2 Measurement Units

Physical quantities are organized in systems of units. This chapter is mainly dedicated to the International System (SI), but other systems, used in specialized fields of physics, are mentioned as well. The chapter ends with an introduction to the dimensions of physical quantities and to the main applications of dimensional analysis.

2.1 Base and Derived Quantities

In Sect. 1.3, it has been shown that every measurement is based on the possibility that some quantities (the additive quantities) can be directly measured by comparison with a unit standard $\mathcal{U}$.

To describe objects and phenomena of the physical world, many quantities are used, both additive and nonadditive, connected by analytical relations. In principle, one could choose an arbitrary unit standard for every additive quantity. This choice would, however, lead to the introduction of inconvenient proportionality factors, and would require us to define and to maintain a large number of measurement standards.

Example 2.1. Let us consider three quantities, length ℓ, time t, and velocity v, which are temporarily labeled $\mathcal{G}_\ell$, $\mathcal{G}_t$, and $\mathcal{G}_v$, respectively. For a uniform motion, the velocity is defined as $\mathcal{G}_v = \Delta\mathcal{G}_\ell/\Delta\mathcal{G}_t$. In principle, one can independently choose the unit standards $\mathcal{U}$ of the three quantities. Possible choices could be the Earth radius $\mathcal{U}_\ell$, the period of the Earth rotation $\mathcal{U}_t$, and the velocity tangential to the equator $\mathcal{U}_v$, respectively. Let us now consider the values X of the quantities $\mathcal{G}$, as defined by (1.1): $\mathcal{G} = X\mathcal{U}$. Velocity is, by definition, the ratio between space and time. According to the independent choice of unit standards, the value of the velocity would be connected to the space and time units through the relation $X_v = (1/2\pi)\, X_\ell/X_t$. In fact, a point fixed at the equator moves with unit velocity $\mathcal{G}_v = 1\,\mathcal{U}_v$, traveling the distance $\mathcal{G}_\ell = 2\pi\mathcal{U}_\ell$ in the unit time $\mathcal{G}_t = 1\,\mathcal{U}_t$.

In this example, the velocity unit is connected to the space and time units through the $(1/2\pi)$ factor. To avoid, or at least to reduce the number of factors different from unity in the relations connecting the units of different physical quantities, it is convenient to arbitrarily choose the unit standards

P. Fornasini, *The Uncertainty in Physical Measurements: An Introduction to Data Analysis in the Physics Laboratory*, DOI 10.1007/978-0-387-78650-6_2,

of only a very small number of quantities. For the other quantities, the units are univocally defined through analytical relations.

Example 2.2. Let us choose as arbitrary unit standards the meter (m) for lengths ℓ and the second (s) for times t; the unit of velocity v is then the meter per second (m/s), defined by the relation $v = \ell/t$.

One distinguishes:

(a) *Base quantities*, for which the unit standard is arbitrarily defined
(b) *Derived quantities*, for which the unit standard is defined through analytical relations that connect them to the base quantities

Establishing a *system of measurement units* consists of:

(a) Choosing a particular partition between base and derived physical quantities
(b) Defining the unit standards of the base quantities

The first attempt at establishing a system of units was made by the French revolutionary government in 1790, and led to the proposal of the Decimal Metric System in 1795. Afterwards, various other systems of units have been introduced, many of which are still in use. The increasing demand of standardization, connected to the development of trade and scientific research, led, since 1895 (Meter Convention), to various international agreements on unit systems. In the last decades, there has been a convergence towards the *International System* (SI), which is treated in Sect. 2.3.

A system of units is said to be

- *Complete,* when all physical quantities can be deduced from the base quantities through analytical relations
- *Coherent,* when the analytical relations defining the derived units do not contain proportionality factors different from unity
- *Decimal,* when all multiples and submultiples of units are powers of ten

2.2 Measurement Standards

The unit standards of the fundamental quantities are physically realized by means of *measurement standards*. There are standards also for many derived quantities. The main properties characterizing a measurement standard are:

(a) *Precision*
(b) *Invariability* in time
(c) *Accessibility*, say the possibility that everyone can have access to the standard

(d) *Reproducibility*, say the possibility of reproducing the standard in case of destruction

One can distinguish two fundamental kinds of measurement standards, *natural standards* and *artificial standards*. The distinction is clarified by the following example.

