2

Newton's Laws of Motion for a Particle Moving in One Dimension

Living cells exchange energy and matter with their surroundings. They reproduce. Often they move about. To understand such basic aspects of life, it is essential to understand how motion is related to force and how force is related to energy. Explaining these relations for an object moving in one dimension is the goal of this and the next two chapters.

Before beginning to read and master the formal discussion of motion that follows in this chapter, however, it is very useful to remind ourselves what it feels like to move at constant velocity and to accelerate. Recall how it feels to ride in a car along a straight flat highway that has recently been resurfaced. If the car's speedometer is fixed at a constant reading you can close your eyes and not know you are moving at all, no matter how fast the speedometer says you are moving. Of course, roads aren't straight and flat for very long stretches. You feel clues that you are moving from the little bumps and turns the car makes. Riding in an elevator is probably a better example. Once the elevator gets going, only the flashing floor numbers give any hint that anything is happening, no matter how fast the elevator is traveling or whether you are going up or down. In both car and elevator examples, when you feel as if you are at rest you are moving in a straight line at a constant rate. This kind of motion is called *constant velocity*. Constant velocity feels exactly like standing still.

When the car turns or goes over a bump or speeds up or slows down, or when the elevator starts or stops, you definitely feel it. All such instances involve change in velocity. Change in velocity is called *acceleration* and acceleration can be felt. If a trinket dangles by a thread from the car's rear view mirror you can see it deflect from hanging vertically at the same instant you feel acceleration. If by some bizarre chance, you are standing on a scale as the elevator starts or stops, the scale's reading will change when you feel the acceleration.

Why you feel acceleration but not constant velocity, why acceleration causes the trinket to deflect and the scale reading to change, all require an explanation. That explanation is contained in Newton's laws of motion, discussed in this chapter. In order to understand the content of Newton's laws, we have to be able to describe motion with quantitative precision. The major goal of this chapter is to demonstrate how a body's interactions with its surroundings can explain changes in its motion. We use the term force to denote a quantitative measure of interaction. The theme of this chapter, then, is that force explains (causes) acceleration. As discussed previously, any macroscopic body is a collection of smaller, more fundamental pieces. A complete understanding of the changes in motion of a macroscopic body requires keeping track of the forces experienced by every subpiece of the body due to every other subpiece (these are called internal forces) and due to every other additional body (external forces). In this chapter, all bodies are treated as particles and all changes in motion arise from external interactions. This simplistic view allows us to develop powerful tools that can subsequently be applied to more general and more realistic behaviors. The chapter ends with a short discussion of diffusion, the random thermal motion of

J. Newman, *Physics of the Life Sciences*, DOI: 10.1007/978-0-387-77259-2_2,

small particles, to contrast this type of motion with that described in the rest of the chapter. Diffusion is an extremely important process in biology, playing a major role in our existence through, for example, gas, nutrient, and waste exchange in the blood.

1. POSITION, VELOCITY, AND ACCELERATION IN ONE DIMENSION

Until we get to Chapter 23, we are interested primarily in phenomena associated with objects that can be seen (perhaps with the aid of a microscope or telescope) with ordinary light. That doesn't narrow our interests very much. On the small end, we can certainly see inside living cells; on the large end we can see clusters of galaxies. All objects that can be seen with light are *composite*, that is, composed of smaller pieces of matter. Organisms, for example, are composed of cells; cells are composed of molecules; molecules are composed of atoms; atoms are composed of nuclei and electrons. As we show in Chapter 6, we can assign to any object a unique point called the object's *center of mass*. The motion of any object can then be thought of as consisting of two parts: motion of the center of mass and motion about the center of mass. For now, just think of the center of mass as the body's "center."

If a body moves so that all of its composite pieces do exactly what the center of mass does—for example, when the center of mass of the body moves 1 m north each composite piece also moves 1 m north—the body is said to undergo a *rigid translation* (see Figure 2.1). An object undergoing a rigid translation can be treated as a *point particle*, a mass without spatial size. Its shape and extent in space are irrelevant.

To start, imagine some object of interest moving along a straight line. The object can be microscopic (such as a protein molecule or a bacterium) or macroscopic (such as a car or even you, yourself). Motion along a line is called *one-dimensional* because only one coordinate, x, say, is needed to describe it. Here, then, x designates the location of the center of mass of a car measured from an arbitrary origin. There are two directions to go along the coordinate axis from its origin. We distinguish between them by saying one is the "positive" direction, the other the "negative." Thus, x is a signed number having units of length.

Whether it is the motion of our car or the motion of a molecule, in practice we measure one position at one time, then another position at another time, and so on, over and over. That is, in any experiment the data we collect are a sample of the motion acquired at discrete instants. This is true irrespective of what apparatus or technique we employ. For example, we (or a policeman) might use radar or sonar to identify where our car is at various moments. Such devices send out a signal and receive its echo, then another signal and its echo, on and on. Between signals we know nothing; there are gaps in the data. The same is true if we videotape a moving object. Video is really a succession of still frames (in the United States, one every thirteenth of a second). We can get detailed information about the object every frame, but nothing in between. The results, consequently, comprise a table of positions (measured with finite precision and limited accuracy) recorded at discrete sampling times. In other words, our experiment yields a finite set of position values $\{x(t_1), x(t_2), x(t_3), \ldots\}$ where $x(t_1)$ is the position measured at time t_1, $x(t_2)$ is the position at time t_2, and so on. Although we believe that our car or a bacterium moves continuously in time (i.e., the closer t_1 and t_2 are to each other, the closer $x(t_1)$ and $x(t_2)$ are to each other), the best we can do, even if (as is frequently the case) we are aided by a high-speed computer with lots of memory, is obtain a broken and punctuated approximation to its theoretical, continuously flowing motion.

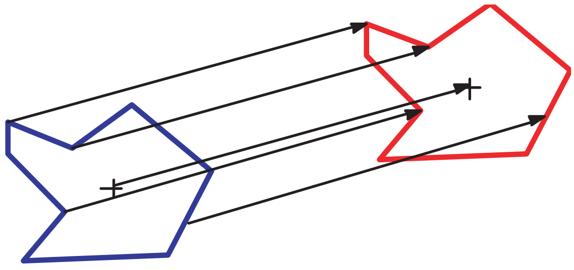


FIGURE 2.1 *An object undergoing rigid translation. All parts do what the center of mass (+) does.*

In this book we use the International Standard (SI) units in which lengths are measured in meters (m), although often we refer to small fractions of meters (e.g., cm, mm, µm, and so on) or large multiples of meters (in particular, km); see Table 2.1.

Table 2.1 *Commonly Used Units of Distance*

Name	*Abbreviation*	*Multiple of a Meter*	*Roughly Comparable to*
Meter	m	1	Length of your arm
Centimeter	cm	10^{-2}	Length of a (new) pencil eraser
Millimeter	mm	10^{-3}	Width of a pencil point
Micrometer	μm	10^{-6}	Length of a cell
Nanometer	nm	10^{-9}	Diameter of a small molecule
Kilometer	km	10^{+3}	Half a mile

A table of numbers is not usually a very useful way to characterize motion. Table 2.2 provides an example. In this table, we see the results of three different observers recording the motion of the same remote control toy model car (Figure 2.2), using the same coordinate system and the same starting time (i.e., the instant they all call $t = 0$ s), but with three different sampling rates (one every 2 s coded in blue, one every 1 s in green, and one every 0.5 s in red, respectively). (The *second*, incidentally, is the SI unit of time, often abbreviated as just s.) There is typically too much to keep track of in a table; it's hard, with tabular information, to see a "big picture."

Table 2.2 *Table of Observations on the Position of a Remote Control Toy Car as a Function of Time*

Observer #1		*Observer #2*		*Observer #3*	
Time (s)	*Position (m)*	*Time (s)*	*Position (m)*	*Time (s)*	*Position (m)*
0	0.890	0	0.890	0	0.890
				0.5	0.663
		1	0.567	1	0.567
				1.5	0.968
2	0.909	2	0.909	2	0.909
				2.5	0.633
		3	0.535	3	0.535
				3.5	1.008
4	0.700	4	0.700	4	0.700

More useful than a table is to make a plot of the data, plotting position $x(t)$ versus time t with t (the independent variable) plotted on the horizontal axis and $x(t)$ (the dependent variable) plotted on the vertical axis, as in Figure 2.3 using the same color codes for the different observers.

In the figure, we have attempted to fill in missing information by interpolating between data points (in this case, by simply "connecting the dots" with straight lines). Interpolation of Observer #1's data (in blue) gives a very crude picture of the car's motion over the interval 0 s to 4 s. Observer #2's data (in green) provides more detail and #3's (in red) even more. By interpolating, we are creating a *model* of the car's motion that will allow us to say something about where the car was at times not observed.

The word "model" is used a lot in physics. A model is a representation or an approximation of a thing, not the thing itself. Some models are better than others: for example, the blue model of the car's motion shown in Figure 2.3 is not as informative or accurate as the red model. The former model has less of a "database" to support it than does the latter. The blue model

FIGURE 2.2 *A remote controlled car whose motion we study.*

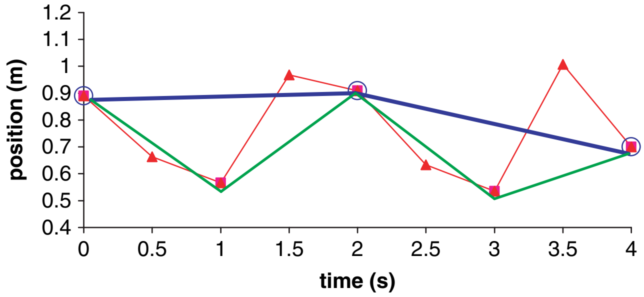


FIGURE 2.3 *The position data of Table 2.2 plotted for each observer.*

can be thought of as "provisional," a kind of first approximation. As we acquire more and more data that model is replaced by more and more sophisticated approximations.

We can imagine that if the observed sampling rate is increased so that data are taken more and more frequently, the resulting plots would more and more define a smoothly continuous curve of some sort. In fact, if we are lucky we might even be able to fit an analytic expression to the data, producing an equation model for the car's *instantaneous position*, $x(t)$, that is, an explicit relationship between position and time that would allow us to determine the car's position at any instant (not just at the times of measurement). Such analytic models are especially useful because they allow us to make predictions about events not yet witnessed.

Given a position record such as that shown in Table 2.2, or, equivalently, in Figure 2.3, we can define a number of useful quantitative tools. First, we have the notion of *distance traveled* in some time interval. The total distance traveled in any interval of time is the sum of the distances traveled during each subinterval of the motion. Furthermore, each contribution is positive, irrespective of in which direction the motion takes place. Formally, distance equals the absolute value of change in position. Thus, according to Observer #1 in Table 2.2, the total distance covered by the car in 4 s is 0.228 m, that is, from a position of +0.890 m out to +0.909 m (a distance of 0.019 m), then back to +0.700 m (an additional distance of 0.209 m). According to #2, the total distance the car travels is 1.204 m, and according to #3 the total distance is 1.938 m. Make sure you understand why.

The *average speed* over a certain time interval is the total distance traveled in that interval divided by the elapsed time. So for the three observers of Table 2.2, #1 assigns to the car's motion an average speed of 0.228 m/s = 0.057 m/s, #2, 0.301 m/s, and #3, 0.485 m/s. (Note that in calculations units are treated as algebraic quantities.)

Next, we introduce the notion of the *displacement*, Δx, in a time interval t_i to t_f ("i" implies "initial", the beginning of the interval, and "f" "final", the end of the interval). (Here, and more generally, the Greek letter Δ [capital "delta"] denotes a difference between two values.) Displacement is the *directed distance*

$$\Delta x = x(t_f) - x(t_i).$$

Displacement can be positive, negative, or zero (as opposed to distance, which is never negative), with the sign indicating the net direction of the associated motion. Thus, in the example of Table 2.2, all three observers agree that the displacement of the car, Δx, for $t_i = 0$ s to $t_f = 2$ s is +0.019 m (displacement in the + direction during this interval), for $t_i = 2$ s to $t_f = 4$ s is −0.209 m (displacement in the − direction during this interval), and for the entire interval from $t_i = 0$ s to $t_f = 4$ s is −0.190 m.

The *average velocity* $\bar{v}$ of our car is defined for a specific interval of time, $\Delta t = t_f - t_i$, as

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2.1}$$

Notice that this expression is different from the average speed, because it is not the distance traveled but the displacement that is in the numerator. Unlike the average speed, which is always positive, the average velocity can be positive, negative, or zero depending on whether Δx is positive (moving to the right), negative (moving to the left), or zero (either there was no motion or the object has returned to its starting point). Again, all three observers in Table 2.2 agree that the car's average velocity is +0.010 m/s from $t_i = 0$ s to $t_f = 2$ s, −0.105 m/s from $t_i = 2$ s to $t_f = 4$ s, and −0.048 m/s from $t_i = 0$ s to $t_f = 4$ s. (Contrast these results with their conclusions about average speeds over the same interval.)

