation is:

$$\frac{V_i^{xx} - V_i^k}{\Delta t} + V_0\frac{V_{i+1}^x - V_i^x}{\Delta x} + g\frac{Y_{i+1}^x - Y_i^x}{\Delta x} \qquad = 0 \qquad (6)$$

or

$$V_i^{xx} \quad = \quad V_i^k - V_0\left(\frac{\Delta t}{\Delta x}\right)\left(V_{i+1}^x - V_i^x\right) - g\left(\frac{\Delta t}{\Delta x}\right)\left(Y_{i+1}^x - Y_i^x\right) \qquad (7)$$

The predictor part for equation Z is:

$$\frac{Y_i^x - Y_i^k}{\Delta t} + D_0\,(\frac{(V_i^k - V_{i-1}^k}{\Delta x} + V0\,\frac{Y_i^k - Y_{i-1}^k}{\Delta x} = 0 \qquad (8)$$

$$Y_i^* = \quad - D0\,\frac{\Delta t}{\Delta x}\,(V_i^k - V_{i-1}^k) - V_0\frac{\Delta t}{\Delta x}(V_i^k - V_{i-1}^k) \qquad (9)$$

$$Y_{i+1}^* = Y_{i+1}^k - D_0\frac{\Delta t}{\Delta x}(\,V_{i+1}^k - V_i^k\,) - V_0\frac{\Delta t}{\Delta x}\,(\,Y_{i+1}^k - Y_i^k) \qquad (10)$$

The corrector part for equation Z is :

$$\frac{Y_i^{*x} - Y_i^k}{\Delta t} + D_0\,((\frac{V_{i+1}^k - V_i^k}{\Delta x}) + V_0\,\frac{Y_{i+1}^* - Y_i^*}{\Delta x} = 0 \qquad (11)$$

or

$$Y_i^{*x} = Y_i^k - D0\frac{\Delta t}{\Delta x}(V_{i+1}^k - V_i^k)V0\frac{\Delta t}{\Delta x}\,Y_{i+1}^* - Y_i^* \qquad (12)$$

The flow velocity in the next time step is given by

$$V_i^{k+1} = \frac{V_i^* + V_i^{*k}}{2} \qquad (13)$$

Substituting Eqns. 4 and 7 into Eq.13 and defining $\Delta t/\Delta x = r$ we obtain:

$$V_i^{k+1} = \frac{V_i^k - V_0 r(V_i^k - V_{i-1}^k) - gr(Y_i^k - Y_{i-1}^k)}{2} + \frac{V_i^k - V_0 1 - (V_{i+1}^* - V_i^*) - gr(Y_{i+1}^* - Y_i^*}{2} \qquad (14)$$

Using Eq.4 and Eq.5 we can express $V_{i+1}^* - V_i^*$ as:

$$V_{i+1}^* - V_i^* = V_{i+1}^k - V_i^k - V_0 1 - (V_{i+1}^k + V_{i-1}^k - 2V_i^k) - gr(Y_{i+1}^k + Y_{i-1}^k - 2Y_i^k) \qquad (15)$$

Similarly by using Eq. 9 and Eq.10 we get:

$$Y^{*}_{i+1} - Y^{*}_{i} = Y^{k}_{i+1} - Y^{k}_{i} - D_0 R - (V^{k}_{i+1} + V^{k}_{i-1} - 2V^{k}_{i}) - V_0 r (Y^{k}_{i+1} + Y^{k}_{i-1} - 2Y^{k}_{i}) \qquad (16)$$

Substituting Eq.15 and Eq.16 into Eq. 14 we have:

$$V^{k+1}_{i} = v^{K}_{I} - \frac{V0t}{2}\left[V^{k}_{i+1} - V^{k}_{i-1} - V_0 t\left(V^{k}_{i+1} + V^{k}_{i-1} - 2V^{k}_{i}\right) - gt\left(Y^{k}_{i+1} + Y^{k}_{i-1} - 2Y^{k}_{i}\right)\right] -$$

$$\upsilon \frac{gt}{2}\left[Y^{k}_{i+1} - Y^{k}_{i-1} - D_0 t\left(V^{k}_{i+1} + V^{k}_{i-1} - 2V^{k}_{i}\right) - Vot\left(Y^{k}_{i+1} + Y^{k}_{i-1} - 2Y^{k}_{i}\right)\right] \qquad (17)$$

The error must also obey equation 17 then:

$$\upsilon^{k+1}_{i} = \upsilon^{k}_{i} - V_0 r\left[\left(\frac{\nu^{k}_{i+1} - \nu_{i-1}}{2}\right) - V_0 r\left(\frac{\upsilon^{k}_{i+1} + \upsilon^{k}_{i-1} - 2\upsilon^{k}_{i}}{2}\right) - gr\left(\frac{\omega^{k}_{i+1} + \omega^{k}_{i-1} - 2\omega^{k}_{i}}{2}\right)\right]$$

$$-gr\left[\left(\frac{\omega^{k}_{i+1} - \omega^{k}_{i-1}}{2}\right) - D_0 r\left(\frac{\upsilon^{k}_{i+1} + \upsilon^{k}_{i-1} - 2\upsilon^{k}_{i}}{2}\right) - V_0 r\left(\frac{\omega^{k}_{i+1} + \omega^{k}_{i-1} - 2\omega^{k}_{i}}{2}\right)\right] \qquad (18)$$