Example 2.3. Let us consider the evolution of the length standard. In 1795, the *meter* was for the first time introduced as the fraction $1/10^7$ of the arc of an Earth meridian from a Pole to the equator (natural standard). In 1799, an artificial standard was built, made by a platinum rule (precision $10 \div 20\,\mu$m). In 1889, the old standard was substituted by a rule made of an alloy 90% platinum + 10% iridium (precision 0.2 μm). In 1960, a natural standard was again introduced, the optical meter, defined as a multiple of the wavelength of the red-orange light emitted by the isotope 86 of krypton (precision 0.01 μm). Finally, since 1983, the definition of the meter has been based on the product between the speed of light and an interval of time.

Natural standards guarantee reproducibility and invariability, although sometimes at the expenses of accessibility.

The standards of the highest precision are called *primary standards.* More accessible, although less precise, *secondary standards* are periodically calibrated against primary standards. Standards of current use, the *working standards*, are in turn calibrated against secondary standards.

2.3 The International System of Units (SI)

The International System (SI) divides the physical quantities into base quantities and derived quantities, and defines the names and symbols of their units. The SI defines the measurement standards of the base quantities. In addition, the SI gives the rules for writing names and symbols of the units, as well as the names and symbols of the prefixes (see Appendix B.1). The SI is complete, coherent, and decimal (with the exception of time measurements).

Base Units

The SI is founded on seven base quantities. Their names, units and symbols are listed in Table 2.1. The official definitions of the base SI units are listed in Appendix B.1.

Time Interval

The unit of time, the *second*, is defined with reference to the period of the electromagnetic radiation that is emitted by the isotope 133 of Cesium (say the isotope whose nucleus contains 55 protons and 78 neutrons) during the

Table 2.1. Base quantities of the International System, their units and symbols.

Quantity	Unit	Symbol
Time interval	second	s
Length	meter	m
Mass	kilogram	kg
Amount of substance	mole	mol
Temperature	kelvin	K
Electric current	ampere	A
Luminous intensity	candela	cd

transition between two hyperfine energy levels of its ground state. The time standard is thus a natural standard.

The ground state of an atom corresponds to the electronic configuration of minimum energy. The splitting of the ground state into hyperfine levels is due to the interaction of electrons with the nuclear magnetic moment; the difference in energy ΔE between the hyperfine levels is much smaller than the difference between the principal levels. During the transition between the two levels of ^{133}Cs labeled $F = 4$, $M = 0$, and $F = 3$, $M = 0$, respectively, electromagnetic waves are emitted with frequency $\nu = \Delta E/h \simeq 10^{10}$ Hz, say in the microwave region (h is the Planck constant). The second is defined as 9 192 631 770 times the period $T = 1/\nu$. The primary standard of time is realized by the cesium clock, whose maximum relative uncertainty is 1×10^{-12}, corresponding to 1 μs every twelve days.

Length

The unit of length, the *meter*, is defined as the distance covered by light in vacuum in 1/299 792 458 seconds. As a consequence of this definition, the exact value $c = 299\,792\,458$ m/s has been attributed, since 1983, to the velocity of light in vacuum, one of the fundamental constants of physics.

Mass

The unit of mass, the *kilogram*, is the mass of a cylindric body made by an alloy of platinum and iridium, that is preserved in Sèvres (France). This is the only artificial standard of the SI. Its relative precision is of the order of 10^{-9}.

Amount of Substance

The *mole* is the amount of substance of a system containing as many elementary entities (atoms, molecules, electrons, etc.) as there are atoms in 0.012 kg of isotope 12 of carbon (say the most abundant isotope of carbon, whose nucleus contains six protons and six neutrons). The number N_0 of elementary entities within a mole is called the *Avogadro number;* its approximate value is $N_0 \simeq 6.022 \times 10^{23}$.

Temperature

The *kelvin* is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. The triple point of water, say the thermodynamic state of equilibrium of the solid, liquid, and gas phases (Fig. 2.1, left), corresponds to a temperature of 273.16 K and a pressure of 610 Pa. The relative precision of the kelvin is 1×10^{-6}.