Average velocity is a statement about the tendency for an object to move over a finite time interval. In between the starting time and the ending time, the object can do lots of interesting things that are not accounted for by the average velocity. Of course, as we increase our sampling rate and make our time interval smaller and smaller, less and less departure from the average motion will occur in an interval of time. This leads us to a still different (more refined) concept, namely, that of *instantaneous velocity*. Imagine starting at some generic time $t_i = t$ with our car at $x(t)$ and going to $x(t + \Delta t)$ at $t_f = t + \Delta t$, some time later. The instantaneous velocity of the car at time t, $v(t)$, is defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \tag{2.2}$$

The symbol "$\lim_{\Delta t \to 0}$" is read, "in the limit as Δt approaches 0." Operationally, it means "make the sampling rate so fast that the average motion and the exact motion in the time interval Δt are indistinguishable." You can think of this as the velocity reported by a car's speedometer.

As we said before, we believe that our car moves continuously in time. Continuous, here, means that we can make a plot of position versus time without ever lifting our pencil off our paper. There are no holes or jumps in such a plot. In other words, we don't believe that our car (no matter how spiffy) is ever at $x(t)$ one instant then at a very different $x(t + \Delta t)$ an extremely short time later. Thus, despite the fact that we are making Δt exceedingly small in the denominator of Equation (2.2)—and therefore seemingly threatening to make $\Delta x/\Delta t$ exceedingly large—Δx in the numerator is also getting smaller and smaller, and the ratio of the two remains nice and finite.

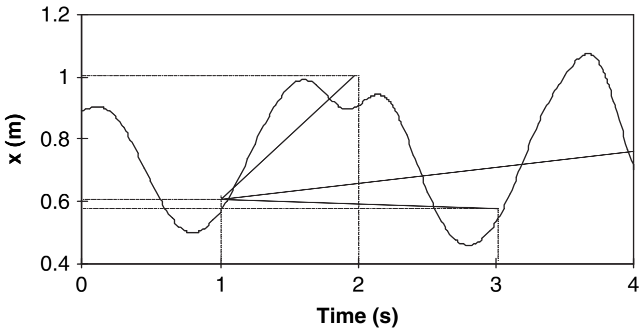
Moreover, we also tacitly believe that the car's motion is *smoothly continuous*. "Smooth" means that there are no instantaneous "jerks." If the car has a nice, finite velocity $v(t)$ at time t, its velocity $v(t + \Delta t)$ is not much different a short time Δt later. As we argue in just a bit, smoothly continuous means a plot with neither holes nor sharp points (cusps).

Well, the formal definition of a velocity at an instant may be clear, but how do we actually use the definition? How, for example, do we assign a number to it? The answers to these questions depend on what information you have at the start. First, suppose another observer has taken a great deal more of the car's position data and fit a smooth curve to the data points. This smooth curve is presented to you as an accurate model of the car's motion at any time. Such a plot is shown in Figure 2.4a.

Let's try to determine, from the curve given to us, the car's instantaneous velocity at $t = 1$ s. The position at 1 s is $+0.567$ m. We take a second time, $t + \Delta t = 4$ s, say, and the corresponding position (read from Figure 2.3 or 2.4a or looked up in Table 2.2) is $+0.700$ m. We conclude that the average velocity over that interval is

$$\bar{v} = \frac{[+0.700 \text{ m}] - [+0.567 \text{ m}]}{4 \text{ s} - 1 \text{ s}} = +0.044 \text{ m/s}.$$

FIGURE 2.4a *Smooth curve of the position versus time for the car.*



Note that this average velocity is the same as the slope of the line connecting the points (1 s, $+0.567$ m) and (4 s, $+0.700$ m) on the graph in Figure 2.4a (because slope is calculated by dividing rise [or fall] in the vertical direction by the corresponding run in the horizontal direction, and, in this case, that is $\Delta x/\Delta t$).

Now, let's take $t + \Delta t$ to be 3 s. Given that $x(3 \text{ s})$ is $+0.535$ m, we calculate the average velocity in this interval to be -0.017 m/s. Then, take $t + \Delta t = 2$ s. The average velocity from 1 s to 2 s is $+0.342$ m/s. Every interval we've

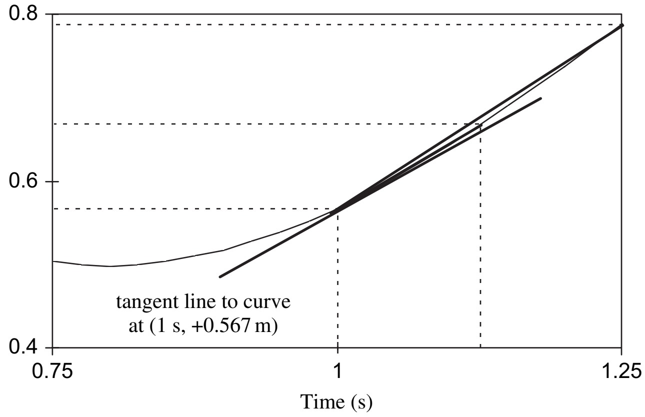


FIGURE 2.4b *Zoom-in around t = 1 s data from Figure 2.4a.*

picked so far has yielded quite a different average velocity. None of these can be said to be the instantaneous velocity at $t = 1$ s, because the Δts aren't very small in any of these examples. Now switch your attention to Figure 2.4b. Here the piece of the plot between $t = 0.75$ s and $t = 1.25$ s is magnified. If we take $t + \Delta t = 1.25$ s, we obtain for an average velocity about

$$\frac{[+0.79 \text{ m}] - [+0.567 \text{ m}]}{1.25 \text{ s} - 1 \text{ s}} = +0.89 \text{ m/s}.$$

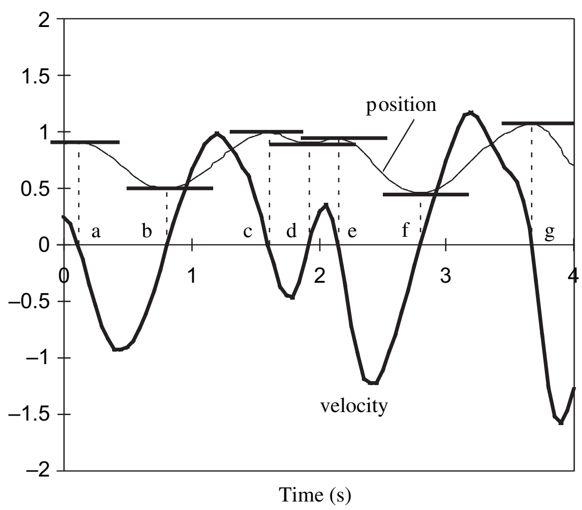
Finally, we take $t + \Delta t = 1.13$. The average velocity in this interval is $+0.82$. These last values are beginning to get closer. We're beginning to hone in on the desired velocity.

We see that the bold line connecting the point (1 s, $+0.567$ m) to the point (1.13 s, $+0.67$ m) is difficult to distinguish from the curve passing through (1 s, $+0.567$ m). If we magnify a piece of a smooth curve enough at any of its points, the curve looks progressively like a little straight line segment at that point. That line segment is called the tangent line to the curve at the point. So, in other words, the smaller and smaller we choose Δt, the closer and closer the line connecting (1 s, $+0.567$ m) to (1 s $+\Delta t$, x(1 s + Δt)) is to being the tangent line to the position versus time curve at the point of interest (i.e., [1 s, $+0.567$ m]). And, the *instantaneous velocity is the slope of the tangent line* at that point (about $+0.66$ m/s for our example).

Given a smoothly continuous position versus time graph (such as Figure 2.4a) we can make a graph of how velocity varies with time by estimating the slope of the tangent line to the curve at successive times and plotting the resulting values. We do this at some selected times and then connect our best estimates in order to obtain a smooth curve for a velocity versus time graph. In principle, one can imagine an automatic calculator that could move along the curve in Figure 2.4a continuously finding the tangent, computing its slope, and then plotting these values as we have done in Figure 2.5.

In Figure 2.5, several tangent lines to the position versus time curve (the lighter curve) are displayed. All have zero slope and the velocity graph at those corresponding times shows zero velocity. The associated instants in time correspond to "turning points," instants where the car changes direction. Between turning points the car moves continuously in one direction. Thus, from instant a to instant b the car moves toward the origin, and from instant b to instant c, the car moves away from the origin. While moving away from the origin (to more positive x-coordinates), the car's velocity is positive (the slope of the tangent line to the position versus time curve at any instant in this interval is positive) and while moving toward the origin (to less positive x-coordinates), the car's velocity is negative. Note that at the moments the car changes direction, its velocity is instantaneously equal to zero; that is, the car is instantaneously at rest.

FIGURE 2.5 *Velocity of the car obtained from its position versus time curve.*



If we had an equation for the curve in Figure 2.4a, that is, an explicit relation between x and t, we could utilize Equation (2.2) to determine an equation for how velocity varies in time. The translation of $x(t)$ into $v(t)$ is the heart of what we call calculus. These days, computers can do this translation for us.

You can see that the velocity of our car portrayed in Figure 2.5 varies in time, much as position does. Because velocity is rate of change of position, it is also useful to define rate of change of velocity. Indeed, as we show in Chapter 3, rate of change of velocity is the centerpiece of Newton's laws of dynamics.

The *average acceleration* is defined, in a similar way to the average velocity, as

$$\bar{a} = \frac{\Delta v}{\Delta t}, \tag{2.3}$$

where $\Delta v = v(t_f) - v(t_i)$. Note that the average acceleration reflects the change of the velocity with time and that in order to calculate the average acceleration from this definition, you must first have a graph of (or equations for) the velocity versus time and then obtain the ratio in Equation (2.3) for the time interval of interest. The average acceleration can be positive, negative, or zero depending on whether v is increasing (Δv is positive), decreasing (Δv is negative), or is the same at the two ends of the time interval of interest (regardless of what occurred during the interval of time). Acceleration is change in velocity per unit time, so its units are velocity units divided by time units: (m/s)/s = m/s^2, for the car example given above.

We define, analogous to instantaneous velocity, the *instantaneous acceleration* (or simply the acceleration) as

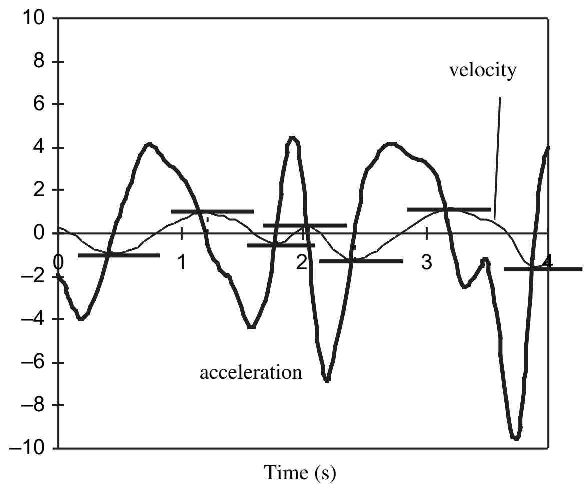


FIGURE 2.6 *Acceleration of the car obtained from the velocity data of Figure 2.5.*

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}. \tag{2.4}$$

Just as velocity at any instant (for motion in one dimension) is the slope of the tangent line to the position versus time curve at that instant, the acceleration at any instant (for motion in one dimension) is the slope of the tangent line to the velocity versus time curve. Thus, if we are given a plot of v versus t, we can approximate a versus t by sketching tangent lines at a number of instants, estimating the respective slopes, plotting those values, then interpolating. Starting with the velocity plot in Figure 2.5, we can then generate an acceleration plot, as in Figure 2.6. We identify several instants at which the acceleration vanishes by noting where the velocity versus time curve has tangent lines with zero slope. Note that the acceleration is not zero when the velocity is zero nor is the velocity zero when the acceleration is zero. The two quantities measure different things and it is important to keep them straight.

Previously, we said that the motion of our car (or any other object) should result in a position versus time graph that is both continuous and smooth, that is, with no holes (discontinuities) or sharp points (kinks). No holes ensures that the position doesn't abruptly change from instant to instant. No kinks ensures that the velocity doesn't abruptly change from instant to instant. The analysis of motion could continue with additional quantities, such as the time-rate-of-change of acceleration, and the time-rate-of-change of that, and so on. Remarkably, such additional quantities are unnecessary for a complete understanding of how objects move about. Newton's laws of motion, the subject of the next section, tell us that acceleration is the most complicated piece of motion analysis apparatus we need.

2. NEWTON'S FIRST LAW OF MOTION

The gist of the preceding section is that there is an intimate mathematical connection among position, velocity, and acceleration. In essence, if we know an object's position over time we can infer what its acceleration must have been; inversely, given its acceleration we can make inferences about its position. Although they are intertwined, mathematics and physics are not the same thing. In this section, we begin to probe the physical rules that underlie the mathematics of motion. Constant velocity can't be felt, but acceleration can be. What you feel when you accelerate is physics. Acceleration is the key that unlocks the secrets of much of the physical universe. *Constant velocity doesn't require an explanation, but acceleration does.*

Perhaps you are puzzled by the last sentence. Everyday experience tells us that to start a body moving we have to give it a push. When we stop pushing, the body comes to rest. In our everyday experience, rest is the natural state of things. In our everyday experience, it is velocity that requires a cause. It took many centuries of human intellectual development before we (that is to say, Galileo, in the seventeenth century) recognized that our common experience is dominated by two phenomena that acting together obscure from us the truth about motion. One of these, gravity, makes things fall down. The other, friction, makes them stop.