Where $\upsilon^{k}_{i} = A(t)e^{mjx}$ *and* $w^{k}_{i} = B(t)e^{mjx}$, *and* $j = \sqrt{-1}$

Defining the amplification factor as

$$\} = \frac{A(t + \Delta t)}{A(t)}$$

and substituting the expression for ν^{k}_{i} and w^{k}_{i} into Eq.18, we have :

$$\}\ Ae^{xmj} = Ae^{xmj} - V_0 r e^{xmj}\left[A\left(\frac{e^{m\Delta xj} - e^{-m\Delta xj}}{2}\right) - AV_0 r\left(\frac{e^{m\Delta xj} - e^{-m\Delta xj}}{2} - 1\right) - \right]$$

$$Bgr\left(\frac{e^{m\Delta xj} - e^{-m\Delta xj}}{2} - 1\right) - gr\, e^{mxj}$$

$$\left[B\left(\frac{e^{m\Delta xj} - e^{-m\Delta xj}}{2}\right) - D_0 rA\left(\frac{e^{m\Delta xj} - e^{-m\Delta xj}}{2} - 1\right) - V_0 rB\left(\frac{e^{m\Delta xj} + e^{-m\Delta xj}}{2} - 1\right)\right] \qquad (19)$$

Introducing δ =mΔx, cos∫= $\frac{e^{m\Delta xj}+e^{-m\Delta xj}}{2}$, sin ∫ = $\frac{e^{m\Delta xj}-e^{-m\Delta xj}}{2}$ into eq. 19 and simplifying, we get.

$$A\left[\}-1+V_0 r\sin\int j-r^2(\cos\int-1)(V_0^2 D_0 g)\right]+B[gr(\sin\int j-2V_0r(\cos\int-1)]=0 \qquad (20)$$

Similarly to equation B, we have

$$Y_i^{k+1}=\frac{Y_i^{*}+Y_i^{*k}}{2} \qquad (21)$$

After substituting Eqns. 4,5,9, and 10 into eq. 21 and simplifying, we obtain

$$Y_i^{k+1}=Y_i^k-\frac{D_0 r}{2}[V_{i+1}^k-V_{i-1}^k-V^0 r(V_{i+1}^k+V_{i-1}^k-2V_i^k)$$

$$-gr(Y_{i+1}^k+Y_{i-1}^k-2Y_i^k)]-\frac{V_0 r}{2}[(Y_{i+1}^k-Y_{i-1}^k-D_0 r(V_{i+1}^k+V_{i-1}^k-2V_i^k)V_0 r(Y_{i+1}^k+Y_{i-1}^k-2Y_i^k \quad (22)$$

Writing Eqn. 22 in terms of the expression for the error, using the trigonometric equalities for cosδ and sinδand simplifying, we arrive to :

A [D_0r(sinδj-2rv_0(cosδ-1)] + B[} -1-r^2(D_0g+v_0^2)]

For a non-trivial solution of A and B, the following condition must be satisfied.

$$\left[\}-1+V_0 r\sin\int j-r^2(\cos\int-1)(V_0^2 D_0 g)\right] gr(\sin\int j-2V_0r(\cos\int-1)$$

D0r(sin∫j-2V0(cos∫-1)) $\}-1-r^2(D_0 g+V_0^2)(\cos\int-1)+V_0 r\sin\int j$

Must be equal to zero. Expanding the determinant and simplifying, we get.

For a decay in the error the modulus of 3 must be less than 1.0. If we define,

Cn = as the Courant Number, the modulus of 3 is :

A plot of the amplification factor, 3, versus the frequency parameter, δ, for different Courant Number shows that the scheme is STABLE if $Cn \leq 1$. If $Cn > 1$ the amplification factor grows larger than 1 and the scheme becomes unstable.

For Gabutti scheme follows the same procedure presented for MaiCormack .

14.7

The stability analysis for Preissman scheme is discussed in the following references:

(a) Lyn, DA and Goodwin P "Stability of a General Preissman Scheme", Journal of Hydraulic Engineering, ASCE, Vol. 113, F=1, 1987

(b) Samuels, P.G and Skeels, CP "Stability limits for Preissman Scheme", Journal of Hydraulic Engineering, ASCE, Vol. 116, N28, 1990

