

---

## Preface

Statistical modeling is a critical tool in scientific research. Statistical models are used to understand phenomena with uncertainty, to determine the structure of complex systems, and to control such systems as well as to make reliable predictions in various natural and social science fields. The objective of statistical analysis is to express the information contained in the data of the phenomenon and system under consideration. This information can be expressed in an understandable form using a statistical model. A model also allows inferences to be made about unknown aspects of stochastic phenomena and to help reveal causal relationships. In practice, model selection and evaluation are central issues, and a crucial aspect is selecting the most appropriate model from a set of candidate models.

In the information-theoretic approach advocated by Akaike (1973, 1974), the Kullback–Leibler (1951) information discrepancy is considered as the basic criterion for evaluating the goodness of a model as an approximation to the true distribution that generates the data. The Akaike information criterion (AIC) was derived as an asymptotic approximate estimate of the Kullback–Leibler information discrepancy and provides a useful tool for evaluating models estimated by the maximum likelihood method. Numerous successful applications of the AIC in statistical sciences have been reported [see, e.g., Akaike and Kitagawa (1998) and Bozdogan (1994)]. In practice, the Bayesian information criterion (BIC) proposed by Schwarz (1978) is also widely used as a model selection criterion. The BIC is based on Bayesian probability and can be applied to models estimated by the maximum likelihood method.

The wide availability of fast and inexpensive computers enables the construction of various types of nonlinear models for analyzing data with complex structure. Nonlinear statistical modeling has received considerable attention in various fields of research, such as statistical science, information science, computer science, engineering, and artificial intelligence. Considerable effort has been made in establishing practical methods of modeling complex structures of stochastic phenomena. Realistic models for complex nonlinear phenomena are generally characterized by a large number of parameters. Since the maximum

likelihood method yields meaningless or unstable parameter estimates and leads to overfitting, such models are usually estimated by such methods as the maximum penalized likelihood method [Good and Gaskins (1971), Green and Silverman (1994)] or the Bayes approach. With the development of these flexible modeling techniques, it has become necessary to develop model selection and evaluation criteria for models estimated by methods other than the maximum likelihood method, relaxing the assumptions imposed on the AIC and BIC.

One of the main objectives of this book is to provide comprehensive explanations of the concepts and derivations of the AIC, BIC, and related criteria, together with a wide range of practical examples of model selection and evaluation criteria. A secondary objective is to provide a theoretical basis for the analysis and extension of information criteria via a statistical functional approach. A generalized information criterion (GIC) and a bootstrap information criterion are presented, which provide unified tools for modeling and model evaluation for a diverse range of models, including various types of nonlinear models and model estimation procedures such as robust estimation, the maximum penalized likelihood method and a Bayesian approach. A general framework for constructing the BIC is also described.

In Chapter 1, the basic concepts of statistical modeling are discussed. In Chapter 2, models are presented that express the mechanism of the occurrence of stochastic phenomena. Chapter 3, the central part of this book, explains the basic ideas of model evaluation and presents the definition and derivation of the AIC, in both its theoretical and practical aspects, together with a wide range of practical applications. Chapter 4 presents various examples of statistical modeling based on the AIC. Chapter 5 presents a unified information-theoretic approach to statistical model selection and evaluation problems in terms of a statistical functional and introduces the GIC [Konishi and Kitagawa (1996)] for the evaluation of a broad class of models, including models estimated by robust procedures, maximum penalized likelihood methods, and the Bayes approach. In Chapter 6, the GIC is illustrated through nonlinear statistical modeling in regression and discriminant analyses. Chapter 7 presents the derivation of the GIC and investigates its asymptotic properties, along with some theoretical and numerical improvements. Chapter 8 is devoted to the bootstrap version of information criteria, including the variance reduction technique that substantially reduces the variance associated with a Monte Carlo simulation. In Chapter 9, the Bayesian approach to model evaluation, such as the BIC, ABIC [Akaike (1980b)] and the predictive information criterion [Kitagawa (1997)] are discussed. The BIC is also extended such that it can be applied to the evaluation of models estimated by the method of regularization. Finally, in Chapter 10, several model selection and evaluation criteria such as cross-validation, generalized cross-validation, final prediction error (FPE), Mallows'  $C_p$ , the Hannan–Quinn criterion, and ICOMP are introduced as related topics.

We would like to acknowledge the many people who contributed to the preparation and completion of this book. In particular, we would like to acknowledge with our sincere thanks Hirotugu Akaike, from whom we have learned so much about the seminal ideas of statistical modeling.

We have been greatly influenced through discussions with Z. D. Bai, H. Bozdogan, D. F. Findley, Y. Fujikoshi, W. Gersch, A. K. Gupta, T. Higuchi, M. Ichikawa, S. Imoto, M. Ishiguro, N. Matsumoto, Y. Maesono, N. Nakamura, R. Nishii, Y. Ogata, K. Ohtsu, C. R. Rao, Y. Sakamoto, R. Shibata, M. S. Srivastava, T. Takanami, K. Tanabe, M. Uchida, N. Yoshida, T. Yanagawa, and Y. Wu.

We are grateful to three anonymous reviewers for comments and suggestions that allowed us to improve the original manuscript. Y. Araki, T. Fujii, S. Kawano, M. Kayano, H. Masuda, H. Matsui, Y. Ninomiya, Y. Nonaka, and Y. Tanokura read parts of the manuscript and offered helpful suggestions. We would especially like to express our gratitude to D. F. Findley for his previous reading of this manuscript and his constructive comments. We are also deeply thankful to S. Ono for her help in preparing the manuscript by L<sup>A</sup>T<sub>E</sub>X. John Kimmel patiently encouraged and supported us throughout the final preparation of this book. We express our sincere thanks to all of these people.

Sadanori Konishi  
Genshiro Kitagawa

Fukuoka and Tokyo, Japan  
February 2007



<http://www.springer.com/978-0-387-71886-6>

Information Criteria and Statistical Modeling

Konishi, S.; Kitagawa, G.

2008, XII, 276 p., Hardcover

ISBN: 978-0-387-71886-6