It's a pity that Galileo didn't have an "air table" to play with. If he had, he wouldn't have had to work so hard to uncover the truth about motion. An air table has many holes in its top, through which jets of air can be squirted. Maybe you've seen an air table at a game arcade (or, perhaps in an introductory physics laboratory). Often hockeylike games are played on them using pucks that are levitated by the squirting air. When the air is turned off and the puck is pushed, it quickly comes to rest. Gravity makes the puck fall to the table and friction makes it stop. When the table is level and the air is on, the puck hovers in one place. The jets of air effectively cancel gravity out and render friction negligible. On a properly leveled air table once a puck is pushed it travels off at constant speed in a straight line, until it hits a sidewall. Between the initial push and when it hits the wall, no additional push is required to keep the puck going. The natural state of a body's motion is constant velocity (zero velocity, i.e., rest, is a special case). No external influence is required to keep the puck moving, however, an influence from outside is certainly required to change its velocity.

Isaac Newton, in his *Principia Mathematica* (1687), greatly extended Galileo's insight that change in motion requires cause. The first of Newton's laws is a kind of statement of faith. It says that

It is possible to find laboratories ("frames of reference") in which a body's acceleration is solely attributable to interactions between that body and other bodies.

In the laboratories of Newton's first law a body never accelerates spontaneously; every acceleration is caused by an interaction. That a body does not spontaneously accelerate is attributed to a property of all material objects called *inertia*. The frames of reference of Newton's first law are said to be *inertial frames*.

It is usually desirable to observe and describe motion in inertial laboratories, because in them every acceleration is caused by identifiable pushes and pulls and, as we show, the associated quantitative analysis is straightforward. Spontaneous accelerations observed in noninertial frames necessitate inventing fictitious causes for their explanation. For example, suppose you jump off the roof of a building (we are not recommending you do this!). You will notice that in the frame of reference you carry with you all objects—such as the building, people standing on the sidewalk below, and the Earth itself—accelerate towards the sky with exactly the same acceleration. There is no identifiable interaction that causes all of these simultaneous spontaneous accelerations. To explain them requires assigning a fictitious cause. You're carrying a poor frame of reference for doing physics, a fact that will be painfully apparent when the upward accelerating ground reaches you. People standing on the sidewalk will offer a simpler picture of what is occurring. They will say that it is you who is accelerating, and that there is an easily identifiable cause: the pull of gravity of the Earth. This situation is general: any frame of reference in which accelerations occur without cause must itself be accelerating.

There is another, perhaps more common, way to state Newton's first law, given our understanding of an inertial reference frame.

In inertial reference frames, objects traveling at constant velocity will maintain that velocity unless acted upon by an outside force; as a special case, objects at rest will remain at rest unless an outside force acts.

It's not hard for us to accept that an object at rest will remain at rest, but it is very hard to accept the fact that an object will move at constant velocity unless an outside force, one originating from another object, acts. Friction is so common in our experience that we often don't realize it is almost always present and acting to slow objects down.

Noninertial frames of reference abound. For example, while driving your car you rapidly accelerate from rest at a stoplight. A box of cookies on the seat next to you spontaneously slides toward the back of the seat and at the same time the trinket hanging from your rear view mirror also spontaneously accelerates to the rear. No object can be found that causes these accelerations. By speeding up, your car becomes an accelerated reference frame. Similarly, if you spin around on a lab stool you will observe all objects in your vicinity orbit around you in circles. Because they travel in circular paths in your reference frame, we show later that they must accelerate. But, again, no object can be identified as the cause of these accelerations. A spinning frame is noninertial.

The latter example draws attention to the following cautionary tale. As the day passes on Earth we see remarkable events in the sky. The sun rises and sets, seemingly orbiting the Earth in a circular path. Then the moon, the stars, and even the most distant galaxies do the same thing. All traveling in circles about the Earth, all, from our vantage point, therefore accelerating. To explain how all of these accelerated motions occur requires a very complicated picture of how the Earth could possibly cause them. A much simpler explanation is that the Earth is spinning: we, on the Earth, live in a noninertial frame of reference. Does that mean we have to leave the Earth in order to observe the validity of Newton's law(s)? That depends on what you want to measure. If you are doing an experiment that is completed in a few minutes and/or is confined to a small region of the Earth, the acceleration of your laboratory is probably ignorable. On the other hand, if you are interested in the motion of large volumes of air moving for hours above the Earth, for example, your acceleration will make what you see more difficult to explain. (The apparent circulation of winds around high and low pressure cells results from the acceleration of the Earth relative to the air. There is no body that can be identified as causing those circulations.)

3. FORCE IN ONE DIMENSION

The acceleration of any body is caused by interactions with other bodies. Dynamics is an exact mathematical formulation of the connection between acceleration and "interaction." How is the qualitative notion of "interaction" made mathematically precise? An interaction is a push or a pull. An interaction has a magnitude, or size, and a direction. In one dimension, say along the x-axis, there are only two choices for direction: along the positive x-axis direction or along the negative direction (right or left along the axis). We call such objects, with both a magnitude and a direction, *vector quantities*; a vector quantity in one dimension is simply a signed number measured in appropriate units. Examples of vector quantities from the first section of this chapter include position, displacement, velocity, and acceleration. Each of these has both a magnitude and a direction associated with it. On the other hand, quantities such as distance traveled or average speed do not have a direction and are called *scalar quantities*. We indicate vector quantities by placing an arrow over their symbol, for example, the acceleration vector $\vec{a}$. The simplest assumption we can make is that a physical interaction also can be represented mathematically by a vector quantity. We call such vectors *forces* and our first goal is to provide an operationally meaningful definition for force.

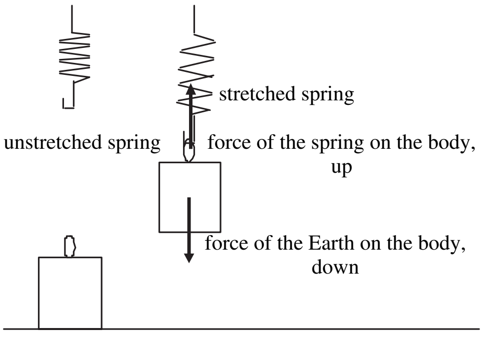
The definition of force we seek relies on a sequence of reasonable assumptions and their logical consequences. First, from our study of kinematics earlier in this chapter, we recall that acceleration, like force, also has a magnitude and a direction and is thus a vector quantity. Everyday experience suggests that when we push an initially resting object in a given direction the object accelerates in that direction. So, we reasonably assume that when a body experiences a single interaction, the vector force (the cause) and the vector acceleration (the result) are parallel and that one is, at most, just a scalar multiple of the other.

Next, suppose a body experiences more than one interaction at any instant. Interactions are represented by force vectors, therefore we assume that the vector sum of the individual forces is equivalent to a single force that would yield the same acceleration. The vector sum in one dimension is simply obtained by adding the signed numbers representing the individual vectors. For example, given two acceleration vectors with magnitudes of 3 and 4 m/s^2, both pointing along the positive x-axis, the vector sum is 7 m/s^2 also along the positive x-axis, whereas if the second vector points along the negative x-axis, the vector sum of the two is $(3 - 4) = -1$ m/s^2, where the negative sign indicates that the direction is along the negative x-axis. Clearly it only makes sense to add two vectors that represent the same physical quantity, for example, accelerations. (Just as you shouldn't add "apples and oranges" because the result mixes the two kinds of fruit together and has no immediate interpretation, adding a force to a velocity doesn't make physical sense either.) Vector addition in one dimension can be generalized to add any number of vectors using simple arithmetic (Just adding positive and negative numbers). If the vectors we are adding are force vectors acting on an object, the vector sum represents the net force on the object. In particular, if a body is at rest or traveling with a constant velocity (i.e., not accelerating) the vector sum of all forces acting on the body must be zero, assuming we are in an inertial reference frame. We can exploit this quite reasonable assumption to develop a method for measuring force.

We know that all objects near the Earth fall if they are not supported. The cause of this downward acceleration is a field force. We say that the Earth is responsible for this force because it exerts a "gravitational pull" on all bodies in its vicinity. It is traditional to call the force of gravity of the Earth on any object the object's *weight*. We often measure weights by using a spring scale, such as the familiar hanging scales in a grocery store. When we place some tomatoes on a grocery scale, the tomatoes cause a spring to stretch and a needle to deflect. The deflection of the needle is taken to be a measure of the "weight" of the tomatoes. This happens primarily because the Earth somehow pulls the tomatoes down toward it and the scale somehow gets in the way and keeps the tomatoes from falling. The word more commonly used by physicists for a pull (or a push) is *force*. The force the Earth exerts on the tomatoes is called *gravity*. There's a wondrous thing about gravity: gravitational pulls exist even though the bodies involved don't touch. The Earth reaches out across empty space and pulls on the tomatoes. (Of course, the space between the Earth and the tomatoes isn't really empty: it's filled with air. But, we can get rid of the air, in a vacuum chamber, for example, and when we do we find that the pull of gravity is almost exactly the same.) Forces that exist across empty space are said to be *field forces*. In the field force picture, the Earth is viewed as creating a "gravitational force field" in the space around it. When the tomatoes are placed in the Earth's field they respond by falling toward the Earth. The scale, on the other hand, is doing something more directly to the tomatoes. It appears to stretch only when it is in direct contact with the tomatoes. The force the scale exerts on the tomatoes is an example of what is called a *contact force*. When the tomatoes hang from the scale without moving, the force down on them by the Earth is said to equal the force up on them by the scale.

This works because of a very useful property of springs. Suspend a simple spring from a fixed support. Attach an object to the free end of the spring and gradually lower the object until it can be let go and remain at rest. In this state of persistent rest, the object is not accelerating so the spring must be exerting an upward (contact) force on the object, balancing out the Earth's downward pull (field) on it. We note that the spring is stretched. The amount by which the spring has been stretched can be used to measure the force it is exerting. (See Figure 2.7.)

FIGURE 2.7 *Spring scale used to measure weight.*



Suppose we have another object that is identical to the one that is already hanging from the spring. (We can check whether the weights of the two objects are identical by suspending them individually from the spring and noting that the stretch is the same in both cases.) Attach the second object to the end of the spring along with the first. We assume that these two bodies together are equivalent to a third body whose weight is twice that of the individuals. As long as the two hanging bodies are not too heavy (so that their combined weight does not permanently deform the spring) the new stretch is observed to be twice that when the spring is supporting just one of the bodies. In other words, the amount of stretch is directly proportional to the weight the spring supports, or, equivalently, the amount of stretch of a spring is a direct measure of how much force the spring exerts. Similarly, if we have two identical springs (two springs that stretch exactly the same amount when the same mass is suspended from each) and we hang a single weight by both springs as in Figure 2.8, we find that they each stretch by half the distance they would stretch if they each supported the full hanging weight. This should make sense because each spring is supporting half the weight with an equal upward force.

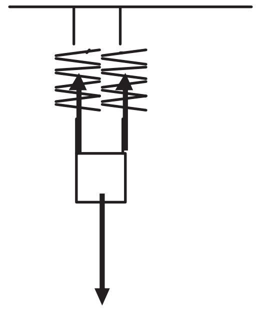


FIGURE 2.8 *Two identical spring forces each supporting half the weight of an object.*

In principle, we can imagine measuring any force on any object by replacing the force we are interested in by an appropriately calibrated, stretched spring (big stiff ones for large forces, and tiny flexible ones for small forces), keeping all other forces as before, and generating the same acceleration as when the replaced force is present. Because a spring exerts a force along its length, the direction of the spring corresponds to the direction of the replaced force and the stretch of the spring determines the force's magnitude.

4. MASS AND NEWTON'S LAW OF GRAVITY

The Earth isn't the only object that creates gravity. Every mass creates a gravitational pull on every other mass. You actually pull the tomatoes you weigh in the grocery toward you a little (and they pull you, too). It's just that the Earth's pull is so much greater than yours, you don't realize you're doing it. Mass plays two roles in producing a gravitational force. First, one mass creates a gravitational field in the space around it. Then, a second mass placed in the field of the first experiences a force due to the first's field. The two masses reciprocate in their pulls. The second makes a field of its own and the first, being in the field of the second, feels a force due to it. We say that a gravitational field has a direction—it points toward the mass making it—and a size, or *magnitude*. Let's call the magnitude of the gravitational field made by a mass M, g_M. The magnitude of the force this field produces when a mass m is placed in it is defined to be $F_{\text{of } M \text{ on } m} = mg_M$. Like mass and length, force has its own SI unit, the *newton* (N). (You don't find the newton in Table 1.1 because force is not defined as a fundamental quantity. It is expressible in terms of mass, length, and time, as we show in the next section. Because it is expressible in terms of fundamental units it is called a *derived unit*.) Gravitational field is gravitational force divided by mass, so the units of gravitational field are newtons per kilogram, N/kg.

We say that a body's weight (near the Earth) is the gravitational force the Earth exerts on that body. Thus, a mass m weighs

$$W_{\text{mass } m} = F_{\text{Earth on } m} = mg_{\text{Earth}} \tag{2.5}$$

SI units of mass (the kg), distance (the m), time (the s), and force (the N) were historically developed to be independent of the Earth's gravitational pull. Thus, a mass of 1 kg does not weigh 1 N, for example. Rather, under the SI conventions, we find that a mass of 1 kg near the Earth actually weighs about 9.8 N. Consequently, we say that the gravitational field of the Earth is about 9.8 N/kg near the Earth's surface.

