
Contents

1	Mathematics for the Semiclassicist	1
1.1	Single-Valued Analytic Functions	1
1.2	Method of Steepest Descent and Asymptotic Methods	2
1.2.1	Stationary-Phase Version	3
1.3	Generalized Variation and Perturbation Theories	4
1.4	Hypergeometric Series	6
1.5	Contour Integral Transforms	11
1.6	Combinatorics	14
1.6.1	Proof via Sister Celine's Technique	15
1.7	Generalized Hypergeometric Functions	16
1.8	Fourier and Laplace Transforms	19
1.8.1	Critical Fourier Transform Relation	19
1.8.2	Critical Laplace Transform Relation	20
2	Semiclassical Phase Integrals	21
2.1	Approximation	21
2.1.1	JWKB Approximation	21
2.1.2	Gans–Jeffreys Asymptotic Connection Formula	24
2.2	Phase Integrals	25
2.2.1	Stokes Phenomenon: One Transition Point	25
2.2.2	Application of JWKB to Coupled Wave Equations	29
2.3	Two and Four Transition Points: Crossing and Noncrossing	44
2.3.1	Introduction	44
2.3.2	Exact Resumming of Asymptotic Relations for Parabolic Cylinder Functions of Large Order and Argument	45
2.3.3	The Crossing Parabolic Model	58
2.3.4	Connection to Bárány–Crothers Phase-Integral Nikitin-Model Analysis	61
2.3.5	Connections to Nakamura and Zhu Phase-Integral Analysis	62
2.3.6	Connections to the Frömans-Lundborg Phase-Integral Analysis	64

2.3.7	Conclusions	65
2.3.8	Curve Crossing Reflection Probabilities in One Dimension	66
2.4	Addition of a Simple Pole	71
2.4.1	Introduction	71
2.4.2	The Semiclassical Scattering Matrix	74
2.4.3	Phase-Integral Treatment	75
2.4.4	Comparison Equation	80
2.4.5	General Phase-Integral Abstraction	83
2.4.6	Discussion	83
2.5	Other Generalizations	85
2.5.1	Four Close Curve-Crossing Transition Points	85
2.5.2	Circuit-Dependent Adiabatic Phase Factors from Phase Integral Theory	88
3	Semiclassical Method for Hyperspherical Coordinate Systems	93
3.1	Wannier's Classical Treatment of Electron Correlation	93
3.2	Differential and Integrated Wannier Cross Sections	98
3.2.1	Conclusions	115
3.3	Doubly Excited States and Their Lifetimes	116
3.3.1	Results	123
3.3.2	Doubly Excited States of He	125
3.4	Divergent Exponents	128
3.4.1	Wannier's Theory	129
3.4.2	The Semiclassical JWKB Approximation	130
3.4.3	Semiclassical Theory when the Exponent Diverges	131
3.4.4	Results, Discussion, and Conclusions	137
4	Ion-Atom Collisions	139
4.1	The Semiclassical Impact Parameter Treatment	139
4.2	Traveling Atomic and Molecular Orbitals	144
4.2.1	Traveling Molecular H_2^+ Orbitals	145
4.2.2	Traveling Molecular HeH^{2+} Orbitals	155
4.2.3	Traveling Atomic Orbitals	171
4.3	Continuum Distorted Waves and Their Generalizations	172
4.3.1	Introduction	172
4.3.2	Charge Transfer	173
4.3.3	Ionization	182
4.3.4	Fully differential cross sections for ionization	197
4.3.5	Generalized Continuum Distorted Waves	210
4.3.6	Double Ionization	215
4.4	Relativistic CDW	219
4.4.1	Antihydrogen Production	231
4.5	Semiclassical Acausality	234
4.5.1	Introduction	234
4.5.2	Generalized Impact-Parameter Treatment	236

4.5.3	Perturbation Theory	238
4.5.4	Discussion and Conclusions	240
5	Diffusion in Liquids and Solids	243
5.1	Single-Domain Ferromagnetic Particles	243
5.2	The Fokker–Planck and Langevin Equations	267
5.2.1	Drift and Diffusion Coefficients	273
5.3	Dielectric Relaxation, Anomalous Diffusion, Fractals, and After Effects	284
5.3.1	Numerical Calculation and Physical Understanding	289
5.4	Nonlinear Response of Permanent Dipoles and After Effects	292
5.4.1	Complex Susceptibility for the Debye and Debye–Fröhlich Models of Relaxation	294
5.4.2	Linear Dielectric Response	297
5.4.3	Dynamic Kerr Effect	299
5.4.4	Nonlinear Dielectric Relaxation	300
5.4.5	Approximate Analytical Formula for the Dynamic Kerr Effect for a Pure Cosinusoid	301
A	Continued Fraction Solutions of Eq. (5.301)	305
B	Mittag–Leffler Functions	309
B.0.1	Properties of Mittag–Leffler Functions	309
B.0.2	Asymptotics of Mittag–Leffler functions	309
B.1	Check on Norm of $x^2(\tau)$	311
C	Nonlinear Response to Alternating Fields	313
	References	321
	Index	337



<http://www.springer.com/978-0-387-74312-7>

Semiclassical Dynamics and Relaxation

Crothers, D.S.F.

2008, XII, 344 p., Hardcover

ISBN: 978-0-387-74312-7