Chapter 16
SEDIMENT TRANSPORT

16.1

Table 16.1: Fraction of individual size classes in the sample

i	D_fi	Ψ_fi	F_fi	D_i	Ψ_i	f_i
1	0.125	-3	0	0.1767767	-2.5	0.01
2	0.25	-2	0.01	0.35355339	-1.5	0.05
3	0.5	-1	0.06	0.70710678	-0.5	0.12
4	1	0	0.18	1.41421356	0.5	0.07
5	2	1	0.25	2.82842712	1.5	0.01
6	4	2	0.26	5.65685425	2.5	0.02
7	8	3	0.28	11.3137085	3.5	0.04
8	16	4	0.32	22.627417	4.5	0.18
9	32	5	0.5	45.254834	5.5	0.2
10	64	6	0.7	90.509668	6.5	0.22
11	128	7	0.92	181.019336	7.5	0.08

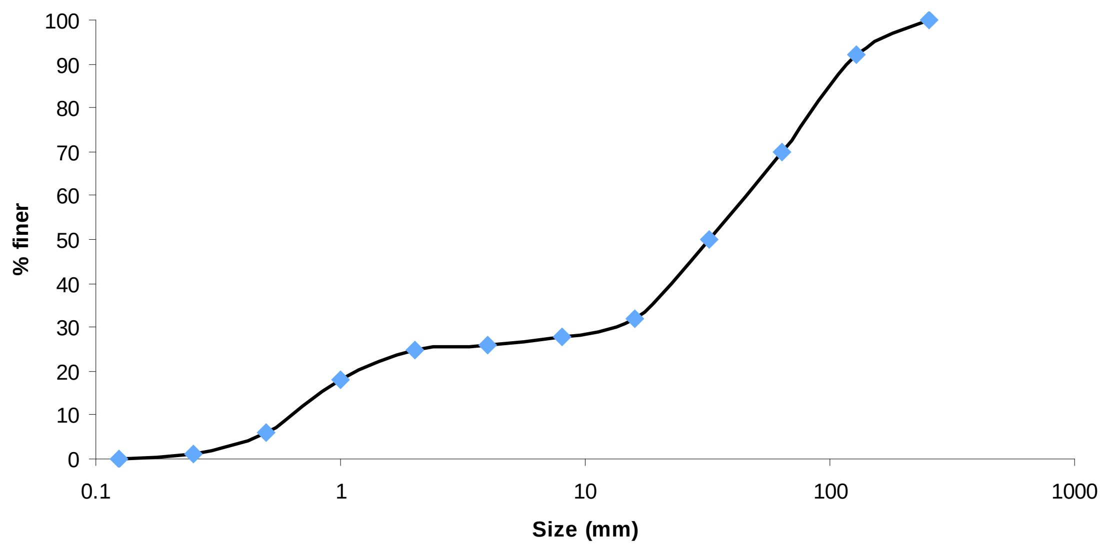


Figure 16.1: Grain size distribution

D_{50} = 32 mm
D_{90} = 126 mm
Ψ_m = 4.02
$D_g = 2\^(\Psi_m)$ = 16.223 mm
σ^2 = 8.19

$\sigma_g = 2\text{^}(\sigma) = 7.269$
Gravel bed river

16.2
Eq. 16-30 is used. The solution is shown is Table 16.2. Columns 1 and 2 are from the problem statement. Column 3 is obtained by R ep = (D/ν).√ (R.g.D). Column 4 is obtained by using Eq. 16-30.

Table 16.2: Non-dimensial critical shear stress by Brownlie equation

Sediment size, D (mm)	R	R_{ep}	Z^*_c
0.25	1.65	15.90327	0.1144
0.5	1.65	44.98125	0.088855
2	1.65	359.85	0.06822
16	1.65	8142.475	0.061262

16.3
Eqs. 16-44, 16-45 and 16-48 are used to calculate the non-dimensional bed load for different values of non-dimensional shear stress. The range of values for the non-dimensional shear stress is from 0.05 to 1.0 with an incremental value of 0.05. Computations for the three methods are shown in Table 16.3. The plots are presented in Fig. 16.3.

Table 16.3: Computation of bed load by different formulas

ζ*	**(i)**	**(ii)**	**(iii)**
0.05	0.001315	0.000453	0
0.1	0.097612	0.044842	0.078728
0.15	0.264451	0.129833	0.278276
0.2	0.47877	0.244542	0.570197
0.25	0.731702	0.384451	0.939737
0.3	1.018054	0.546814	1.377492
0.35	1.334301	0.729718	1.876806
0.4	1.677846	0.931722	2.43265
0.45	2.046669	1.151688	3.041052
0.5	2.439143	1.388686	3.698775
0.55	2.853921	1.641934	4.403111
0.6	3.289862	1.910764	5.151755
0.65	3.745984	2.194596	5.942714
0.7	4.221432	2.492918	6.774238
0.75	4.715448	2.805275	7.644781
0.8	5.22736	3.131258	8.55296
0.85	5.756564	3.470498	9.497528
0.9	6.30251	3.822657	10.47736
0.95	6.864701	4.187428	11.49141
1	7.44268	4.564525	12.53875

