

Preface

The first S.E.R.C. (Science and Engineering Research Council of the Department of Science and Technology, Government of India, New Delhi) School on Special Functions was sponsored by DST (Department of Science and Technology), New Delhi, and conducted by CMS (Centre for Mathematical Sciences) for six weeks in 1995. The second S.E.R.C. School on Special Functions and Functions of Matrix Argument was sponsored by DST, Delhi, and conducted by CMS for five weeks during the period 29th May to 30th June 2000. In the second School, the main lectures were given by Dr. H.L. Manocha of Delhi, India, Dr. S. Bhargava of Mysore, India, Dr. K. Srinivasa Rao and Dr. R. Jagannathan of Chennai, India and Dr. A.M. Mathai of Montreal, Canada. Supplementary lectures were given and problem sessions were supervised by Dr. K.S.S. Nambooripad and Dr. S.R. Chettiyar of CMS, Dr. R.N. Pillai (retired), Dr. T.S.K. Moothathu and Dr. Yageen Thomas of the University of Kerala and Dr. E. Krishnan of the University College Trivandrum. Lecture Notes were brought out as Publication No.31 of CMS soon after the School was completed. The first two Schools were conducted in Trivandrum (Thiruvananthapuram) area. Dr. A.M. Mathai was the Director and Dr. K.S.S. Nambooripad was the Co-Director of these two Schools.

The third S.E.R.C. School on Special Functions and Functions of Matrix Argument: Recent Developments and Recent Applications in Statistics and Astrophysics, sponsored by DST, Delhi, was conducted for five weeks from 14th March to 15th April 2005 by CMS at its Pala Campus, Kerala, India. This time DST, wanted the lecture notes to be collected from the main lecturers in advance, compiled and distributed prior to the start of the School. The main lectures for the 3rd School were given by Dr. Hans J. Haubold of the Office of Outer Space Affairs of the United Nations, Dr. Serge B. Provost, Professor of Actuarial Sciences and Statistics of the University of Western Ontario, Canada, Dr. R.K. Saxena of Jodhpur, India, Dr. S. Bhargava of Mysore, India and Dr. A.M. Mathai of Montreal, Canada. Supplementary lectures were given by Dr. A. Sukumaran Nair (Chairman, CMS), Dr. K.K. Jose (Director-in-Charge, CMS Pala Campus), Dr. R.N. Pillai, Dr. Yageen Thomas, Dr. V. Seetha Lekshmi, Dr. Alice Thomas, Dr. E. Sandhya, Dr. S. Satheesh and

Dr. K. Jayakumar. Problem sessions were supervised by Dr. Sebastian George and other lecturers. Extra training in the use of statistical packages, LATEX and Mathematica/Maple were given by Joy Jacob, Seemon Thomas and K.M. Kurian of the Department of Statistics of St. Thomas College, Pala. A special lecture sequence on Matlab was conducted by Alexander Haubold of Columbia University, U.S.A.

The 4th S.E.R.C. School on Special Functions and Functions of Matrix Argument: Recent Advances and Applications to Stochastic Processes, Statistics and Astrophysics, sponsored by DST, Delhi, was conducted by CMS for five weeks from 6th March to 7th April 2006 at its Pala Campus. The main lectures were scheduled to be given by Dr. Hans Joachim Haubold of the Office of Outer Space Affairs of the United Nations, Dr. P.N. Rathie of the University of Brasilia, Brazil, Dr. S. Bhargava of Mysore, India, Dr. R.K. Saxena of Jodhpur, India, Dr. K.K. Jose of Pala, India and Dr. A.M. Mathai of Montreal, Canada. Supplementary lectures were given by Dr. A. Sukumaran Nair (Chairman, CMS), Dr. K.S.S. Nambooripad (Director-in-Charge, CMS Trivandrum Campus), Dr. R. Y Denis of Gorakhpur, India, Dr. N. Unnikrishnan Nair (former Vice-Chancellor of Cochin University of Science and Technology, Kerala, India), Dr. Yageen Thomas and Dr. K. Jayakumar. One week was devoted to Stochastic Processes and recent advances in this area in the 4th School. Problem sessions were supervised by Dr. Joy Jacob, Dr. Sebastian George and other lecturers.

