Preface

This research monograph develops the Hamilton-Jacobi-Bellman (HJB) theory via the dynamic programming principle for a class of optimal control problems for stochastic hereditary differential equations (SHDEs) driven by a standard Brownian motion and with a bounded or an unbounded but fading memory. These equations represent a class of infinite-dimensional stochastic systems that become increasingly important and have wide range of applications in physics, chemistry, biology, engineering, and economics/finance. The wide applicability of these systems is due to the fact that the reaction of real-world systems to exogenous effects/signals is never "instantaneous" and it needs some time, time that can be translated into a mathematical language by some delay terms. Therefore, to describe these delayed effects, the drift and diffusion coefficients of these stochastic equations depend not only on the current state but also explicitly on the past history of the state variable.

The theory developed herein extends the finite-dimensional HJB theory of controlled diffusion processes to its infinite-dimensional counterpart for controlled SHDEs in which a certain infinite-dimensional Banach space or Hilbert space is critically involved in order to account for the bounded or unbounded memory. Another type of infinite-dimensional HJB theory that is not treated in this monograph but arises from real-world application problems can often be modeled by controlled stochastic partial differential equations. Although they are both infinite dimensional in nature and are both in the infancy of their developments, the SHDE exhibits many characteristics that are not in common with stochastic partial differential equations. Consequently, the HJB theory for controlled SHDEs is parallel to and cannot be treated as a subset of the theory developed for controlled stochastic partial differential equations. Therefore, the effort for writing this monograph is well warranted.

The stochastic control problems treated herein include discounted optimal classical control and optimal stopping for SHDEs with a bounded memory over a finite time horizon. Applications of the dynamic programming principles developed specifically for control of stochastic hereditary equations yield an infinite-dimensional Hamilton-Jacobi-Bellman equation (HJBE) for finite time horizons discounted optimal classical control problem, a HJB variational inequality (HJBVI) for optimal stopping problems, and a HJB quasi-variational inequality (HJBQVI) for combined optimal classical-impulse control problems. As an application to its theoretical developments, characterizations of pricing functions in terms of the infinite-dimensional Black-Scholes equation and an infinite-dimensional HJBVI, are derived for European and American option pricing problems in a financial market that consists of a riskless bank account and a stock whose price dynamics depends explicitly on the past historical prices instead of just the current price alone. To further illustrate the roles that the theory of stochastic control of hereditary differential systems played in real-world applications, a chapter is devoted to the development of theory of combined optimal classical-impulse control that is specifically applicable to an infinite time horizon discounted optimal investment-consumption problem in which capital gains taxes and fixed plus proportional transaction costs are taken into consideration. To address some computational issues, a chapter is devoted to Markov chain approximations and finite difference approximations of the viscosity solution of infinite-dimensional HJBEs and HJBVIs. It is well known that the value functions for most of optimal control problems, deterministic or stochastic, are not smooth enough to be a classical solution of HJBEs, HJBVIs, or HJBQVIs. Therefore, the theme of this monograph is centered around development of the value function as the unique viscosity solution of these equations or inequalities.

This monograph can be used as an introduction and/or a research reference for researchers and advanced graduate students who have special interest in theory and applications of optimal control of SHDEs. The monograph is intended to be as much self-contained as possible. Some knowledge in measure theory, real analysis, and functional analysis will be helpful. However, no background material is assumed beyond knowledge of the basic theory of Itô integration and stochastic (ordinary) differential equations driven by a standard Brownian motion. Although the theory developed in this monograph can be extended with additional efforts to hereditary differential equations driven by semi martingales such as Levy processes, we restrain our treatments to systems driven by Brownian motion only for the sake of clarity in theory developments.

This monograph is largely based on the current account of relevant research results contributed by many researchers on controlled SHDEs and on some research done by the author during his tenure as a faculty member at the University of Alabama in Huntsville and more recently as a member of the scientific staff at the U.S. Army Research Office. Most of the material in this monograph is the product of some recently published or not-yet-published results obtained by the author and his collaborators, Roger Youree, Tao Pang, and Moustapha Pemy. The list of references is certainly not exhaustive and is likely to have omitted works done by other researchers. The author apologizes for any inadvertent omissions in this monograph of their works.

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