Why is the condition "near the Earth's surface" important? Well, it turns out that the strength of a mass's gravitational field gets weaker the farther away one is from the mass.

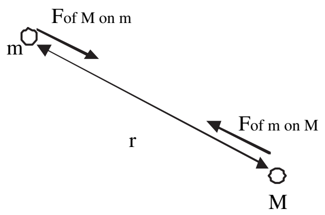


FIGURE 2.9 *Two masses attracting each other by the gravitational force.*

Very careful measurements in the laboratory show that if the centers of two uniform (i.e., no holes or irregularities), spherical masses, M and m, are separated by a distance r, then M pulls m with a gravitational force whose magnitude is given by (see Figure 2.9)

$$F_{M\text{ on }m} = G\frac{Mm}{r^2} \tag{2.6}$$

The quantity G is independent of which masses are interacting and any other physical condition. It is a so-called "universal constant" and in SI units its value is close to 6.67×10^{-11} $N\text{-}m^2/kg^2$. Equation (2.6) is known as *Newton's law of universal gravitation*. If we divide both sides of Equation (2.6) by m we get the gravitational field produced by M at a distance r from its center:

$$g_M = G\frac{M}{r^2} \tag{2.7}$$

Although Equations (2.6) and (2.7) are rigorously correct for uniform spherical masses, they can be applied to arbitrary shaped masses to obtain approximate values for gravitational forces and fields.

Example 2.1 What is the order of magnitude of the mass of the Earth?

Solution: The Earth is approximately a sphere with radius $R_E = 6.38 \times 10^6$ m (about 4000 mi) ~ 10^7 m. At the Earth's surface the r in Equation (2.7) is $r \sim 10^7$ m and we know that $g_{\text{Earth}} \sim 10$ N/kg at the surface. So, solving Equation (2.7) for M, we find $M_{\text{Earth}} \sim (10\text{ N/kg})(10^7\text{ m})^2/(10^{-10}\text{ N-m}^2/\text{kg}^2) \sim 10^{25}$ kg. (Make sure you see how the units work out. A careful calculation yields 5.98×10^{24} kg.) In other words, by making a laboratory measurement of G (and a measurement of R_E) it is possible to "weigh the Earth."

Example 2.2 What is the gravitational field of a typical person 1 m from the person?

Solution: The point of this example is to obtain an approximate value we can compare with the Earth's field. Thus, we treat the person as if she were a sphere of radius less than 1 m and take some typical value for mass, such as ~ 10^2 kg (remember, 1 kg weighs 2.2 pounds). One meter from the center of a 10^2 kg sphere the gravitational field due to that mass is ~$(10^{-10}\ N\text{-}m^2/kg^2)(10^2\text{ kg})/(1\text{ m})^2 \sim 10^{-8}$ N/kg. Compared with the Earth's field this is a tiny value. No wonder a person weighing tomatoes doesn't affect the tomatoes very much.

Example 2.3 What is an accurate value of the Earth's gravitational field at an altitude of 300 km (about the altitude of the Space Shuttle when it is in orbit)?

Solution: Here we want to do a formal calculation to compare with 9.8 N/kg. Recall that in Equation (2.6) or (2.7) r is the distance from the center of the sphere causing the field. An "altitude" is a distance above the surface of the Earth, so that r equals R_{Earth} + 300 km. Now, a km is 1000 m, so 300 km = 3×10^5 m = 0.3×10^6 m and, therefore, $r = 6.38 \times 10^6\text{ m} + 0.3 \times 10^6\text{ m} = 6.68 \times 10^6$ m. Putting this value into Equation (2.7) along with $M_{\text{Earth}} = 5.98 \times 10^{24}$ kg results in a gravitational field equal to 8.9 N/kg. In other words, where the Shuttle orbits, the Earth's gravitational pull is only about 9% less than at the Earth's surface. A Shuttle astronaut who weighs 150 pounds on Earth weighs about 137 pounds in orbit. The pull of Earth's gravity is what keeps weather and communications satellites and even the moon orbiting the Earth. The Earth's gravitational pull doesn't suddenly stop at the top of the atmosphere; it extends, in principle, "to infinity," getting weaker as r gets bigger as $1/r^2$.

The last statement may run counter to what you've heard or read about astronauts in orbit. In orbit, things are said to be "weightless." You've surely seen video of astronauts floating about aboard the Shuttle. If a 150 pound astronaut tried to step on a scale while in orbit, he wouldn't succeed in getting a reading, because the scale would float away. The resolution to the seeming contradiction that an astronaut can be apparently "weightless" and yet weigh 137 pounds requires knowing something about Newton's laws of motion, a topic we are just beginning to explore.

Thus far in this section we have been discussing the gravitational attraction of masses. Historically, in such discussions mass was referred to as gravitational mass, a property that produces gravitational fields leading to gravitational forces. We now turn to a seemingly different property of mass, inertia.

As mentioned previously, the fact that bodies are reluctant to accelerate is said to result from an intrinsic property of matter called inertia. A body's inertia can be assigned a numerical value, referred to as its mass. It is a remarkable law of nature that if two bodies experience the same net force (which we can check with calibrated springs) the ratio of the magnitudes of the resulting accelerations, a_1/a_2, has the same numerical value irrespective of what forces are acting, how the bodies were initially moving, or any other external aspect of the measurement (such as the time of day, the temperature, where the experiment is performed, and so on). With the same net force acting on each body, this ratio depends only on which two bodies' accelerations are being compared. The ratio must be directly related to an intrinsic property of the bodies. Furthermore, there is a kind of reciprocity between "heaviness" and acceleration: if body 1 feels heavier than body 2 (so that intuitively it would seem to have more mass) the ratio a_1/a_2 is less than 1, and vice versa. We define the ratio of the mass of body 2 to that of body 1 to be the numerical value of a_1/a_2 determined by exposing both to the same net force; that is,

$$\frac{m_2}{m_1} \equiv \frac{a_1}{a_2}. \tag{2.8}$$

More massive objects will experience smaller accelerations for the same force, with the accelerations inversely related to the respective masses. The unit for mass is the kilogram (kg, defined below). When used with the meter and second, the kilogram defines the SI (Système International) units (formerly known as the mks system of units). We can define the mass (m_2, say) of an object through this equation by using a standard of mass as another object ($m_1 = 1$ kg) and by measuring the accelerations of the two objects under the action of the same force (m_2 would then be just a_1/a_2 in kg).

Example 2.4 A body with mass equal to 1 kg is pulled across a leveled air table by a spring with constant stretch of 1 cm. The resulting acceleration of the 1 kg mass is observed to be 0.30 m/s^2. A second body of unknown mass is pulled by the same spring with the same constant stretch. The observed acceleration of the second mass is 0.45 m/s^2. What is the mass of the second body?

(*Continued*)

Solution: We assume that under the conditions cited, both bodies experience the same overall force due to the spring. Because the second body has a higher acceleration, we expect it has a mass less than 1 kg. We let $m_1 = 1$ kg and m_2 be the unknown mass. Then using Equation (2.8) we have

$$\begin{aligned} m_2 &= (0.30\ \text{m/s}^2 / 0.45\ \text{m/s}^2) \cdot (1\ \text{kg}) \\ &= 0.67\ \text{kg}. \end{aligned}$$

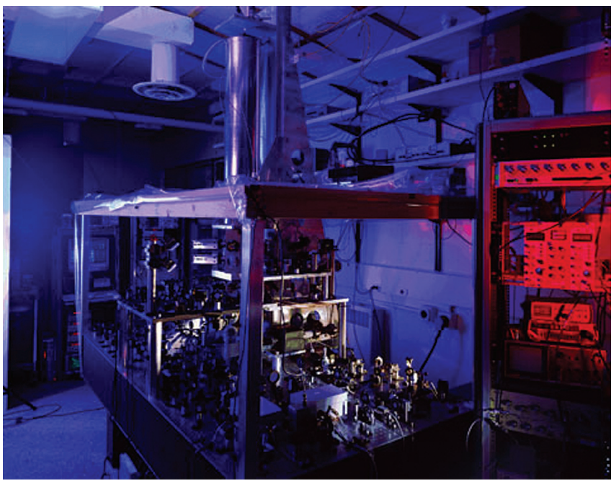
The procedure outlined above could be used, in principle, to measure the mass of any object. Of course, this is not done in practice because interactions (such as collisions) have the nasty potential for altering our standard and because the force that would impart a nice acceleration to an electron would imperceptibly perturb the motion of a kilogram. In practice, a wide range of secondary mass standards has to be used to measure unknown masses.

The standard kilogram (kg) is a platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures. Incidentally, standards for the meter and second are defined more reproducibly: the second is defined as the time needed for 9,192,631,770 vibrations of a cesium atom (a so-called atomic clock) and the meter is defined as the distance traveled by light in a vacuum in a time of 1/299,792,458 s (Figure 2.10). This, in fact, defines the speed of light in vacuum to be exactly $c = 299{,}792{,}458$ m/s. In other words, the speed of light was so well determined that in 1983 the meter was redefined so as to fix the speed of light.

Although fractions and multiples of kilograms suffice for quantifying mass in many situations, in the microworld of atoms and molecules another mass unit is more useful: the atomic mass unit (u) is defined to be exactly 1/12 of the mass of a neutral "carbon twelve" atom (an atom with 6 protons, 6 neutrons, and 6 electrons, often designated by the symbol ^{12}C). The atomic mass unit is preferred over kilograms when dealing with molecules because $1\ \text{u} = 1.66 \times 10^{-27}$ kg, and the latter is a very small and ungainly number with which to deal. The term dalton (D) is sometimes used to denote the same mass unit.

To recap this section on mass, we have discussed mass from two seemingly different approaches: gravitational mass, through Newton's law of gravity, which produces gravitational fields and forces on other masses, and inertial mass, defined through the acceleration produced by forces acting on the mass. Gravitational mass is a "static" mass with no motion required, gravitational fields and forces depending only on gravitational masses and distances. Inertial mass, on the other hand, is a "dynamic" mass, defined in terms of the acceleration response of the inertial mass to a given force of any kind. It is not necessarily apparent that these two concepts should lead to the exact same number for the mass of an object, but we have used the same symbol m for each because it has been shown that these masses have the same value to within better than 1 part in 10^{12}. This equivalence of inertial and gravitational mass has been a subject of discussion and experiment since Galileo and is still under active research.

FIGURE 2.10 *An atomic clock at NIST (National Institute of Standards and Technology) with an accuracy of about 1 s in 20 million years.*



5. NEWTON'S SECOND LAW OF MOTION IN ONE DIMENSION

Newton's first law tells us that in an inertial frame of reference a body accelerates only when it experiences a net force due to all other bodies. Equipped with the definitions of force and mass given above, the idea embodied in Newton's first law—that acceleration has a cause—can be made more precise. Thus, *Newton's second law of motion* says that

In an inertial frame of reference, the acceleration of a body of mass m, undergoing rigid translation, is given by

$$\vec{a} = \frac{\vec{F}_{\text{net on m}}}{m}, \tag{2.9}$$

where $\vec{F}_{\text{net on m}}$ is the net external force acting on the body (i.e., the sum of all forces due to all bodies other than the mass m that push and pull on m).

Embedded in Newton's second law are several important notions. (1) The law says that when the acceleration of a body arises from forces, the acceleration is caused by agents outside the body. A body cannot accelerate itself. Acceleration requires external force. (2) When there is a net (unbalanced) force on a body, the acceleration is in the same direction as the net force. The constant of proportionality that converts force into acceleration is the reciprocal of the body's mass. For a given force, the larger the mass, the smaller the acceleration, and vice versa. (3) Finally, as stated here, Newton's second law is applicable to a body in rigid translation, a body whose extent in space is ignorable, a point particle. For bodies that are tumbling or flexing or breaking into pieces the law of motion stated above has to be clarified and supplemented in ways we examine later.

Note that according to Equation (2.9), force has the units of mass times acceleration. Thus, in SI units one unit of force is equal to 1 kg-m/s^2. Because of the central role that force plays in describing nature, force units are given their own name. Honoring the founder of dynamics, 1 kg-m/s^2 is defined as 1 newton (1 N). (For calibration, a quarter pound hamburger with its bun, but minus the tomato and pickle, weighs about 1 N.)

Mass should be carefully distinguished from weight. *Mass is an intrinsic property of an object whereas weight is the magnitude of the force of the gravitational pull of the Earth.* If a body is in free fall, Equation (2.9) says

$$a = g = \frac{F_{\text{gravity}}}{m}, \tag{2.10}$$

where g is the magnitude of the acceleration due to gravity (9.8 m/s^2 near the Earth's surface). The force F_{gravity} is due to the pull of the Earth on the body whose mass is m. The magnitude, mg, of the gravitational force is also called the body's *weight*. A 1 kg mass thus weighs 9.8 N, because, for such a body, $F_{\text{gravity}} = 1\text{ kg} \times 9.8\text{ m/s}^2$. Note that weight exists whether or not the object is actually accelerating downward with acceleration g. A 1 kg body resting on a table near the surface of the Earth still weighs 9.8 N; the downward pull of the Earth on it must be canceled by an upward force of 9.8 N exerted by the table to keep it at rest. The weight of an object will vary depending on its location. For example, an object on the moon's surface weighs only about 1/6 what it does on Earth. This difference is due to the difference in the gravitational pull of the moon and has to do both with the moon's mass and radius compared to those of the Earth.