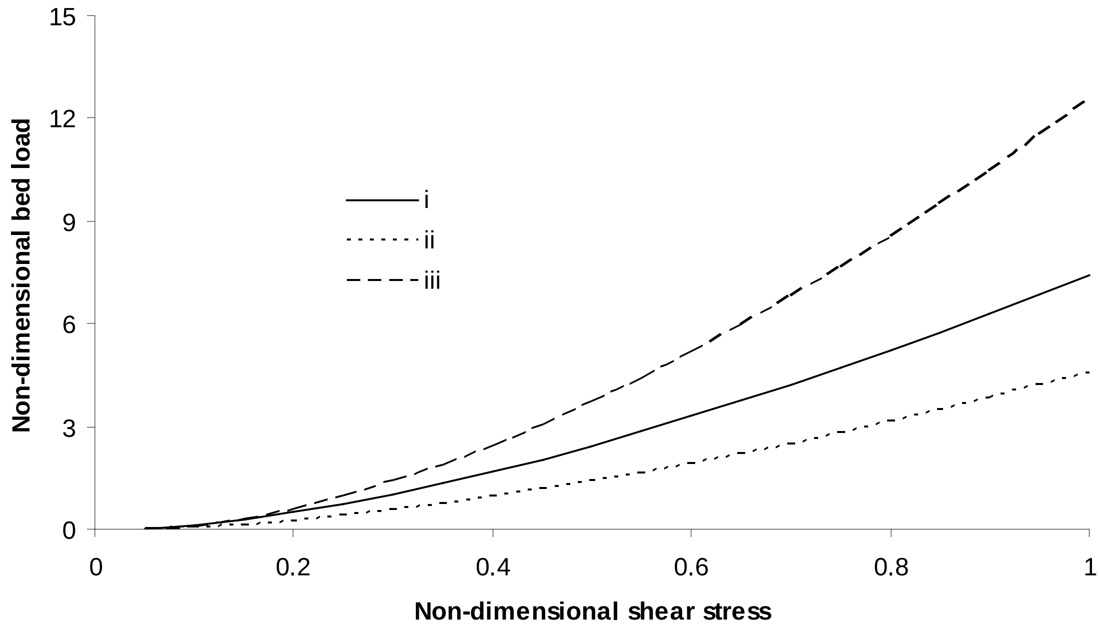


Fig. 16.3 Variation of the non-dimensional bed load with non-dimensional shear stress

16.4

Eq. 16-83 is valid for uniform (normal) flow condition:

$$U = \alpha_r \frac{\sqrt{g}}{C_f^{1/6}} H^{2/3} S^{1/2}$$

Discharge per unit width = $q_w = U.H$

Thus,

$$q_w = \alpha_r \frac{\sqrt{g}}{C_f^{1/6}} H^{5/3} S^{1/2}$$

Or,

$$H = \left(\frac{C_f^{1/3} q_w^2}{\alpha_r^2 g S} \right)$$

16.5

Considering sand to be between 0.65 mm and 2 mm, $F_s = 0.19$, from Table 16.1.

Wilcok-Crowe model:

S = 0.012; $q_w = 4\ m^2/s$

16.6

Eq. 16-80 is used. $\alpha_r = 8.32$ and $k_s = 3D_{90}$ is used. The computations are presented in Table 16.6. A plot between the depth and discharge is shown in Fig. 16.6.

Table 16.6: Computation of discharge

H	S	D_90	K_s	C_f	U	UH
0.9	0.00005	0.000425	0.001275	0.002	0.501	0.451
1	0.00005	0.000425	0.001275	0.002	0.541	0.541
1.1	0.00005	0.000425	0.001275	0.002	0.579	0.637
1.2	0.00005	0.000425	0.001275	0.001	0.617	0.740
1.3	0.00005	0.000425	0.001275	0.001	0.654	0.850
1.4	0.00005	0.000425	0.001275	0.001	0.689	0.965
1.5	0.00005	0.000425	0.001275	0.001	0.725	1.087
1.6	0.00005	0.000425	0.001275	0.001	0.759	1.215
1.7	0.00005	0.000425	0.001275	0.001	0.793	1.348
1.8	0.00005	0.000425	0.001275	0.001	0.827	1.488
1.9	0.00005	0.000425	0.001275	0.001	0.860	1.633
2	0.00005	0.000425	0.001275	0.001	0.892	1.784
2.1	0.00005	0.000425	0.001275	0.001	0.924	1.940
2.2	0.00005	0.000425	0.001275	0.001	0.956	2.102
2.3	0.00005	0.000425	0.001275	0.001	0.987	2.270
2.4	0.00005	0.000425	0.001275	0.001	1.018	2.442
2.5	0.00005	0.000425	0.001275	0.001	1.048	2.620

