Preface

Of what use is mathematics? Hasn't everything in mathematics already been discovered? These are natural questions often asked by undergraduates. The answers provided by their professors are often quite brief. Most university courses, pressed for time and rigidly structured, offer little opportunity to present and study actual applications and real-world examples.

Even more high-school students ask the same questions with more insistence. Teachers in these schools generally work under even tighter constraints than university professors. If they are able to competently respond to these questions it is probably because they received good answers from their teachers and professors. And if they do not have the answers, then whose fault is it?

The genesis of this text

It is impossible to introduce this text without first discussing the course in which it originated. The course "Mathematics and Technology" was created at the Université de Montréal and taught for the first time in the winter semester of 2001. It was created after observing that most courses in the department neglect to present real applications. Since its creation the course has been open to both undergraduate mathematics students and future high-school teachers.

Since no appropriate text or manual for the course we envisioned existed, we were led to write our own course notes, from which we taught. We got so caught up in writing these notes that they quickly grew to the size of a textbook, containing much more material than could possibly be taught in one semester. Despite the two of us being career mathematicians, we must admit that we both knew little or nothing about most of the applications presented in the following chapters.

The goal of the "Mathematics and Technology" course

The primary goals of the course are to demonstrate the active and evolving character of mathematics, its omnipresence in the development of technologies, and to initiate students into the process of modeling as a path to the development of various mathematical applications.

Although a few of the included subjects fall outside the strict domain of technology, we hope to make it clear that, yes, mathematics is useful, and it plays a major role in everyday technologies. Several of the subjects treated in this text are still being actively developed, and this allows students to see, often for the first time, that the field of mathematics remains open and dynamic.

Since the students taking our course include future high school-teachers, it is important to stress that the point is not simply to provide them with examples and applications that they can repeat to their future students, but rather to give them the tools to formulate and develop real-world examples appropriate to their students. They should be instilled with the feeling that they are teaching a subject that is intrinsically elegant, of course, but whose applications have helped shape our physical environment and our understanding of it.

The choice of subjects

In choosing subjects we have paid particular attention to the following points:

- The applications should be recent or affect the students' day-to-day life. Moreover, contrary to the mature mathematics typically taught in other undergraduate courses, some of the mathematics used should be modern or even still in development.
- The mathematics should be relatively elementary and if it exceeds the typical first-year undergraduate curriculum (calculus, linear algebra, probability theory), the missing pieces must be covered within the chapter. A special effort is made to make extensive use of high-school-level mathematics, particularly Euclidean geometry. Basic high-school and undergraduate mathematics form a remarkable toolkit, provided they are well understood and mastered, allowing students to readily explore their wide applications and, often for the first time, to discover their power when used together.
- The level of mathematical sophistication required should remain at a minimum: ideas are a scientist's most precious commodity, and behind most technological successes there lies a brilliant yet sometimes elementary observation.

As a result, the mathematics used in the book covers a very wide spectrum:

- Lines and planes appear in all of their forms (regular equations, parametric equations, subspaces), often in unexpected ways (using the intersection of several planes to decode a Reed–Solomon encoded message).
- A large number of subjects make use of basic geometric objects: circles, spheres, and conics. The concept of *locus of points* in Euclidean geometry is often repeated, for example in problems where we calculate the position of an object through triangulation (Chapter 1 on GPS, and Chapter 15 on *Science Flashes*).
- The different types of affine transformations in the plane or in space (in particular rotation and symmetries) appear several times: in Chapter 11 on image compression using fractals, in Chapter 2 on mosaics and friezes, and in Chapter 3 on robot motion.
- Finite groups appear as symmetry groups (Chapter 2 on mosaics and friezes) and also in the development of primality tests in cryptography (Chapter 7).

- Finite fields make an appearance in Chapter 6 on error-correcting codes, in Chapter 1 on GPS and in Chapter 8 on random-number generation.
- Chapter 7 on cryptography and Chapter 8 on random-number generation both make use of arithmetic modulo n, while Chapter 6 on error-correcting codes makes use of arithmetic modulo 2.
- Probability theory appears in several unexpected places: in Chapter 9 on *Google*'s *PageRank* algorithm, and in the construction of large prime numbers in Chapter 7. It is also used more classically in Chapter 8 on random-number generation.
- Linear algebra is omnipresent: in Chapter 6 on Hamming and Reed–Solomon codes, in Chapter 9 on the *PageRank* algorithm, in Chapter 3 on robot motion, in Chapter 2 on mosaics and friezes, in Chapter 1 on GPS, in Chapter 12 on the JPEG standard, etc.

