Preface

The theory in these notes was taught between 2002 and 2005 at the graduate schools of Ecole Normale Supérieure de Cachan, Ecole Polytechnique de Palaiseau, Universitat Pompeu Fabra, Barcelona, Universitat de les Illes of Balears, Palma, and University of California at Los Angeles. It is also being taught by Andrès Almansa at the Facultad de Ingeneria, Montevideo.

This text will be of interest to several kinds of audience. Our teaching experience proves that specialists in image analysis and computer vision find the text easy at the computer vision side and accessible on the mathematical level. The prerequisites are elementary calculus and probability from the first two undergraduate years of any science course. All slightly more advanced notions in probability (inequalities, stochastic geometry, large deviations, etc.) will be either proved in the text or detailed in several exercises at the end of each chapter. We have always asked the students to do all exercises and they usually succeed regardless of what their science background is. The mathematics students do not find the mathematics difficult and easily learn through the text itself what is needed in vision psychology and the practice of computer vision. The text aims at being self-contained in all three aspects: mathematics, vision, and algorithms. We will in particular explain what a digital image is and how the elementary structures can be computed.

We wish to emphasize why we are publishing these notes in a mathematics collection. The main question treated in this course is the visual perception of geometric structure. We hope this is a theme of interest for all mathematicians and all the more if visual perception can receive –up to a certain limit we cannot yet fix– a fully mathematical treatment. In these lectures, we rely on only four formal principles, each one taken from perception theory, but receiving here a simple mathematical definition. These mathematically elementary principles are the *Shannon-Nyquist principle*, the *contrast invariance* principle, *the isotropy principle* and the *Helmholtz principle*. The first three principles are classical and easily understood. We will just state them along with their straightforward consequences. Thus, the text is mainly dedicated to one principle, the Helmholtz principle. Informally, it states that *there is no perception in white noise*. A white noise image is an image whose samples are identically distributed independent random variables. The view of a white sheet of paper in daylight gives a fair idea of what white noise is. The whole work will be to draw from this impossibility of seing something on a white sheet a series of mathematical techniques and algorithms analyzing digital images and "seeing" the geometric structures they contain.

Most experiments are performed on digital every-day photographs, as they present a variety of geometric structures that exceeds by far any mathematical modeling and are therefore apt for checking any generic image analysis algorithm. A warning to mathematicians: It would be fallacious to deduce from the above lines that we are proposing a definition of geometric structure for all real functions. Such a definition would include all geometries invented by mathematicians. Now, the mathematician's real functions are, from the physical or perceptual viewpoint, impossible objects with infinite resolution and that therefore have infinite details and structures on all scales. Digital *signals*, or *images*, are surely functions, but with the essential limitation of having a finite resolution permitting a finite sampling (they are band-limited, by the Shannon-Nyquist principle). Thus, in order to deal with digital images, a mathematician has to abandon the infinite resolution paradise and step into a finite world where geometric structures must all the same be found and proven. They can even be found with an almost infinite degree of certainty; how sure we are of them is precisely what this book is about.

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