Equation (2.9) can be used to extract acceleration information from known forces or force information from known acceleration. For example, if all the forces acting on a particle of a given mass are known at every instant, the acceleration of that particle for every instant can be determined from the forces. Then, by measuring the particle's position and velocity at any one time, this dynamically inferred acceleration can be used (along with the methods we study in the next chapter) to predict the entire future motion of the particle, as well as deduce its entire past motion. Alternatively, if a complete record of a particle's motion is available, the particle's acceleration for every instant can be calculated from kinematics and forces required to produce that motion can then be determined.

Example 2.5 Television pictures are created by the collisions of a narrow beam of rapidly moving electrons with phosphor molecules on the screen of the picture tube. Suppose an electron (mass = 9.1×10^{-31} kg) in a TV is released from rest. After release it experiences a constant electrical force of 0.001 pN (where 1 pN = 1 piconewton = 10^{-12} N). What is the electron's acceleration under this force?

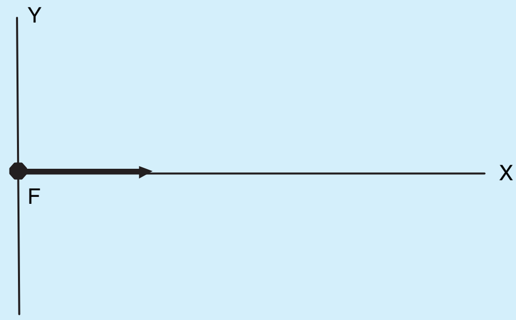


FIGURE 2.11 *An electron, initially located at the origin experiences a constant force F.*

Solution: We choose a coordinate system with the x-axis lined up along the direction of the constant force and with the origin where the electron is released (see Figure 2.11). The magnitude of the acceleration is found from Newton's second law

$$a_x = F_x/m = 0.001 \times 10^{-12} \text{ N}/\ 9.1 \times 10^{-31} \text{ kg} = 1.1 \times 10^{15} \text{ m/s}^2.$$

Because the force is constant throughout this region of space, the acceleration remains constant there as well, always pointing along the x-axis. Note that gravity pulls the electron toward the Earth with an acceleration equal to about 10 m/s^2. The electrical force on the electron in this picture tube is about 10^{14} times larger than gravity! TV designers don't have to worry about gravity making their pictures sag.

Newton's second law has a wonderful range of validity and usefulness. It can be used to aim electrons to make a better TV picture. It can tell us how macromolecules vibrate and tumble in a cell when DNA is undergoing replication. It allows us to design more effective brakes to make cars safer. With it we can calculate the trajectories of planets and rocket-launched satellites to explore the bodies of our solar system. (A powerful example of such calculations is the collision of the comet Shoemaker–Levy 9 with the planet Jupiter in which the collision time was predicted with tremendous accuracy (Figure 2.12).) Newton's second law is arguably one of the central ideas of all of physics. You certainly could do less important things than practice the mantra, "Acceleration is net force over mass; acceleration is net force over mass, ..."

6. NEWTON'S THIRD LAW

According to Newton's second law, acceleration requires force from outside. Swimming fish, flying birds, and human bicyclists all accelerate because something pushes on them, according to the second law. At first, that may sound preposterous. For example, think of what it feels like to increase your speed while running. You feel strain in the muscles of your legs. Or, accelerate your car to pass on a highway. You have to push down the gas pedal. Obviously, in both cases something internal is causing the acceleration.

Well, that's not exactly correct. Suppose you are asked to exert the same strain in your legs but instead of running on a dry track you are placed on a beach with loosely packed, dry sand. The same effort doesn't result in nearly the same acceleration. If you are placed instead on an ice rink, the same effort produces even less of an outcome. Finally, if you were put in a space suit and placed in the vacuum of space outside the Space Shuttle, moving your legs with the same strain as before would produce no acceleration at all. Clearly, moving your legs is important in producing acceleration, but what you are standing on is also important. You have to be able to push against something. That is equally true for fish and birds and accelerating cars.

The reconciliation of examples of apparent self-propulsion with Newton's second law, which says that self-propulsion is impossible, requires another law of motion:

When one body exerts a force on a second body, the second exerts a force in the opposite direction and of equal magnitude on the first; that is,

$$\vec{F}_{2\text{ on }1} = -\vec{F}_{1\text{ on }2}$$

This law, *Newton's third law of motion*, is sometimes referred to as the law of action–reaction: every "action" generates an equal and opposite "reaction." Thus, the feet of a runner do not accelerate the runner. Rather, the feet exert a force on the track, and it is the reaction force of the track back on the feet that accelerates the runner. When you run on a track a given effort leads to a certain push on the Earth; the Earth pushes back on you and that push results in your acceleration. When you run in loose sand, or on ice, you can't exert the same force on the Earth as you can by pushing on a dry track; the weaker push by you on the Earth is reciprocated with a weaker push back, and, therefore, less acceleration. In space, running doesn't result in an acceleration because there is nothing to push against and therefore nothing to push on you.

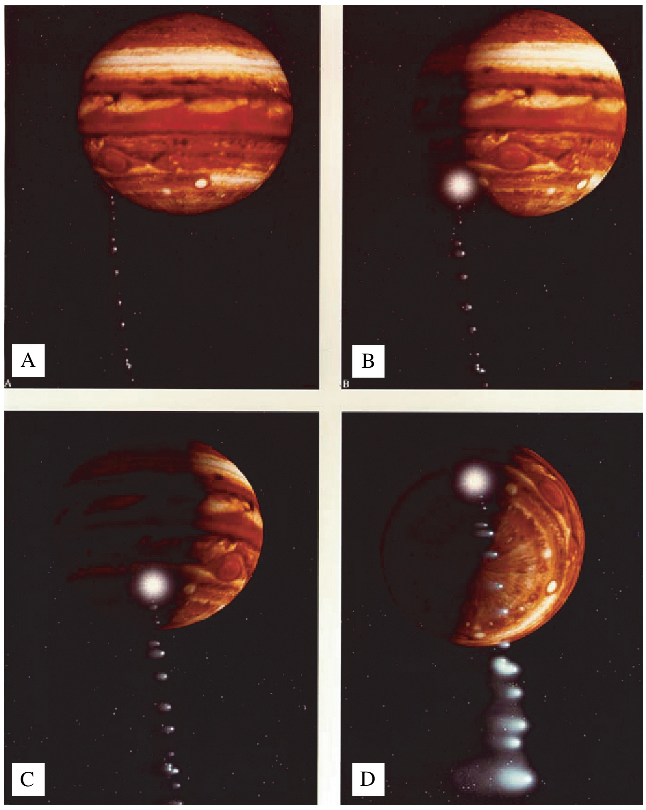


FIGURE 2.12 *Time series showing the collision of a comet with Jupiter in July 1994 as detected by the Galileo satellite probe; the comet, made from over 20 fragments, had been tracked for a year and the location and time of the impact, the first-ever observed collision of two solar system objects, had been calculated very precisely.*

Example 2.6 Newton's third law can be a source of confusion to someone who is thinking about such things for the first time. Here's an example. A young woman kicks a soccer ball 30 m downfield. But how? (Caution: The reasoning that follows contains an error! Can you spot it?) That is, Newton's third law says that the force of her foot on the ball is exactly countered by a reaction force exerted by the ball on her foot. The two are equal in magnitude and oppositely directed. The sum of two equal and opposite forces is zero, so according to Newton's second law, if there is no net force, no acceleration is possible. But, of course the ball does go downfield, so what goes on?

Solution: The wording of this problem illustrates a common pitfall in applying Newton's laws of motion. You have to be careful about identifying what is the body of interest and what are its surroundings. If we are interested in the flight of the soccer ball, then we have to keep track of the forces on the ball, and only those forces. If we are interested in the motion of the woman's foot, then we have to keep track of the forces on her foot. The foot exerts force on the ball and the ball accelerates as a result. The ball exerts a force on the foot and the foot accelerates (slows down) as a result. The two forces are equal and oppositely directed, however, they act on different bodies and each produces its own acceleration. The two don't act together on any one body and the fact that they add up to zero is irrelevant for understanding what happens to the ball.

You may be tempted, in thinking about this example, to say something like, "Well, the ball goes downfield because the woman is more powerful or more massive than the ball." Resist that temptation if you feel it creeping up on you. Keep in mind that a not very powerful nor massive 50 kg woman can easily accelerate a 1000 kg car (in neutral, with its brakes off, on a horizontal surface) by pushing it.

Example 2.7 Two ice skaters, a 90 kg father and his 40 kg daughter standing face to face and holding hands, push off from each other with a constant force of 20 N (Figure 2.13). Find their accelerations during the time they are pushing each other.

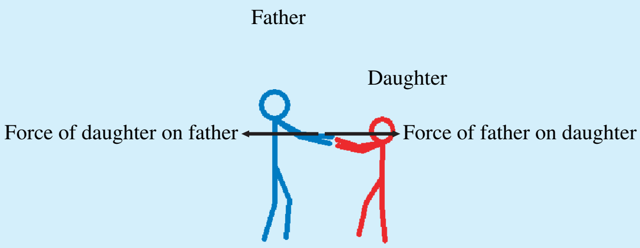


FIGURE 2.13 *Two ice skaters pushing off from each other.*

Solution: Each skater exerts a 20 N force on the other. Assuming there are no other horizontal forces acting, the man's acceleration will be a_{man} = 20 N/90 kg = 0.22 m/s^2 to the left, whereas the girl's acceleration will be a_{girl} = 20 N/40 kg = 0.5 m/s^2 to the right. These accelerations occur only during the time when the skaters are pushing against each other. Note that no matter which person (or both) actually takes the active role in doing the pushing, the force on each person has the same magnitude.

Example 2.8 A book lies at rest on a horizontal table. Identify all forces acting on the book and for each identify the appropriate reaction force.

Solution: The forces labeled "1" and "3" in Figure 2.14 are forces on the book. Forces "2" and "4" are exerted by the book in reaction to "1" and "3". Force "1" is the book's weight. It is due to the Earth's gravitational field. If the Earth pulls on the book, Newton's third law says that the book must pull back on the Earth with a force of equal magnitude. The reaction force to "1" is a gravitational pull exerted by the book on the Earth, and is labeled "2" in the figure. Its magnitude is the same as the book's weight. The force "3" is an upward force exerted by the table on the book because of contact between the table and the book. We know there is such a force because we know the book lies at rest, so the net force on it must be zero. When the force exerted on the book by the table is added to the force exerted on the book by the Earth, the two cancel. Clearly, the upward force of the table on the book must also have the same magnitude as the book's weight. The reaction force to "3" is a contact force, "4," exerted by the book on the table. It points down and it, too, has the same magnitude as the book's weight but it is not the book's weight. If suddenly a hole bigger than the book opened in the table below it, both "3" and "4" would suddenly disappear, but the book's weight "1" and the reaction force "2" would still exist.

So, if the force "2" is due to a gravitational pull of the book how come the Earth doesn't accelerate toward the book with an acceleration g? Newton's

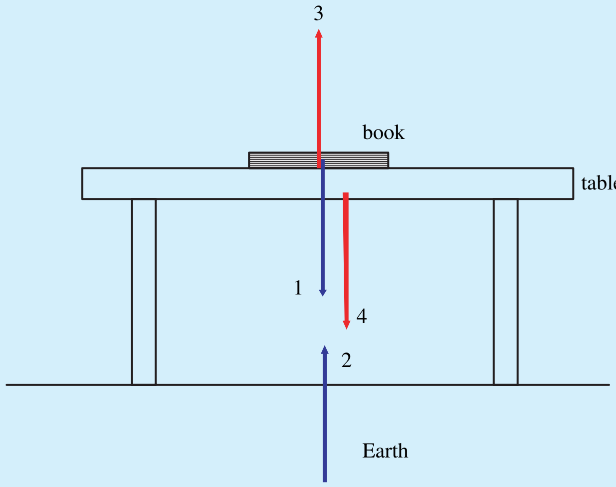


FIGURE 2.14 *Forces involved with a book on a table. Forces 1 and 3 act on the book, whereas 3 and 4, and 1 and 2 represent action–reaction pairs (see discussion of Example 2.8).*

third law says that action–reaction forces are equal, not the accelerations they produce! To find out about those, use Newton's second law: the magnitude of the Earth's acceleration is the magnitude of the force on it divided by the Earth's mass. In other words,

$$a_{\text{Earth}} = \frac{F_{\text{book on Earth}}}{M_{\text{Earth}}} = \frac{m_{\text{book}}\, g}{M_{\text{Earth}}} = \left(\frac{m_{\text{book}}}{M_{\text{Earth}}}\right) g$$

(remember, the magnitude of the force exerted by the book is equal to the book's weight) and because the ratio of the mass of the book to the mass of the Earth is on the order of 10^{-25} the book's pull on the Earth produces a negligible acceleration. Of course, if the book had a lot more mass—like that of another planet—and was as close to the Earth as the book (fortunately, the pull of gravity also depends on distance) then the acceleration of the Earth would not be negligible. But, that's another story.