The 5th S.E.R.C. School in this sequence, titled “Special Functions and Functions of Matrix Argument: Recent Advances and Applications in Stochastic Processes, Statistics, Wavelet Analysis and Astrophysics” was conducted by CMS at its Pala Campus from 23rd April to 25th May 2007. The lecture notes for the 5th School were assembled by November 2006 and were printed and distributed as Publication No.34 in the Publications Series of CMS. The main lectures were scheduled to be given by Dr. Hans J. Haubold (Office of Outer Space Affairs of the United Nations, Austria), Dr. H.M. Srivastava (University of Victoria, Canada), Dr. R.Y. Denis (Gorakhpur University, India), Dr. R.K. Saxena (Jodhpur University, India), Dr. S. Bhargava (Mysore University, India), Dr. D.V. Pai (IIT Bombay, India), Dr. Yageen Thomas (University of Kerala, India), Dr. K.K. Jose (Mahatma Gandhi University, India), Dr. J.J. Xu (China/Canada), Dr. K. Jayakumar (Calicut University, India) and Dr. A.M. Mathai (Canada/India). But Dr. H.M. Srivastava, Dr. S. Bhargava and Dr. Xu could not reach the venue on time due to unexpected emergencies. Supplementary lectures were given by Dr. A. Sukumaran Nair (Chairman, CMS), Dr. R.N. Pillai (former Head, Department of Statistics, University of Kerala, India), Dr. N. Mukunda (IISc, Bangalore, India) and the problem sessions were supervised by Dr. Sebastian George, Dr. Joy Jacob, Dr. Seemon Thomas and the lecturers. For the 2005, 2006 and 2007 Schools, Dr. A.M. Mathai was the Director and Dr. K.K. Jose was the Co-Director of the Schools.

The participants for the S.E.R.C. Schools are selected on all-India basis. All the expenses of the selected candidates, total number of seats is 30, including travel, accommodation, food, lecture materials, local travels etc are met by the DST, Delhi, Government of India. Foreign participation is allowed under the conditions that the

participants must come with their own return air tickets and must attend all the lectures, and problem sessions and take all the examinations and tests from the beginning till the end. All their local expenses are met and lecture materials are provided by the Schools. There is no fee for attending the Schools. The Schools are mainly research orientation courses aimed at young faculty members in colleges and universities across India, below 35 years of age, and fresh graduates with M.Sc, M.Phil, Ph.D degrees below 30 years of age, in Mathematics/Statistics/Theoretical Physics. In most of the Indian universities rigid compartmentalization is the order and as a result a M.Sc graduate in mathematics may not have even taken a very basic course in probability and statistics. Even though basic differential and integral calculus and matrix theory are required subjects for statistics and physics students, these students may have forgotten these subjects because they teach the compartmentalized topics in their own areas and do not usually do research even in their narrow areas of their own fields. For maintaining their jobs and getting regular increments in their salaries and all other monetary and other benefits, research work and further reading and learning process are not required of them. As a result, the quality of teaching and the information passed on to the students go down from year to year. In order to remedy this situation a little bit, S.E.R.C. Schools in various areas were started by DST, Delhi. Dr. A.M. Mathai was asked to run S.E.R.C. Schools in mathematics. For taking up a challenging research problem in any applied area a good background in basic mathematics, probability and statistics is required. In order to give the basic ideas in probability and statistics and to bring a number of topics in the area of Special Functions and Functions of Matrix Argument and their applications to the current research level, these S.E.R.C. Schools on Special Functions were established. The current sequence of five Schools really achieved the aim and almost all the participants in the first four Schools have become research oriented towards a career in research and teaching, and the participants of the 5th School are also expected to follow the same footsteps of their predecessors.

Chapter 1 introduces elementary classical special functions. Gamma, beta, psi, zeta functions, hypergeometric functions and the associated special functions, generalizations to Meijer's G and Fox's H-functions are also examined here. Discussion is confined to basic properties and some applications. Introduction to statistical distribution theory is given here. Some recent extensions of Dirichlet integrals and Dirichlet densities are also given. A glimpse into multivariable special functions such as Appell's functions and Lauricella functions is also given. Special functions as solutions of differential equations are also examined here.

Chapter 2 is devoted to fractional calculus. Fractional integrals and fractional derivatives of various kinds are discussed here. Then their applications to reaction-diffusion problems in physics, input-output analysis and Mittag-Leffler stochastic processes are examined here. Chapter 3 deals with q-hypergeometric or basic hypergeometric functions and Chapter 4 goes into basic hypergeometric functions and Ramanujan's work on elliptic and theta functions. Chapter 5 examines the topic of Special Functions and Lie Groups.