Temperature is a nonadditive quantity. The absolute thermodynamic temperature is defined in relation to the efficiency of the Carnot cycle; its measurement corresponds to the measurement of the ratio between two additive quantities, for example, two heat quantities.

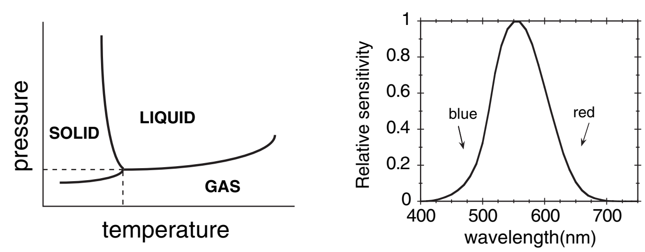

Fig. 2.1. Left: schematic representation of the phase diagram of water. Right: sensitivity curve of the human eye as a function of light wavelength.

Electric Current Intensity

The definition of the unit of electric current intensity, the *ampere*, refers to the force F per unit length ℓ between two parallel conductors placed at a distance d and carrying the same current I:

$$F/\ell = 2k_m \, I^2/d \; .$$

A numerical value 10^{-7} is attributed to the constant k_m. One ampere corresponds to the current that produces a force of 2×10^{-7} N per meter of length.

For the practical realization of standards, one prefers to rely on the Ohm law $I = V/R$, and obtain the ampere as a ratio between the units of potential difference (volt) and resistance (ohm). The standards of volt and ohm are realized by means of two quantum phenomena, the Josephson effect and the quantum Hall effect, respectively.

Luminous Intensity

Luminous intensity is the base quantity of photometry, say the discipline dealing with the sensitivity of the human eye to electromagnetic radiation.

The luminous intensity corresponds to the flux of energy radiated by a source within the unit solid angle, weighted by the average sensitivity of the human eye (Fig. 2.1, right). Photometric measurements are relevant in the fields of astronomy, photography, and lighting. The unit of luminous intensity, the *candela*, corresponds to the intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity of 1/683 watt per steradian.

Derived Units

The derived quantities are defined in terms of the seven base quantities via a system of simple equations. There are no conversion factors different from one (the SI is coherent). Some derived units have special names and are listed in Appendix B.1. Here are some relevant examples.

Example 2.4. Acceleration is a derived quantity. By definition, it is the derivative of velocity with respect to time. Its unit is the ratio between the unit of length and the square of the unit of time, say $1\,\mathrm{m\,s^{-2}}$. The unit of acceleration has no special name.

Example 2.5. Plane angle and *solid angle* are derived quantities. Their units have special names, *radian* and *steradian*, respectively. The radian (rad) is the plane angle that subtends, on a circumference centered on its vertex, an arc whose length is equal to the radius. The steradian (sr) is the solid angle that subtends, on a sphere centered on its vertex, a spherical cap whose area is equal to the square of the radius.

Example 2.6. Force F is a derived quantity. By the fundamental law of dynamics, $F = ma$, the unit of force is referred to the units of mass and acceleration. The unit of force has a special name, the *newton* (N), and is defined as $1\,\mathrm{N} = 1\,\mathrm{Kg\,m\,s^{-2}}$.

2.4 Other Systems of Units

In spite of the SI being an internationally adopted complete system, several other systems are still in use. We consider here the systems most relevant for physics.

cgs Systems

In cgs systems, the fundamental mechanical quantities are length, mass, and time, as in the SI. The corresponding units are centimeter, gram, and second (whence the acronym cgs). The differences between cgs and SI systems are limited, for mechanical quantities, to multiplicative factors, powers of 10, in the values of base and derived quantities, and to the name of units.

The substantial difference between cgs and SI systems concerns the electromagnetic quantities. While the SI has a base quantity for electromagnetism (the electric current intensity), in csg systems all electromagnetic units are derived from mechanical units. Several different cgs systems exist, depending on the relation that is used to define electromagnetic units as a function of mechanical units.

The *electrostatic cgs system* defines the electric charge unit (statcoulomb) through the Coulomb law

$$F_e = K_e \, q_1 \, q_2 / r^2 \, , \tag{2.1}$$

by imposing $K_e = 1$, dimensionless.

