Preface

The purpose of this monograph is to offer a comprehensive presentation of methods for solving systems of linear algebraic equations that typically arise in (but are not limited to) the numerical solution of partial differential equations (PDEs). Focus is on the finite element (or f.e.) method, although it is not presented in detail. It would, however, help the readers to be familiar with some basic knowledge of the finite element method (such as typical error estimates for second-order elliptic problems). There are a number of texts that describe the finite element method with various levels of detail, including Ciarlet [Ci02], Brenner and Scott [BS96], Braess [B01], Ern and Guermond [EG04], Solin [So06], and Elman, *et al.* [ESW06]. The presentation here utilizes matrix–vector notation because this is the basis of how the resulting solution methods are eventually implemented in practice. The choice of the material is largely based on the author's own work, and is also aimed at covering a number of important achievements in the field that the author finds useful one way or another. Among those are the most efficient methods, such as multigrid (MG), especially its recently revived "algebraic" version (or AMG), as well as domain decomposition (DD) methods. The author found a common ground to present both as certain block-matrix factorizations. This framework originates in some more classical methods such as the (block-) approximate (or incomplete) LU (or block-ILU) factorization methods. This led to the somewhat unusual title of the book. The approach, as well as the specific topics covered, should offer a different view on topics covered in other books that deal with preconditioned iterative methods.

This book starts with a motivational introductory chapter that describes the class of matrices to which this book is mainly devoted and sets up the goals that the author tries to achieve with the remainder of the text. In particular, it describes sparsity, conditioning, assembly from local element matrices, and the Galerkin relation between two matrices coming from discretization of the same PDE on coarse and fine meshes (and nested finite element spaces). The introduction ends with a major strong approximation property inherited from the regularity property of the underlining PDE. A classical two-grid method is then introduced that is illustrated with smoothing iterations and coarse-grid approximation. The motivational chapter also contains some basic facts about matrix orderings and a strategy to generate a popular nested dissection ordering arising from certain element agglomeration algorithms. The element agglomeration is later needed to construct a class of promising algebraic multigrid methods for solving various PDEs on general unstructured finite element meshes. Also discussed is the important emerging topic in practice of how to generate the f.e. discretization systems on massively parallel computers, and a popular mortar f.e. method is described in a general algebraic setting. Many other auxiliary (finite element and numerical linear algebra) facts are included in the seven appendices of the book.

The actual text starts with some basic facts about block-matrices and introduces a general two-by-two, block-factorization scheme followed by a sharp analysis. More specific methods are then presented. The focus of the book is on symmetric positive definite matrices, although extensions of some of the methods, from the s.p.d. case to nonsymmetric, indefinite, and saddle-point matrices, have been given and analyzed. In addition to linear problems, the important case of problems with constraints, as well as Newton-type methods for solving some nonlinear problems, are described and analyzed. Some of the topics are only touched upon and offer a potential for future research. In this respect, the text is expected to be useful for advanced graduate students and researchers in the field. The presentation is rigorous and self–contained to a very large extent. However, at a number of places the potential reader is expected to fill in some minor (and obvious) missing details either in the formulation and/or in the provided analysis.

Specific comments due to Yvan Notay, David Silvester, Joachim Schöberl, Xiao–Chuan Cai, Steve McCormick, and Ludmil Zikatanov are gratefully acknowledged. Special thanks are due to Tzanio Kolev for his comments and for providing numerous illustrations used throughout the book.

The author is thankful to Arnold Gatilao for his invaluable help with editing major part of the text.

Finally, the help of Vaishali Damle, Editor, Springer is greatly appreciated.

Portions of this book were written under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

Livermore, California
March 2007 *Panayot S. Vassilevski*

Springer

http://www.springer.com/978-0-387-71563-6

Multilevel Block Factorization Preconditioners
Matrix-based Analysis and Algorithms for Solving Finite Element Equations
Vassilevski, P.S.
2008, XIV, 530 p. 34 illus., Hardcover
ISBN: 978-0-387-71563-6