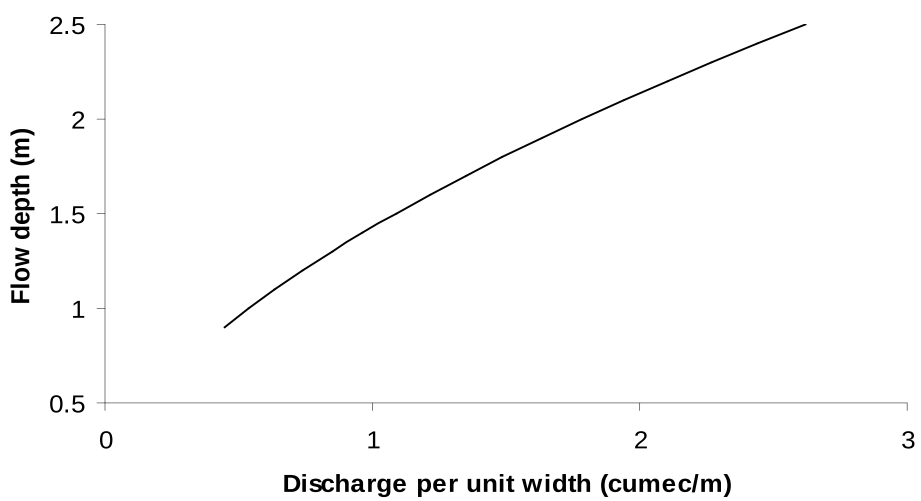


Fig. 16.6: Depth-discharge relationship

Chapter 17
SPECIAL TOPICS

17.1

Given: Reservoir with the following data:

(i) Spillway outflow = 50 $H^{1.5}$, where H = head above the spillway crest in feet.

(ii) Reservoir with vertical sides and surface area of 300000ft^2

(iii) The inflow increases linearly from zero at t = 0 to 500 ft^3/s at t = 15 min. Then linearly decreases to 100 ft^3/s in 10 min. to remain constant afterwards.

(iv) Reservoir at spillway crest elevation at t=0

Solution:

To solve this problem, follow the procedure described in Section 17.4

For a given outflow, the spillway head can be obtained as :

$$H = (O/H)^{0.667}$$

Then, the corresponding storage is S = 300000 Hm^3. If a Dt of 5 minutes is selected, then the relation 0 vs 0+25/Dt is easily obtained. Table 17.1a presents the computations. Figure 17.1a shows this relation.

With the inflow hydrograph and Fig. 17.1a the routing is done by following the steps of the procedure given in section 17.4

Table 17.1a

Relationship O vs 2S/Dt +O

Dt = 5 min

O	H	S	2S/Dt	O+2S/Dt
cfs	ft	ft3	ft3	ft3
0	0	0	0	0
1	0.074	22103.9	147.4	148.4
2	0.117	35087.8	233.9	235.9
3	0.153	45978.1	306.5	309.5
4	0.186	55698.6	371.3	375.3
5	0.215	64632.5	430.9	435.9
10	0.342	102598.0	684.0	694.0
20	0.543	162864.6	1085.8	1105.8
30	0.711	213413.2	1422.8	1452.8
40	0.862	258532.0	1723.5	1763.5
50	1.000	300000.0	2000.0	2050.0
60	1.129	338773.2	2258.5	2318.5
70	1.251	375439.9	2502.9	2572.9
75	1.310	393111.7	2620.7	2695.7
80	1.368	410394.9	2736.0	2816.0
85	1.424	427321.4	2848.8	2933.8
90	1.480	443919.0	2959.5	3049.5
95	1.534	460212.0	3068.1	3163.1
100	1.587	476221.4	3174.8	3274.8
105	1.640	491966.1	3279.8	3384.8
110	1.692	507462.8	3383.1	3493.1
115	1.742	522726.3	3484.8	3599.8
120	1.793	537770.1	3585.1	3705.1

Table 17.1b

Computation of Reservoir Routing

Dt = 5 min

Time	Inflow	I1 + I2	2S/Dt - O	2S/Dt + O	Outflow
min	cfs	cfs	cfs	cfs	cfs
0	0.0	166.7	0.0	166.7	0.0
5	166.7	500.0	164.3	664.3	1.2
10	333.3	833.3	645.5	1478.8	9.4
15	500.0	750.0	1417.4	2167.4	30.7
20	250.0	350.0	2059.0	2409.0	54.2
25	100.0	200.0	2281.8	2481.8	63.6
30	100.0	200.0	2348.2	2548.2	66.8
35	100.0	200.0	2410.2	2610.2	69.0
40	100.0	200.0	2467.2	2667.2	71.5
45	100.0	200.0	2519.6	2719.6	73.8
50	100.0	200.0	2567.6	2767.6	76.0
55	100.0	200.0	2611.6	2811.6	78.0
60	100.0	200.0	2652.0	2852.0	79.8
65	100.0	200.0	2689.0	2889.0	81.5
70	100.0	200.0	2722.8	2922.8	83.1
75	100.0	200.0	2753.6	2953.6	84.6
80	100.0	200.0	2781.8	2981.8	85.9
85	100.0	200.0	2807.6	3007.6	87.1
90	100.0	200.0	2831.2	3031.2	88.2
95	100.0	200.0	2852.8	3052.8	89.2
100	100.0	200.0	2872.4	3072.4	90.2
105	100.0	200.0	2890.4	3090.4	91.0
110	100.0	200.0	2906.8	3106.8	91.8
115	100.0	200.0	2921.8	3121.8	92.5