The *electromagnetic cgs system* defines the current unit (abampere) through the law of electrodynamic interaction between currents

$$F_m = 2 \, K_m \, I_1 \, I_2 \, \ell / d \, , \tag{2.2}$$

by imposing $K_m = 1$, dimensionless.

The *Gauss symmetrized cgs system* uses the electrostatic cgs units for electrical quantities and the electromagnetic cgs units for magnetic quantities. The symmetrized cgs system is still frequently used in theoretical physics.

Practical Systems

In the past, many practical units have been introduced in different fields of science and technology. After the introduction of the SI, practical units should not be used. Several exceptions limited to specialistic fields are, however, still accepted. Let us list here some non-SI units that are frequently used in physics; other examples can be found in Appendix B.1.

The *atomic mass unit* (u) is 1/12 of the mass of an atom of carbon 12, say the isotope whose nucleus contains six protons and six neutrons. The approximate value is $1\,\mathrm{u} \simeq 1.66 \times 10^{-27}\,\mathrm{kg}$.

The *electronvolt* (eV) is the energy gained by an electron when crossing an electric potential difference of 1 V. The approximate value is $1\,\mathrm{eV} \simeq 1.602 \times 10^{-19}\,\mathrm{J}$.

The *astronomic unit* (au), roughly corresponding to the distance between the Earth and Sun, is used for expressing distances within the solar system. Its approximate value is $1\,\mathrm{au} \simeq 1.496 \times 10^{11}\,\mathrm{m}$.

Plane angles are often measured using the *degree* (°) and its nondecimal submultiples: the minute, $1' = (1/60)^\circ$, and the second, $1'' = (1/3600)^\circ$.

The *ångström* (Å) is often used to measure distances at the atomic level: $1\,\text{Å} = 0.1\,\mathrm{nm} = 10^{-10}\,\mathrm{m}$.

British Systems

In some countries, such as Great Britain (UK) and the United States of America (USA), some British units are still in use (a partial list can be found in Appendix B.3).

British systems are not decimal. For example, the base unit of length is the *inch*, and its most important multiples are the *foot*, corresponding to 12 inches, and the *yard*, corresponding to three feet.

Some units have different values in the UK and in the USA. For example, the gallon, unit of volume, corresponds to $4.546\,\mathrm{dm}^3$ in the UK and $3.785\,\mathrm{dm}^3$ in the USA.

Natural Systems

In some specialistic fields of physics, it is customary to use peculiar units, which are called *natural* because they refer to particularly relevant fundamental quantities.

The *Hartree atomic system* is often used when describing phenomena at the atomic level. Its base quantities are:

- *Mass:* the unit is the electron rest mass m_e (in SI, $m_e \simeq 9.109 \times 10^{-31}\,\mathrm{kg}$)
- *Electric charge:* the unit is the electron charge e (in SI, $e \simeq 1.602 \times 10^{-19}\,\mathrm{C}$)
- *Action* (product of energy and time): the unit is the quantum of action h (Planck constant) divided by 2π, $\hbar = h/2\pi$ (in SI, $\hbar \simeq 1.054 \times 10^{-34}\,\mathrm{J\,s}$)

The *Dirac system* is often used in elementary particle physics. Its base quantities are:

- *Mass:* the unit is again the electron rest mass, m_e
- *Velocity:* the unit is the velocity of light in vacuum, c (in SI, $c = 299\,792\,458\,\mathrm{m\,s}^{-1}$)
- *Action:* the unit is again $\hbar = h/2\pi$

2.5 Dimensional Analysis

The choice of the base units reflects on the numerical values of both base and derived quantities. Dimensional analysis deals with this topic, and represents a useful tool for testing the soundness of equations connecting physical quantities.