7. DIFFUSION

An E. coli bacterium typically swims in a straight line for some distance, during which time its flagella undergo a coordinated helical motion driven by a rotary molecular motor located in the membrane at the flagella attachment sites (we study this molecular motor further in Section 3 in Chapter 7; see also Figure 1.2 for a cartoon sketch). In response to external stimuli of, for example, nutrient or oxygen level, the molecular motor may reverse and cause the flagella to become uncoordinated, resulting in a characteristic "twiddling" motion in which the bacterium randomly gyrates about, before finally taking off in a straight-line trajectory in some other direction. *E. coli* have been shown to respond to variations in environmental factors, being attracted to higher levels of nutrients and oxygen and repelled by poisons; this response is known as chemotaxis. If the *E. coli* are either killed or have their flagella removed they are no longer motile but they still move due to a phenomenon known as *Brownian motion*, named after Robert Brown who in 1827 noticed the random thermal motions of

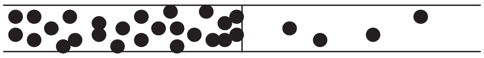


FIGURE 2.15 *Diffusion will tend to equalize the numbers of molecules in the left and right sides of the initially sharp boundary.*

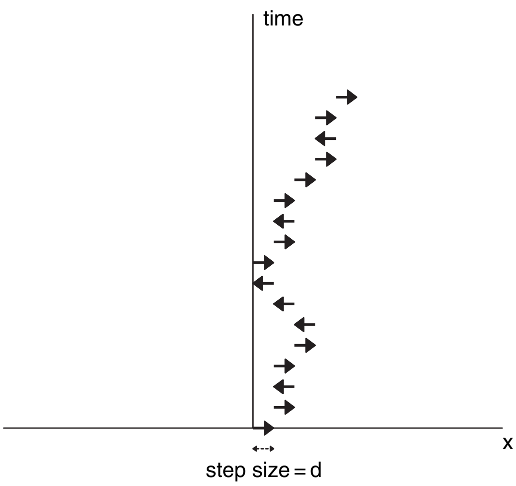
suspended pollen grains under a microscope. Rapid and numerous collisions of solvent molecules with the *E. coli* produce random erratic motions. The Brownian motions of such "killed" *E. coli*, as well as the random motions of the solvent molecules themselves, are examples of a general process known as *diffusion*, which is the term for such thermally driven motions at the molecular level.

Although diffusion appears, at first glance, to be random and incapable of resulting in useful or interesting results, diffusive phenomena abound in the biological and physical world. In biology, diffusion is the process that controls both the exchange of oxygen in the hemoglobin of our red blood cells and the elimination of wastes in our kidneys. Whenever molecules move from one place to another without the expense of energy specifically earmarked for that motion, it is by diffusion; for example, diffusion controls the passive transport of molecules across a membrane and stored chemical energy is required for the process known as active transport.

Often when there are concentration differences across macroscopic distances diffusion will play a role in reducing those differences. In these cases, even though the motion of each individual molecule may be random in direction, the collective motion that affects the local concentration of molecules can be directed. For example, in the case of one-dimensional diffusion, suppose there is a sharp spatial boundary in the concentration of some molecules as shown in Figure 2.15. Then even though any particular molecule is equally likely to move left or right, as time evolves, the variation tends to disappear because, on average, there are more molecules in the higher concentration region moving into the lower concentration region. Examples of just this type of diffusion are the oxygen and waste transport in the blood and kidneys cited above. In general when there are initial concentration variations and no active, energy-consuming processes occurring, diffusion tends to result in a uniform final state. We show the connection of this randomization process to the science of thermodynamics in Chapter 13.

The mathematics of diffusion in one dimension can be described by a related problem known as the *random walk*. Suppose that one starts at the origin and takes equal length steps in either the positive or negative x-direction with equal probability (this is also known as the drunkard's walk problem). Without regard for the details of the mathematics, it is clear that the average position of the person after many steps is still at the origin since positive or negative steps are equally likely and the average is simply computed by adding up the (plus and minus) displacements. On the other hand, it should also be clear that as time goes on, it will become more and more possible that the person will be found farther away from the origin. We can characterize this motion by calculating the average of the squares of the displacements, because these will all be positive quantities and cannot average away to zero. A calculation shows that this mean square displacement, $<(\Delta x)^2>$, is given by

FIGURE 2.16 *One-dimensional random walk with equal step size and time interval.*



$$<(\Delta x)^2> = Nd^2,$$

where N is the number of steps, d is the step size, and the brackets $< >$ indicate taking the average value (Figure 2.16).

The one-dimensional diffusion of a "killed" *E. coli* can be solved using mathematics similar to the random walk problem, but clearly the step size and number of steps do not directly apply. The analogous equation for the mean square displacement of a diffusing bacterium is given by

$$<(\Delta x)^2> = 2\,Dt,$$

where t is the elapsed time and D is a constant known as the diffusion coefficient, which is a property of the size and shape of the bacterium as well as of the viscosity (a measure of "stickiness") and temperature of the liquid medium in which the bacterium is found. It turns out that as this result is generalized to two (or three) spatial dimensions of motion, the mean square displacement has an additional 2 Dt (or 4 Dt), so that in three dimensions

$$<(\Delta r)^2> = 6\,Dt. \tag{2.11}$$

The square root of the mean square displacement (known as the *root mean square* or *rms displacement*) is thus proportional to $\sqrt{t}$, a result that is very different from the linear t-dependence for a particle moving with constant velocity. Although diffusing particles may move rapidly over short times, because of their constant random changes in direction, the overall average displacements change much more slowly with time. The characteristic $\sqrt{t}$ signature of displacements in diffusion appears often in our discussions of many physical as well as biophysical phenomena. For example, we show that electrical and thermal conductivities are closely related to the diffusion of loosely bound electrons in a metal.

Example 2.9 The diffusion coefficient for sucrose in blood at 37°C is 9.6×10^{-11} m^2/s. (a) Find the average (root mean square) distance that a typical sucrose molecule moves (in three dimensions) in 1 h. (b) Now find how long it takes for a typical sucrose molecule to diffuse from the center to the outer edge of a blood capillary of diameter 8 μm.

Solution:

(a) Simple substitution finds the rms distance to be equal to

$$\sqrt{6Dt} = \sqrt{6 \cdot 9.6 \times 10^{-11}\,\text{m}^2/\text{s} \cdot 3600\,\text{s}} = 1.4 \times 10^{-3}\,\text{m}.$$

(b) This is a problem in two dimensions (in a cross-sectional plane of the capillary), so that from the above discussion, the relationship between the mean square distance and the time is $<(\Delta r)^2> = 4\,Dt$. Substituting $\Delta r = 4$ μm $= 4 \times 10^{-6}$ m, we find that

$$t = \frac{<(\Delta r)^2>}{4\,D} = \frac{(4 \times 10^{-6}\,\text{m})^2}{4 \cdot 9.6 \times 10^{-11}\,\text{m}^2/\text{s}} = 0.04\text{ s}.$$

Note that this answer for the time scales as the square of the capillary radius and so increases by a factor of 4 for a capillary of twice the radius. This example demonstrates why capillaries need to be so small in order to carry out efficient exchange of food and wastes between the blood and surrounding tissue.

CHAPTER SUMMARY

In one dimension, starting with the concept of displacement Δx, velocity and acceleration are defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \quad \textit{and} \tag{2.2}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}, \tag{2.4}$$

where the average values over a time interval Δt are equal to these expressions without taking the limit.

The gravitational force between any two masses is given by Newton's universal law of gravity,

$$F_{M\text{ on }m} = G\frac{Mm}{r^2}. \tag{2.6}$$

(*Continued*)

For a mass near the Earth's surface, this force is equal to its weight,

$$W_{\text{mass } m} = F_{\text{Earth on } m} = mg_{\text{Earth}}, \qquad (2.5)$$

with $g_{\text{Earth}} = 9.8 \text{ m/s}^2$.

Newton's second law states that

$$\vec{a} = \frac{\vec{F}_{\text{net on m}}}{m} \qquad (2.9)$$

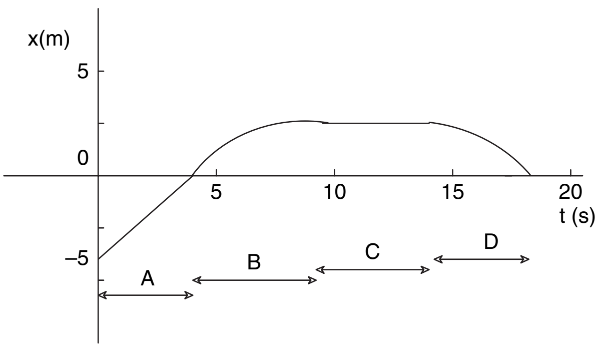
and in the absence of a net force, the acceleration must be equal to zero, a statement equivalent to Newton's first law. The third law is a statement that all forces arise from interactions between pairs of objects; the two forces (action and reaction) each act on one of the objects and are equal in magnitude, but opposite in direction.

Unlike directed motion, diffusion is a random thermal process in which the average displacement is zero, however, the mean squared displacement is given by

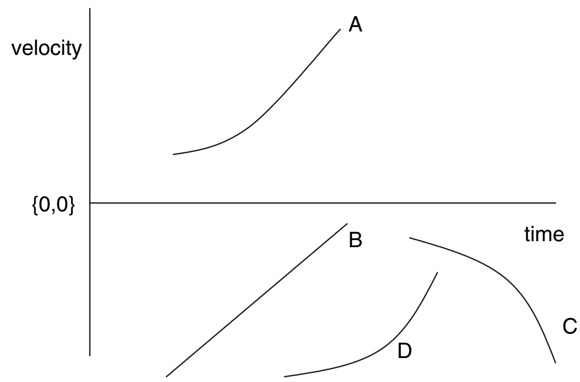
$$<(\Delta r)^2> = 6\,Dt. \qquad (2.11)$$

QUESTIONS

1. As a car moves steadily down a road, we can deduce the motion of the car by following the motion of only one piece, for example, the corner of a fender or the license plate. However, the motion of the piece only conveys complete information about the rigid structure of the car. Describe the motion through space of each of the following as a car moves forward: a tire air valve, the tip of a working windshield wiper, the top of an engine piston, and the label on a fan belt.
2. As a person runs, describe the motion through space of a wrist, a kneecap, and an elbow.
3. In the figure the position of an object is shown as a function of time. Indicate whether the velocity and acceleration in each labeled interval are positive, zero, or negative.



4. In the figure the velocity of four different objects is shown as functions of time. Indicate whether the velocity and acceleration for each labeled object are positive, zero, or negative.



5. Is the average velocity during an interval of time always equal to the sum of the initial and final velocities of the time interval divided by two? If not, give an example showing why not.
6. When an object free falls, does it travel equal distances in equal time intervals? Does its velocity increase by equal amounts in equal time intervals?
7. In each of the following situations, first identify all the forces acting on the object and then, for each force, identify the reaction force and its source:
   (a) A bird flying through the air
   (b) A horse pulling a cart
   (c) A person riding in an elevator that is accelerating upwards
   (d) A hot air balloon hovering in place
   (e) A ladder leaning against a wall.
8. A VW bug has a terrible head-on collision with an 18-wheeler truck. Which vehicle experiences the greatest force on impact? The greatest acceleration?
9. Tell whether the following pairs of forces are action–reaction pairs, and include a statement about your reasoning.

(a) The weight of a fish and the buoyant force holding it up
(b) The centripetal force on a protein molecule in a centrifuge and the force the protein exerts on the solvent surrounding it
(c) The weight of a free-fall skydiver and his frictional drag after reaching a terminal velocity
(d) The thrust on a jellyfish and the force the jellyfish exerts on the jet of water it expels
(e) The frictional force that allows you to walk and the force you exert horizontally on the Earth

10. Describe some situations in which forces act on an object but there is no motion. How can this occur?

11. What is the difference between mass and weight?

12. Which of the following situations involve field forces and which contact forces: a tug-of-war, moving paper clips around with a horseshoe magnet, riding a Ferris wheel, getting a shock when you reach for a door knob, a ball falling through the air, a train rolling on tracks, a levitated train traveling at over 340 min/h.

13. Two equal masses attract each other with a gravitational force of 18 pN. If their separation is tripled what will the gravitational force between them be?

14. A mass produces a gravitational field g at a point. If the mass is doubled and moved twice as far away from the point, what will the new gravitational field be?

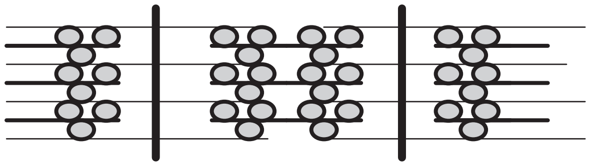
15. Discuss how you think scientists were able to determine the mass of the sun.

16. Explain why even though an astronaut in orbit around the Earth is weightless, she must exert a force in order to propel herself across the spaceship.

17. A person riding on the "whip" at an amusement park watches an ice skater coast by. The ice skater believes that she is coasting in a straight line at a constant speed. How does the person on the "whip" describe her motion? This same person believes that Newton's first law is violated for the ice skater. Why is he wrong?