Using this book as a course text

The text is written for students who have a familiarity with Euclidean geometry and have mastered multivariable calculus, linear algebra, and elementary probability theory. We hope that we have not implicitly assumed any other background knowledge. Working through the text nonetheless requires a certain scientific maturity: it involves integrating a variety of mathematical tools in a setting different from the one in which they were originally taught. For that reason, undergraduates in their junior or senior years are the ideal audience for the course.

The text presents applications in two forms: the main chapters (all except Chapter 15) are long and detailed, while the *Science Flashes* (sections of Chapter 15) are short and narrow in scope. Readers will notice a certain unity in the form of the longer chapters: the first sections describe the application and the underlying mathematical problem; this is followed by an exploration of simple cases of the problem and, if necessary, a development of the required mathematics. We call these parts the *basic* portion of the chapter. Afterward, one or more sections may explore more-complicated examples, provide more details to the mathematical tools discussed earlier, or simply discuss the fact that mathematics alone is not always sufficient! We refer to this latter part of a chapter as the *advanced* portion. Each application is typically covered in 5–6 hours of class: two hours for the basic theory, two hours for examples and exercises and, if time permits, one or two hours for advanced topics. Often we are able only to touch briefly on the advanced material, unless a second week is spent on the chapter. Each *Science Flash* can be treated in an hour of class or even assigned as an exercise without being preceded by any theory development. During a single semester we aim to cover a significant part of 8 to 12 chapters and a handful of *Science Flashes*. Another option is to significantly reduce the number of chapters being covered and to dig further into their advanced sections.

We are thus forced to select subjects as a function of their intrinsic interest or the students' mathematical knowledge. The chapters not selected or the advanced portions of those that were covered are natural points of departure for course projects. Self-guided students who are reading this text on their own may simply jump from chapter to chapter as the mood strikes them. Each chapter is (mathematically) independent (or very nearly so), and any links among them are explicitly stated.

One last note for professors using this book as a course text. Teaching this course has forced us to revise our usual pedagogical methods: here no subject is prerequisite for further courses, the definitions and theorems are not the ultimate goals of the course, and the problems are not drill. These factors can cause some anxiety on the students' side. Moreover, we are not specialists in any of the technologies we discuss here. So we had to revise our teaching. We try to make as many links as possible to the technology. We encourage students to participate in the course. This allows us to check their background relative to the mathematical tools being used. As for exams, we choose to reassure them from the beginning by stating that the exams are open book, noncumulative, and limited to the basic material. Emphasis is put on simple mathematical modeling and problem solving. Our sets of exercises focus on these skills.

Using this book as a self-directed reader

During the writing of this text we have always been passionate about presenting the mathematics underlying technology and demonstrating both its intrinsic beauty and power. We believe that this text will be of interest to any reader, from young scientist to experienced mathematician, curious to understand the mathematics that drives technological innovation. Since the chapters are largely independent, the reader can hop from subject to subject at will. Hopefully, the reader will be equally interested in the many historical notes scattered throughout the text and, who knows, even find time to work through a few of the exercises.

The contributions of Hélène Antaya and Isabelle Ascah-Coallier

The first draft of Chapter 14 on the calculus of variations was written by Hélène Antaya during a summer internship at the end of her junior college. Chapter 13 on computing with DNA was written the following summer by Hélène Antaya and Isabelle Ascah-Coallier while they were supported by an Undergraduate Student Research Award from the National Sciences and Engineering Research Council (NSERC) of Canada.

How to use the chapters

For the most part, chapters are independent. The beginning of each chapter contains a brief "how-to," describing the required basic knowledge, the relationships between the sections, and, if necessary, their relative difficulty.

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Acknowledgments

The genesis of the “Mathematics and Technology” course and accompanying course notes can be traced back to the winter of 2001. We had to learn a variety of subjects that we knew only incidentally or not at all, and also had to construct sets of exercises and student projects. Throughout the many years of evolution of this text we have asked numerous questions that required a great level of explanation. We would like to thank those who have supported us in this endeavor. Their assistance has helped us reduce the inevitable ambiguities and errors; we are responsible for any that remain, and we invite our readers to report any they may find.

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