Dimensions of Physical Quantities

Let us suppose that the unit of length, the meter, is substituted by a new unit, L times smaller; for example, the centimeter is $L = 100$ times smaller. As a consequence of the new choice of the length unit, the values

of length	are multiplied by	L
of time	are multiplied by	$L^0 = 1$
of volume	are multiplied by	L^3
of velocity	are multiplied by	L

The exponent of L is the *dimension* with respect to length:

Length	has dimension	1	with respect to length
Time	has dimension	0	with respect to length
Volume	has dimension	3	with respect to length
Velocity	has dimension	1	with respect to length

The dependence of the value X of whichever base or derived quantity on the units of the base quantities $A, B, C, \ldots$ is symbolically expressed by a *dimensional equation*:

$$[X] = [A]^{\alpha}\ [B]^{\beta}\ [C]^{\gamma} \ldots \tag{2.3}$$

where α, β, γ are the dimensions of X with respect to A, B, and C, respectively.

Dimensional analysis is mainly used in mechanics: here we consider only the dimensions with respect to length, mass, and time, which are symbolized by L, M, T, respectively. For example, the dimensions of velocity are expressed by the equation

$$[v] = [L]^1\ [T]^{-1}\ [M]^0\ , \tag{2.4}$$

and the dimensions of work and energy are expressed by the equation

$$[W] \ = \ [E] = [L]^2\ [T]^{-2}\ [M]^1\ . \tag{2.5}$$

Quantities characterized by the same dimensions are said to be dimensionally homogeneous.

Dimensionless Quantities

Some quantities have zero dimension with respect to all base quantities:

$$[L]^0\ [T]^0\ [M]^0\ . \tag{2.6}$$

It is the case of pure numbers (3, $\sqrt{2}$, π, ...) and of dimensionless quantities, say quantities defined by the ratio between two homogeneous quantities. The value of dimensionless quantities does not depend on the particular choice of the base units.

Example 2.7. Plane angles are dimensionless quantities; their value measured in radians is the ratio between the length of the arc and the length of the radius. Also solid angles are dimensionless quantities; their value measured in steradians is the ratio between two squared lengths.

Example 2.8. The absolute density of a substance is the ratio between its mass and its volume: $\rho = m/V$. The dimensions of the absolute density are given by $[\rho] = [L]^{-3}[T]^0[M]^1$. The relative density of a substance is the ratio between its absolute density and the absolute density of water at the temperature of 4°C. The relative density is a dimensionless quantity.

Principle of Dimensional Homogeneity

The usefulness of dimensional analysis is founded on the principle of dimensional homogeneity, stating that it is possible to sum or equate only dimensionally homogeneous quantities. Otherwise stated, any equation between physical quantities

$$A + B + C + \ldots = M + N + P + \ldots \tag{2.7}$$

is true only if $A, B, C, \ldots, M, N, P, \ldots$ are dimensionally homogeneous monomials. In particular, transcendental functions (sin, cos, exp, log, etc.) are dimensionless, and their arguments must be dimensionless.

Applications of Dimensional Analysis

Let us consider here the most important applications of dimensional analysis.

Test of Equations

The dimensional homogeneity is a *necessary condition* for the correctness of equations connecting physical quantities, such as (2.7). This means that, for the equation to be true, all terms must have the same dimensions. Dimensional homogeneity is thus the first test of validity of any theoretical relationship.

Example 2.9. Let us consider the oscillations of a mass suspended from a spring. The position x of the mass depends on time t according to a sinusoidal law. The dependence of x on t cannot be expressed by $x = \sin t$, because: (a) the argument t of the sine function is not dimensionless; (b) the sine function is dimensionless, and x has the dimension of length. A valid expression is $x = A\sin(\omega t)$, where A is a constant with the dimension of length, and ω is a constant with the dimension of an inverse time.

The dimensional homogeneity, however, is not a *sufficient condition* for the correctness of an equation, since:

1. Dimensional analysis cannot test the correctness of numerical values

2. There exist some quantities that, in spite of being dimensionally homogeneous, have a completely different meaning (e.g., mechanical work and the momentum of a force).