Chapters 6 to 9 are devoted to applications of Special Functions to various areas. Applications to stochastic processes, geometric infinite divisibility of random variables, Mittag-Leffler processes, α -Laplace processes, density estimation, order statistics and various astrophysics problems, are dealt with in these chapters.

Chapter 10 is devoted to Wavelet Analysis. An introduction to wavelet analysis is given here. Chapter 11 deals with the Jacobians of matrix transformations. Various types of matrix transformations and the associated Jacobians are given here. Chapter 12 is devoted to the discussion of functions of matrix argument. Only the real case is considered here. Functions of matrix argument and the pathway models, recently introduced by Mathai (2005), are also discussed here, along with their applications to various areas.

In all the S.E.R.C. Schools conducted under the Directorship of Dr. A. M. Mathai a serious effort is made so that the participants absorb the materials covered in the Schools. The classes started at 8.30 am and went until 6.00 pm. The first lecture of 08.30 to 10.30 was followed by a problem session from 10.30 to 13.00 hrs on the materials covered in the first lecture. The second lecture of the day was from 14.00 to 16.00 hrs followed by problem session from 16.00 to 18.00 hrs. At the end of every week a written examination was conducted, followed by a personal interview of each participant by the lecturer of that week in the form of an oral examination. Cumulative grades of such weekly examinations appeared in the final certificates distributed to them. The main aim was to inculcate in them a habit of long and sustained hard work, which would help them in their careers whatever they may be. During the first week the participants, especially the teachers from colleges and universities, found it difficult to adjust to the routine of long hours of hard work but starting from the second week, in all the Schools, the participants started enjoying, especially the problem sessions, because for the first time, they started understanding and appreciating the meanings and significance of theorems that they learnt or memorized when they were students.

The lecture notes are written up in a style for self-study. Each topic is developed from first principles with lots of worked examples and exercises. Hence the material in this book can be used for self-study and will help anyone to understand the basic ideas in the area of Special Functions and Functions of Matrix Argument and they will be able to make use of these results in their own problems in applied areas, especially in Statistics, Physics and Engineering problems. Applications in various areas are illustrated in this book. Insights into recent developments in the applications of fractional calculus, in the developments in various other topics are also given in the book so that the readers who are interested in any of the topics discussed in the book can directly go into a research problem in the topics.

Several people have contributed enormously for the success of the S.E.R.C. Schools and in making the publications of four Lecture Notes and this final publication of summarized lecture notes possible. Dr. B.D. Acharya, Advisor to Government of India and Dr. Ashok K. Singh of the Mathematical Sciences Division of DST, New Delhi, are the driving force behind the re-energized mathematical activities in India

now. They were kind enough to pursue the matter and get the funds released for running the Schools as well as for the preparation of various publications, including this one. Since the basic materials for this publication are supplied by various lecturers in the form of their lecture notes, there will be some overlaps. Very obvious inconsistencies are removed but some overlapping materials are left there to make the discussions self-contained. Dr. R.K. Saxena, Dr. S. Bhargava, Dr. H.J. Haubold, Dr. P.N. Rathie, Dr. K.K. Jose, Dr. K. Jayakumar, Dr. H.L. Manocha, Dr. R. Jagannathan, Dr. K. Srinivasa Rao, Dr. K.S.S. Nambooripad, Dr. Serge B. Provost, Dr. Yageen Thomas, Dr. D.V. Pai, and Dr. A.M. Mathai are thanked for making their notes available in advance for the S.E.R.C. Schools. Most of the material was typeset at CMS office by Miss K.H. Soby, Dr. Joy Jacob, Seemon Thomas, Dr. Sebastian George, Dr. K.K. Jose and Dr. A.M. Mathai. Part of the material was typeset by Barbara Haubold of the United Nations, Vienna Office, fully free of charge as a voluntary service to the Schools. Notes and programs for a series of lectures on Matlab were supplied to CMS by Alexander Haubold of Columbia University, USA. Those notes are not included in this book to keep the materials within the focus of the book. Dr. Sebastian George, Dr. Joy Jacob, Dr. Seemon Thomas, K.M. Kurian, Jaisymol Thomas and Ashly P. Jose, who spent a lot of time in running problem sessions, in running separate sessions on the use of statistical packages and Maple program, use of LATEX etc and for checking the typed materials, are thanked. CMS would like to thank each and every one who helped to make S.E.R.C. Schools on Special Functions a grand success and who helped to make this publication possible.

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