Fig.17.1a Relationship O vs 2S/Dt + O

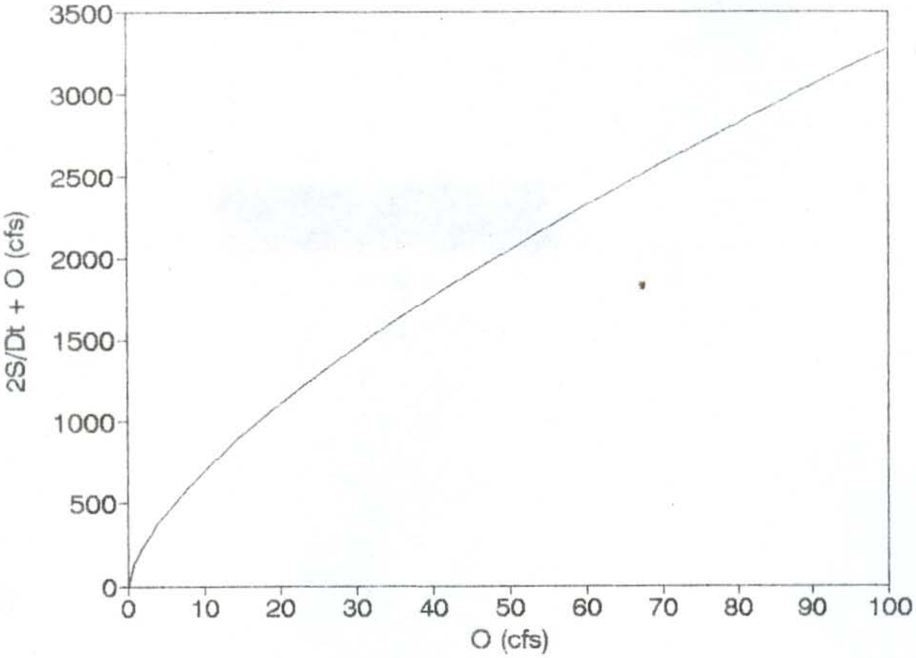


Fig.17.1b Routing Through Reservoir

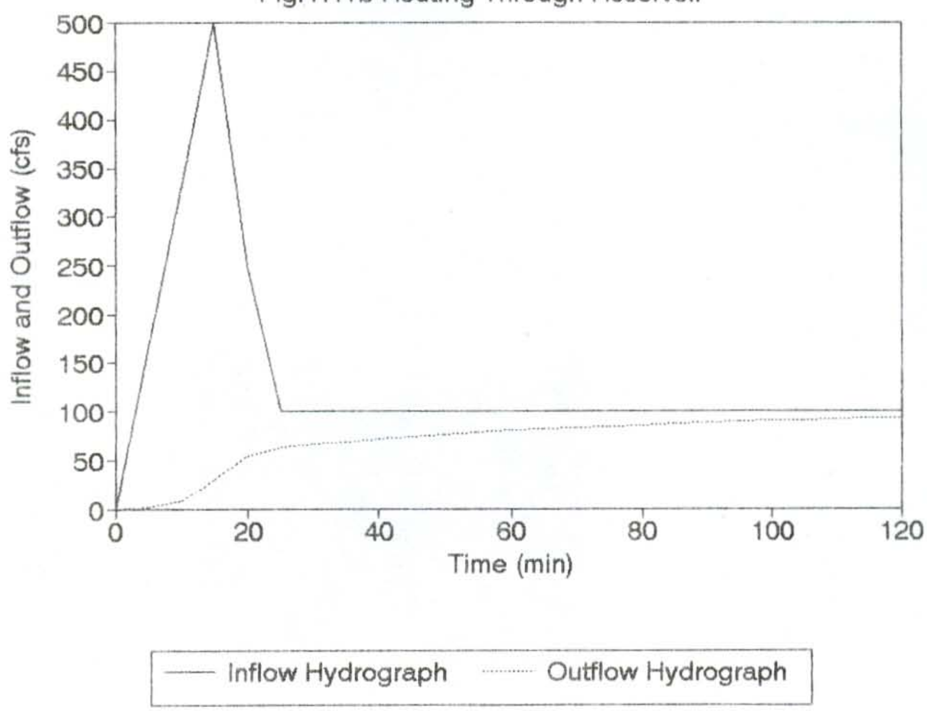


Table 17.1b shows the computations. The inflow and outflow hydrographs are presented in Fig. 17.1b

17.3

Given,

Detention pond with the following characteristics:

(i) Spillway crest level = 10 ft.

(ii) Spillway discharge equation, $Q = 100(E-10)^{1.5}$, E in ft.

(iii) Pond surface area at El. 0 ft = 200000 ft^2 and it increases linearly to 300000 ft^3 at El.40 ft.