18. Muscle basically consists of interdigitating thick and thin filaments that interact via cross-bridges (the "heads" of myosin molecules). Because the force a myosin head exerts on an actin thin filament is equal and opposite to the force the actin exerts back on the myosin head and thereby the thick filament, how can the muscle generate any force?

19. The detailed structure of a muscle fiber includes a series of Z-lines with actin thin filaments of opposite polarity on either side and with thick filaments not attached to the Z-lines as shown. The cross-bridge interactions tend to shorten the distance between neighboring Z-lines when a muscle contracts, but should not a given Z-line feel symmetric forces from the equivalent thin filament interaction on either side, and hence not feel a net force?



20. In each of the following cases, identify the interaction pairs of forces and draw a free-body diagram of the object in italics: (a) a *book* resting on a table; (b) a *book* resting on a table with a paperweight on top of the book; (c) a *cart* being pulled by a horse along a level road; (d) a heavy *picture* being pushed horizontally against the wall to hold it in place.

21. What causes diffusion? If a container is kept perfectly still, without any vibrations on it whatever (e.g., covered, in a draft-free room, atop a granite block mounted on shock absorbers) will diffusion occur within it?

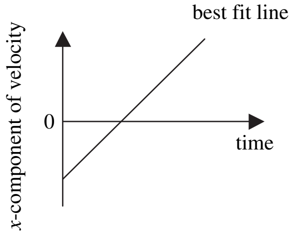
22. Why doesn't a drop of dye, when added to water, simply grow outward uniformly from the position at which it is first placed? (Or does it?) If you carefully put one drop of cream atop a mug of coffee, what happens to it? Is there any way to keep the added drop from diffusing?

MULTIPLE CHOICE QUESTIONS

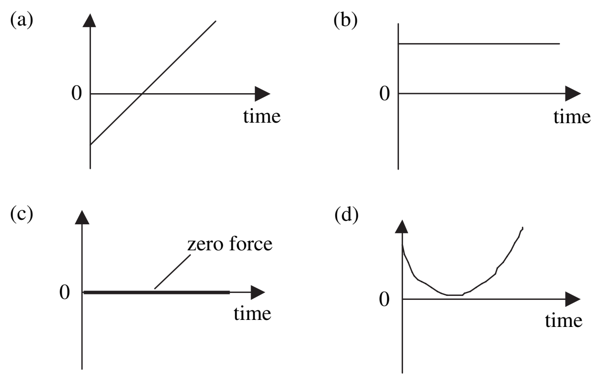
1. The x-position of a particle is sampled every 0.5 s, as in the following table.

Time (s)	*x-Position (m)*
0.0	+3.0
0.5	+2.2
1.0	+3.0
1.5	+1.0
2.0	−0.5

Which one of the following must be true? (a) The x-component of the average velocity in the interval 0.0 s to 1.0 s is 0.0 m/s. (b) The average speed in the interval 0.0 s to 1.0 s is 0.0 m/s. (c) The x-component of the instantaneous velocity at 1.0 s is +3.0 m/s. (d) The x-component of the instantaneous velocity throughout the interval 1.0 s to 2.0 s is always negative.



2. The x-component of a particle's velocity is sampled every 0.5 s. The data are fit with a straight line as shown in the figure to the right. Assuming the fit is a good approximation to the motion, which of the following best represents the x-component of the net force on the particle as a function of time?



3. A 9.8 N force causes a 1 kg mass to have an acceleration of 9.8 m/s^2. This situation is most closely related to Newton's (a) first law of motion, (b) second law of motion, (c) third law of motion, (d) law of universal gravitation.

4. A woman weighing 500 N stands in an elevator that is traveling upward. At a given instant the speed of the elevator, as well as that of the woman, is 10 m/s and both are decreasing at the rate of 2 m/s^2. At that instant, the floor of the elevator exerts a force on the woman that is (a) about 400 N, pointing up, (b) 500 N, pointing up, (c) 500 N, pointing down, (d) about 600 N, pointing up.

5. A soccer ball approaches a soccer player with a speed of 10 m/s. The player heads the ball with the net result that the ball travels off in the opposite direction with a speed of 15 m/s. The player stays more or less in place. During the time the player's head is contact with the ball the head exerts an average force of magnitude 100 N. Which one of the following is true concerning the magnitude of the average force the ball exerts on the player's head during that time? (a) It must be about zero because the head doesn't move much. (b) It's hard to say from the information given, but it certainly must be less than 100 N or else the ball wouldn't reverse direction. (c) Nothing can be said about the magnitude of the force because neither the mass of the ball nor the time of contact is given. (d) It's 100 N.

6. A bicyclist rides for 20 s along a straight line that corresponds to the $+x$-axis covering a distance of 400 m. She then turns her bike around; that takes another 20 s. Finally, she rides back to where she started (400 m in the $-x$-direction) for 40 s. The average velocity for this trip is (a) 0, (b) +3, (c) +10, (d) +15 m/s.

7. A ball is thrown directly upward. After leaving the hand the ball is observed to be at a height A and rising. A little while later, the ball is at height B and is instantaneously at rest. Later still the ball is observed to be height C and falling. All during the flight the ball is in free-fall. The acceleration of the ball (a) points up at A, is 0 at B, and points down at C; (b) points up during each portion of the flight; (c) is zero during each portion of the flight; (d) points down during each portion of the flight.

8. An object is thrown straight up. At the top of its path (a) the velocity is zero and the acceleration is zero, (b) the velocity is zero and the acceleration is equal to the weight, (c) the velocity is down and the acceleration is equal to g, (d) the velocity is zero and the acceleration is equal to g.

9. Newton's law of gravitation says that the magnitude of the gravitational force of a body of mass M on a body of mass m is GMm/r^2. The fundamental dimensions of Newton's Gravitational Force are (a) $[M][L][T]^{-2}$, (b) $[M]^2[L]^{-2}$, (c) $[M][L][T]^{-1}$, (d) $[M][L]^2[T]^{-2}$. (Here [M] represents mass, [L] length, and [T] time.)

10. Given that the Earth is about 1.5×10^{11} m from the sun and takes a year (about 3.1×10^7 s) to make one revolution around the sun, the Earth's orbital speed around the sun is (a) 4.8×10^3 m/s, (b) 2.3×10^{15} m/s, (c) 3.0×10^4 m/s, (d) 7.3×10^{14} m/s.

11. Agnes is in an elevator. Andy, sitting on the ground, observes Agnes to be traveling upward with a constant speed of 5 m/s. At one instant Agnes drops a pen from rest. Immediately after, the acceleration of the pen according to Agnes is (a) 10 m/s^2, down, (b) 0, (c) 15 m/s^2, down, (d) 5 m/s^2, up.

12. As in the previous question, Agnes is in an elevator that Andy (attached to the ground) sees traveling upward. This time Andy sees the elevator's speed increasing by 5 m/s every second. Agnes stands on a scale in the elevator and sees the reading to be 750 N. After the elevator comes to a complete stop, Agnes is still on the scale. The reading now is (a) 250 N, (b) 500 N, (c) 750 N, (d) 1000 N.

13. As I apply the brakes in my car, books on the passenger seat suddenly fly forward. That is most likely because (a) the car is not an inertial reference frame, (b) the seat supplies a forward push to make the books accelerate, (c) there is a strong gravitational field generated by the brakes, (d) there is a strong magnetic field generated by the brakes.

14. A particle of mass m_1 collides with a particle of mass m_2. All other interactions are negligible. The ratio of the acceleration of mass m_1 to the acceleration of mass m_2 at any instant during the collision (a) is small at first, then reaches a maximum value, then goes back to a small value, (b) depends on whether m_1 and m_2 stick together in the collision, (c) depends on how fast each of the particles is initially moving, (d) is always the constant value m_2/m_1.

15. A 10 kg and a 4 kg mass are acted on by the same magnitude net force (which remains constant) for the same period of time. Both masses are at rest before the force is applied. After this time, the 10 kg mass moves with a speed v_1 and the 4 kg mass moves with a speed v_2.

Which of the following is true? (a) v_1 is equal to v_2, (b) the ratio v_1/v_2 is equal to 5/2, (c) the ratio v_1/v_2 is equal to 2/5, (d) the ratio v_1/v_2 is equal to $(2/5)^2$.

16. Can an object's velocity change direction when its acceleration is constant? (a) No, this is not possible because it is always speeding up. (b) No, this is not possible because it is always speeding up or always slowing down, but it can never turn around. (c) Yes, this is possible, and a rock thrown straight up is an example. (d) Yes, this is possible, and a car that starts from rest, speeds up, slows to a stop, and then backs up is an example.

17. Can an object have increasing speed while its acceleration is decreasing? (a) No, this is impossible because of the way in which acceleration is defined. (b) No, because if acceleration is decreasing the object will be slowing down. (c) Yes, and an example would be an object falling in the absence of air friction. (d) Yes, and an example would be an object released from rest in the presence of air friction.

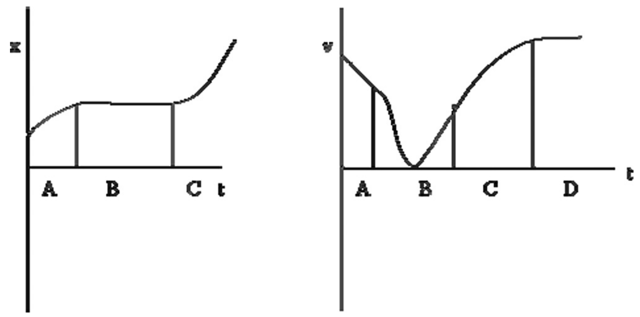
Questions 18–21 concern interpreting the two graphs below.

18. In which interval of the x versus t graph (A, B, or C) is the acceleration negative?

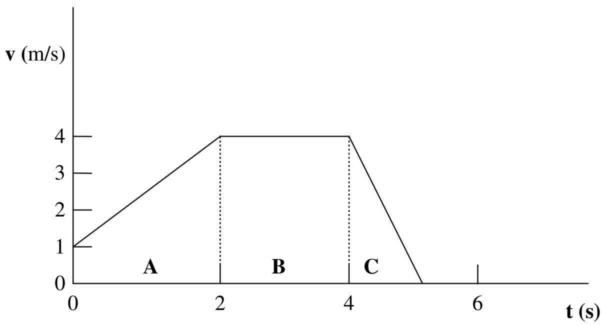
19. In which interval of the x versus t graph (A, B, or C) is the velocity constant?

20. In which interval of the v versus t graph (A, B, C, or D) is the acceleration constant but nonzero?

21. In which interval of the v versus t graph (A, B, C, or D) is the acceleration only positive?



Questions 22 and 23 refer to the following diagram.

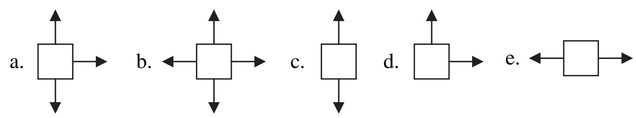


22. If the above graph is for a 4 kg object, the forces acting during each of these three intervals (A, B, C) are given (in Newtons) by (a) (6, 0, 16), (b) (−6, 0, 16), (c) (3/2, 0, −4), (d) (6, 0, −16), (e) (3/2, 0, −16).

23. If the object described by the above graph starts at the origin at $t = 0$, where will it be at $t = 4$ s? (a) $x = 11$ m, (b) $x = 13$ m, (c) $x = 8$ m, (d) $x = 4$ m, (e) $x = 22$ m.

24. A person is holding up a picture by pushing it horizontally against a vertical wall. The reaction force to the weight of the picture is (a) the normal force on the picture, (b) the pull upwards on the Earth equal to the weight, (c) the frictional force on the picture at the wall equal to the weight, (d) the frictional force on the wall by the picture, (e) the normal force on the wall by the picture.

25. Which of the following represents the correct free-body diagram for a helium (floats in air) balloon held by a string that is tied to a seat inside the passenger compartment of a train traveling to the right at a constant 60 mph?



26. A cart is being pulled along a horizontal road at constant velocity by a horse. What is the reaction force to the horse pulling on the cart? (a) the normal force of the ground on the cart, (b) the weight of the cart, (c) the friction force on the cart equal to the pull of the horse, (d) the equal backwards pull on the horse.

27. An object is thrown straight up. At the top of its path the net force acting on it is (a) greater than its weight, (b) greater than zero but less than the weight, (c) instantaneously equal to zero, (d) equal to its weight.

28. A trained seal at the circus sits on a chair and balances a physics book on its nose. On top of the book sits a basketball. Which of the objects exerts a force on the basketball? (a) the book only; (b) both the seal and the book; (c) the seal, the book, and the chair; (d) none of the above.

29. A large truck runs into a small car and pushes it 20 m before stopping. During the collision (a) the truck exerts a larger force on the car than the car exerts on the truck; (b) the truck exerts a smaller force on the car than the car exerts on the truck; (c) the truck and car exert equal forces on each other; (d) the car doesn't actually exert a force on the truck; the truck just keeps going.