(iv) Inflow for t < 10 min. is 5t (t in seconds). After 10 min. the inflow remains constant at 3000 fs.

(v) Pond level at t = 0 at El. 8 ft.

Compute : Outflow hydrograph from the pond until t = 20 min.

Solution :

(i) Express the pond surface area and the storage volumen as a function of the spillway head elevation E, this is :

A = 225000 + (e-10)2500

S = $\int_0^z A dz = 200000 z + 1250 z^2$

Therefore, for any value of E the previous relations give the area and the storage volume. The spillway discharge equation gives the outflow corresponding to the given E. These three variables, A, S and Q are computed in the first columns (columns 2 to 4 in Table 17.3a). The last column is the at flow-storage relation 2S/Dt + 0 where an interval of one minute was chosen for the routing. Figure 17.3 a shows the relation 0 Vs 2S/Dt + 0

(ii) Following the procedure outlined in section 17.4, Tble 17.3 is obtained. The last column in this table is the water elevation over the spillway corresponding to the particular time of the routing.

(iii) Figure 17.3b shows a plot of the inflow and outflow hydrographs. It also shows the spillway head during the routing time.

Table 17.3a

Reservoir Characteristics

Elev. z	Area	Outflow,O	Volume,S	2S/60+O
ft	ft2	cfs	ft3	cfs
8	220000	0.00	1680000	56000.0
9	222500	0.00	1901250	63375.0
10	225000	0.00	2125000	70833.3
10.58	226450	44.17	2255921	75241.5
11.22	228050	134.75	2401361	80180.1
11.92	229800	266.04	2561608	85653.0
12.61	231525	421.66	2720765	91113.8
13.25	233125	585.90	2869453	96234.3
13.85	234625	755.42	3009778	101081.4
14.4	236000	922.95	3139200	105563.0
14.91	237275	1087.98	3259885	109750.8
15.37	238425	1244.40	3369296	113554.3
15.79	239475	1393.21	3469655	117048.4
16.18	240450	1536.32	3563241	120311.0
16.53	241325	1668.67	3647551	123253.7
16.84	242100	1788.89	3722482	125871.6
10.58	226450	44.17	2255921	75241.5

Table 17.3b
Computation of Reservoir Routing

Time	I	I1 + I2	2S1/dt-O1	2S2/dt+O	O1	Elev, z
min	cfs	cfs	ft3	ft3	ft3	ft
0	0	300	56000.0	56300.0	0.0	8.00
1	300	900	56300.0	57200.0	0.0	8.06
2	600	1500	57200.0	58700.0	0.0	8.16
3	900	2100	58700.0	60800.0	0.0	8.37
4	1200	2700	60800.0	63500.0	0.0	8.66
5	1500	3300	63500.0	66800.0	0.0	9.02
6	1800	3900	66800.0	70700.0	0.0	9.46
7	2100	4500	70700.0	75200.0	0.0	9.98
8	2400	5100	75111.7	80211.7	44.2	10.58
9	2700	5700	79942.2	85642.2	134.8	11.22
10	3000	6000	85110.1	91110.1	266.0	11.92
11	3000	6000	90266.8	96266.8	421.7	12.61
12	3000	6000	95095.0	101095.0	585.9	13.25
13	3000	6000	99584.1	105584.1	755.4	13.85
14	3000	6000	103738.2	109738.2	923.0	14.40
15	3000	6000	107562.3	113562.3	1088.0	14.91
16	3000	6000	111073.5	117073.5	1244.4	15.37
17	3000	6000	114287.0	120287.0	1393.2	15.79
18	3000	6000	117214.4	123214.4	1536.3	16.18
19	3000	6000	119877.1	125877.1	1668.7	16.53
20	3000	3000	122299.1	125299.1	1789.0	16.84

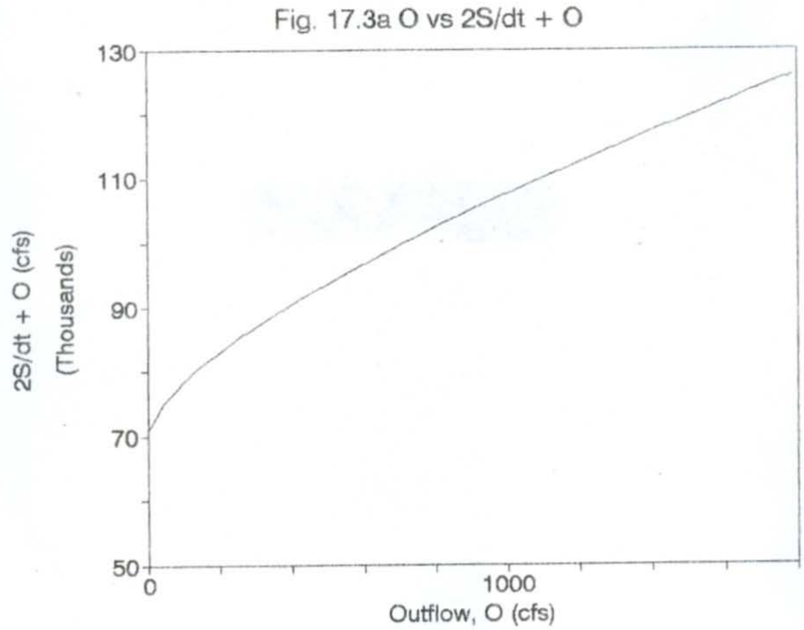


Fig. 17.3a O vs 2S/dt + O
2S/dt + O (cfs)
(Thousands)
130
110
90
70
50
0
1000
Outflow, O (cfs)

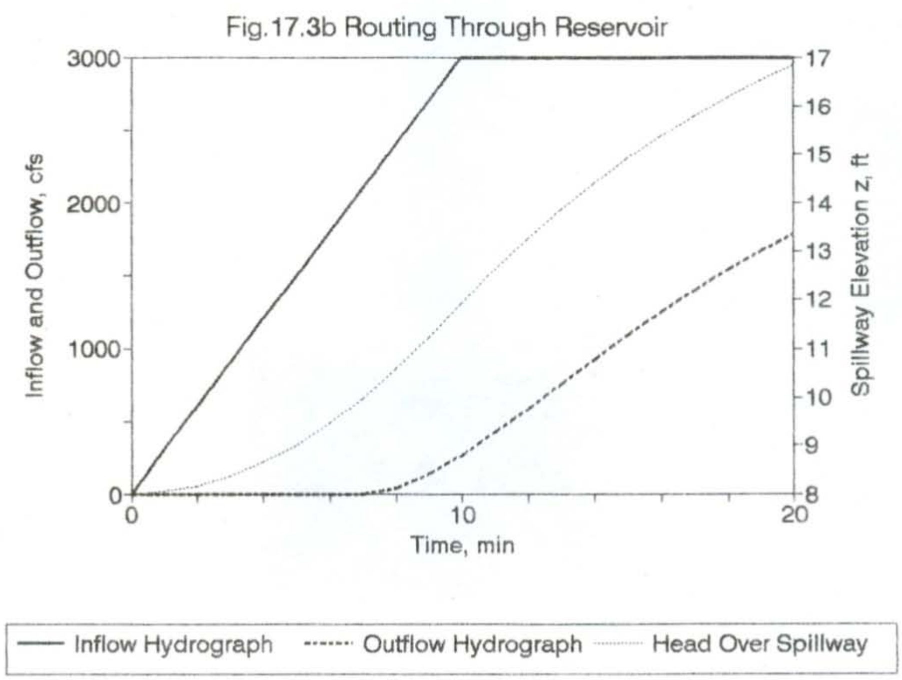


Fig.17.3b Routing Through Reservoir
Inflow and Outflow, cfs
3000
2000
1000
0
0
10
20
Time, min
17
16
15
14
13
12
11
10
9
8
Spillway Elevation z, ft
Inflow Hydrograph
Outflow Hydrograph
Head Over Spillway

17.4 Given

X = 0.5 and a $\frac{\Delta t}{\Delta x}$ = 1, in the Muskingum-Cunge model.

Show that the wave does not attenuate as if is routed through a channel reach.

Solution :

Compute the constants for the Muskingum-Cunge model

$$Co = \frac{0.5\Delta t - \alpha \Delta x / a}{0.5\Delta t + (1-\alpha)\Delta x/a} = \frac{0.5\Delta t - 0.5\,\Delta x/a}{0.5\,\Delta t + 0.5\,\Delta x/a}$$

$$= \frac{a\Delta t - \Delta x}{a\Delta t + \Delta x}$$

$$Co = \frac{(a\frac{\Delta t}{\Delta x} - 1)}{(a\frac{\Delta t}{\Delta x} + 1)} = 0 \quad \text{because } a\frac{\Delta t}{\Delta x} = 1$$

$$C1 = \frac{0.5\,\Delta t + (\alpha \Delta x / a)}{0.5\Delta t + (1-\alpha)\Delta x/a} = \frac{0.5\Delta t + 0.5\,\Delta x/a}{0.5\,\Delta t + 0.5\,\Delta x/a} = 1$$

And

$$C2 = \frac{-0.5\,\Delta t + (1-\alpha)\Delta x / a}{0.5\Delta t + (1-\alpha)\Delta x/a} = \frac{-0.5\Delta t - 0.5\,\Delta x/a}{0.5\,\Delta t + 0.5\,\Delta x/a}$$

$$= \frac{-\Delta t + \Delta x/a}{\Delta t + \Delta x/a}$$

But $\Delta x/a$ = Δt, therefore, Cz = c

Subsequently, in the routing expression (eq. 17.12) the values of C0, C1 and Cz, we get,

O^{k+1} = Co I^{k} + 1 +C_1 I^k + C_2 O^k

O^{k+1} = I^k

From this equality we conclude that the inflow at time 't' is exactly the outflow at time t +Δt and the wave is not attenuated.

http://www.springer.com/978-0-387-30174-7

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