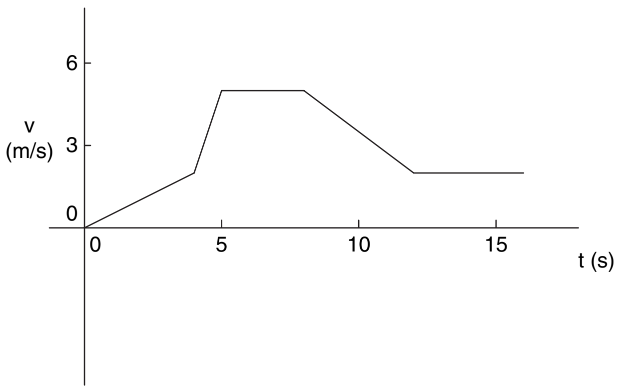
30. A car weighing 10,000 N initially traveling at 30 m/s crashes into a 100 N garbage can, initially at rest, sending it flying. During the time the car is in contact with the can it exerts a force of 3000 N on the can. During the time of contact the can exerts (a) a force of 3000 N on the car, (b) a force considerably less than 3000 N on the car, (c) a force considerably greater than 3000 N on the car, (d) no force on the car.

31. As a protein diffuses in a thin long tube (effectively 1-dimensional motion) starting from $x = 0$, its average position $<x>$ and its mean square position $<x^2>$ change with time t according to (a) $<x> = <x^2> = 0$, (b) $<x> = 0$; $<x^2> \propto t^2$, (c) $<x> \propto t$; $<x^2> \propto t^2$, (d) $<x> \propto t$; $<x^2> \propto t$, (e) $<x> = 0$; $<x^2> \propto t$.

32. At a turning point in the motion of an object: (a) the velocity can be positive or negative but the acceleration must be instantaneously zero, (b) the velocity must be instantaneously zero, but the acceleration can be positive or negative, (c) both the velocity and acceleration must be instantaneously zero, (d) the velocity and acceleration must have opposite signs (i.e., one positive and the other negative), (e) none of the above is true.

PROBLEMS

1. Shown is a plot of velocity versus time for an object originally at rest at the origin. Develop the corresponding plot for acceleration.



2. (a) Using the data given, plot position versus time for $t = 0$, 4, and 8 s. Calculate the velocity for each interval [0,4] and [4,8] and determine that the average acceleration between these two time intervals is zero.

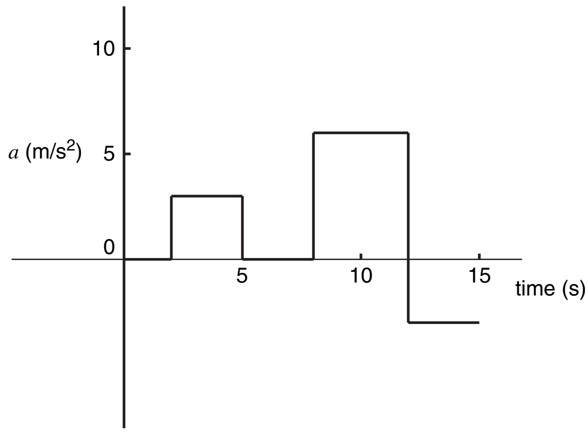
T, seconds	0	1	2	3	4	5	6	7	8
x, meters	1	7.25	9	7.75	5	2.25	1	2.75	9

(b) Now plot all nine data points. Calculate velocity again, this time for all eight time intervals from [0,1] through [7,8]. Calculate the average accelerations for the time intervals [0,2], [2,4], [4,6], [6,8] starting with the velocities just previously calculated.

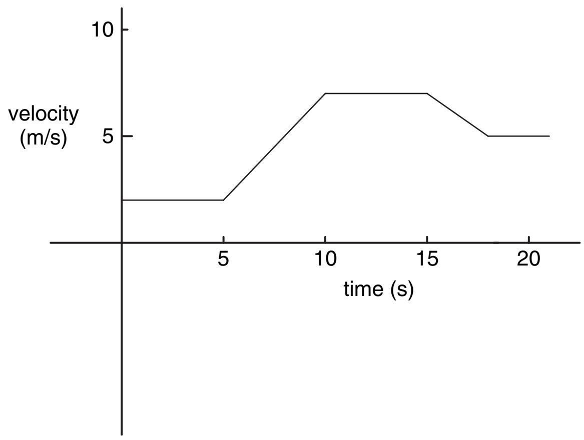
(c) Note that the given data are from the functional expression $x(t) = t^3/4 - 3t^2 + 9t + 1$. Deduce that the data describe the motion of an object that moves forward, stops and backs up, stops again, and moves forward with increasing speed.

(d) Do you see how use of 4 s time intervals misses the details of motion that is more fully described by the use of shorter time intervals? Where is the slope of the $x(t)$ curve positive? Where negative? Where zero? What is the physical meaning of the sign of the slope of the $x(t)$ curve? If the slope of the $x(t)$ curve changes sign, what does that say about the velocity and the acceleration of the object?

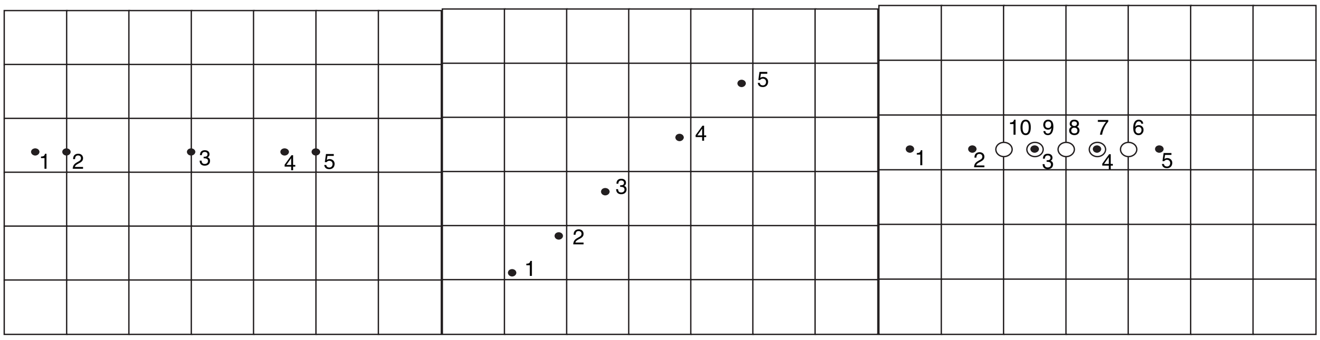
3. Shown is a plot of acceleration versus time for an object. Assuming that its initial position and initial velocity are both zero in magnitude, for how long after $t = 12$ s, must the acceleration of -3 m/s^2 persist, in order that the object be brought to rest?



4. Shown is a plot of velocity versus time for a particle starting at the origin. Sketch a plot of the acceleration corresponding to the time interval for which velocity is shown.



5. A microbiologist observes the motion of a microorganism within a slide sample. Photographic records are snapped at 5 s intervals and the successive positions of the organism are shown. Calculate the average velocities and accelerations corresponding to the appropriate 5 s intervals, assuming the grid line spacing is 25 μm, for each of the three sets of records. Such quantitative investigations of biological motion can reveal important information about the organism. We show later that the measurement of acceleration can indicate how much force certain organs of locomotion are capable of generating. If the organism moves by expelling fluid, we may be able to determine the amount of fluid ejected per unit time and its expulsion velocity.



6. A 1400 kg car accelerates uniformly from rest to 60 mph in 6 s. Find the net force needed to produce this motion.
7. A car accelerates from rest uniformly to 30 mph in 5 s, travels at a constant 30 mph for 0.3 mi, and then decelerates to rest in 6 s.
   (a) What is the average velocity for each interval and for the entire trip?
   (b) What is the displacement for each interval and for the total trip?
   (c) What is the average acceleration for the entire trip?
8. A 0.1 kg mass stretches a linear spring by 10 cm. If three identical masses are hung together from two such identical springs (as in Figure 2.8), by how much will each spring stretch?
9. A Boeing 737 jet plane lands with a speed of 60 m/s (about 135 mi/h) and can decelerate at a maximum rate of 5 m/s^2 as it comes to rest.
   (a) What is the minimum time needed before the plane will come to rest?
   (b) Could this plane land on a runway that is 2800 feet long?
10. A person throws a set of keys upward to his friend in a window 9.2 m above him. The keys are caught 3.0 s later by the friend's outstretched hand.
   (a) With what initial velocity were the keys thrown?
   (b) What was the velocity of the keys just before they were caught?
11. Suppose that a 1 kg block attached to a light rope free-falls (with acceleration g) from rest for 5 s before someone grabs the rope.
   (a) What velocity will the block have when the rope is grabbed?
   (b) In order to stop the block after an additional 5 s, what must be the constant acceleration of the block?
   (c) With what force must the rope be pulled upward to stop the block in those 5 s?
12. What is the acceleration of a 5 kg package being lowered to the ground by a light rope in which there is a tension of 25 N?
13. A truck moves through a school zone at a constant rate of 15 m/s. A police car sees the speeding truck and starts from rest just as the truck passes it. The police car accelerates at 2 m/s^2 until it reaches a maximum velocity of 20 m/s. Where do the police and the truck meet and how long does it take?
14. A person of mass 60 kg stands on top of a table located 1/2 m above the floor and then walks off the edge of the table.
   (a) Draw a free-body diagram of this situation.
   (b) During the time the person is falling to the floor, what is the upwards acceleration of the Earth as seen by the person?
   (c) As seen by the person, through what distance does the Earth move up towards her in this time?
15. The planet Pluto travels once around the sun every 248 years at a mean distance from the sun of 5890×10^6 km. Find its orbital speed around the sun (in m/s).
16. What is the gravitational field on the surface of the moon? Take the mass of the moon as 7.4×10^{22} kg and its radius as 1.74×10^6 m and calculate g as a fraction of that on the Earth's surface.
17. What is the gravitational force of the sun on Pluto with a mass of 1.5×10^{22} kg (less than the moon) and a mean distance from the sun of 5890×10^6 km?

18. Suppose your normal weight is 1200 N standing on a bathroom scale. If you stand on that same scale in an elevator in a skyscraper that is accelerating upwards at 1 m/s^2, what will the scale read?

19. An eagle soaring overhead has a weight of 120 N. If the area of each wing is 1.7 m^2, find the force per unit area required to support the eagle while it soars.

20. The electron in a hydrogen atom is attracted to the proton in the nucleus with an electrical force of 8.2×10^{-8} N. What is the acceleration (magnitude and direction) of the electron? (According to classical physics this acceleration keeps the electron orbiting the nucleus.)

21. Two astronauts are out for a space walk near their shuttle. They have masses of 120 kg and 140 kg suited up in their space suits and are attached to the shuttle by umbilical cords. With both initially at rest with respect to the shuttle, if the 140 kg astronaut pushes the other one with a 20 N force for 1 s,

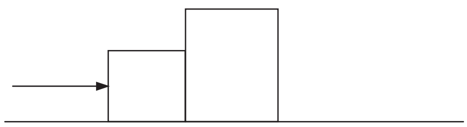
(a) What is the acceleration of the 120 kg astronaut during this 1 s?
(b) What is the acceleration of the 140 kg astronaut during the same 1 s?
(c) What velocity will each have after the 1 s interval with respect to the shuttle?
(d) If the umbilical is 10 m long, how long will it be before they each feel another force from the tug of the umbilical?

22. A heavy 40 kg crate sits on a shelf and is connected by a taut rope to the ceiling. If it is pushed off the shelf so that it is suspended freely find

(a) The net force on the crate.
(b) The tension force in the rope supporting the crate.
(c) If the rope is cut, what is now the net force on the crate?

23. Two heavy crates (of 10 kg and 20 kg mass) sit touching on a smooth surface of ice as shown. If a 20 N force pushes on the 10 kg crate as shown:

(a) What is the acceleration of both blocks?
(b) What is the net force on the 20 kg block?
(c) What force does the 20 kg block exert on the 10 kg block?
(d) What is the origin of the force in part c?
(e) Repeat the problem if the two blocks are physically interchanged (in parts (b) and (c) interchange the two masses as well) and the same force pushes the 20 kg block.



24. A 0.01 g water strider, an insect that can "walk on water," propels itself with its six legs to travel along at 0.5 m/s.

(a) What vertical force must the surface tension of water provide to each foot?
(b) If the insect is able to travel at constant velocity by overcoming a total resistive force from the water of 10^{-6} N, find the horizontal force from the water on each leg as the bug "walks."

25. A single nonmotile cell is confined to a thin capillary tube so that it essentially undergoes one-dimensional diffusion with a diffusion coefficient of 10^{-9} cm^2/s. Find (a) the time it takes for the cell to diffuse a distance of 1 cm (express your answer in hours), and (b) the rms distance the cell will travel in 1 s (expressed in μm). Why don't your answers to (a) and (b) scale linearly so that 3600 s/h multiplied by the answer to (b) would give a 1 cm distance?

26. As cells crawl along a surface in tissue culture their cytoplasm is observed to undergo "retrograde" flow in the direction opposite to the motion of the leading edge of the cell. When this motion is studied by imaging the cell in a microscope and making a movie of the motion, a feature in the cytoplasm is observed to travel a distance of 1.1 μm in 25 s. What is the speed of this retrograde flow?

Physics of the Life Sciences
Newman, J.
2008, XVII, 720 p., Hardcover
ISBN: 978-0-387-77258-5