Example 2.10. We want to determine the trajectory of a missile that is thrown at an angle θ with respect to the horizontal, with initial velocity v_0. After some kinematical calculations, we find the following equation,

$$z \;=\; -\,\frac{g}{2\,v_0\,\cos^2\theta}\,x^2 \;+\; x\,\cos\theta\,, \qquad \text{[wrong!]} \tag{2.8}$$

where x and z are the horizontal and vertical coordinates, respectively. A dimensional check shows that (2.8) is wrong. We repeat the calculations and find

$$z \;=\; -\,\frac{g}{2\,v_0^2\,\cos^2\theta}\,x^2 \;+\; x\,\cos\theta\,; \qquad \text{[wrong!]} \tag{2.9}$$

the equation is now dimensionally homogeneous; in spite of this, it is still wrong. The right equation is

$$z \;=\; -\,\frac{g}{2\,v_0^2\,\cos^2\theta}\,x^2 \;+\; x\,\tan\theta\,. \tag{2.10}$$

The exchange of $\tan\theta$ by $\cos\theta$ in the last term of (2.9) led to a dimensionally homogeneous wrong equation.

In order to use dimensional analysis for testing equations, it is necessary to perform calculations in literal form. Numerical values should be inserted only at the end of the test.

Deduction of Physical Laws

In particular cases, dimensional analysis allows one to find analytical relations between different quantities characterizing a physical phenomenon. Obviously, no information on numerical values can be obtained, and dimensionless constants are neglected.

Example 2.11. The period $\mathcal{T}$ of a pendulum can depend, in principle, on the mass m and length ℓ of the pendulum, on the acceleration of gravity g, and on the amplitude θ_0 of the angular oscillation. The dimensional dependence of $\mathcal{T}$ on m, ℓ, and g can be expressed as

$$[\mathcal{T}] \;=\; [m]^{\alpha}\,[\ell]^{\beta}\,[g]^{\gamma}\,, \tag{2.11}$$

say, taking into account the dimensions of m, ℓ, and g,

$$[T] \;=\; [M]^{\alpha}\,[L]^{\beta+\gamma}\,[T]^{-2\gamma}\,. \tag{2.12}$$

The principle of dimensional homogeneity demands that

$$\alpha = 0 \,, \quad \beta + \gamma = 0 \,, \quad \gamma = -1/2 \,, \tag{2.13}$$

whence

$$T \;=\; C\,\sqrt{\ell/g}\,, \tag{2.14}$$

where C is a dimensionless constant. It is important to note that one cannot determine, by dimensional analysis, the possible dependence of the period on the amplitude θ_0 (dimensionless), nor the value of the constant C.

Actually, the period of a pendulum can easily be determined, including the value of the dimensionless constant C and the dependence on the amplitude θ_0, by properly solving the equation of motion. The foregoing application of dimensional analysis to the determination of the period of a pendulum has thus mainly didactical interest.

In the case of very complex physical systems, however, for which a complete theory does not exist or is exceedingly complicated (such as in some problems of fluid dynamics), dimensional analysis can represent a very useful tool.

Physical Similitude

Large complex systems are often studied with the help of models in reduced scale (e.g., in applications of hydraulic and aeronautic engineering, elasticity, and heat transmission). A basic problem is to evaluate how the scale reduction affects the different properties of the model with respect to the real system. For example, a reduction of linear dimensions by a factor $L = 100$ produces a reduction of volumes by a factor 10^6, which corresponds to a reduction of masses by the same factor 10^6, if the densities are the same in the model and in the real system.

Dimensional analysis is of great help in this task, because it accounts for the influence of the reduction of base units on the values of derived quantities. In particular, dimensionless quantities, such as relative density or more specialized quantities (e.g., the Reynolds number or the Mach number) are very effective, because their values do not depend on scale factors.

Problems

2.1. The unit of mechanical work is the joule in the International System, and the erg in the cgs systems. Calculate the conversion factor between joule and erg.

2.2. The measure of a plane angle in degrees is $\theta = 25^\circ\, 7'\, 36''$. Express the measure of the angle in radians.

2.3. The mass of an iron atom is 55.847 atomic mass units (average value of the masses of the different isotopes, weighted by their natural abundance). Calculate the mass, in kilograms, of three moles of iron atoms.

2.4. The interaction between the two atoms of a hydrogen molecule H_2 can be assimilated, to a good approximation, to an elastic harmonic spring. The potential energy is $E_p(x) = kx^2/2$, where $x = r - r_0$ is the deviation of the instantaneous distance r from the equilibrium distance $r_0 = 0.74\,\text{Å}$, and k is the elastic constant, whose value is $k = 36\,\text{eV}/\text{Å}^2$.

In classical mechanics, the elastic constant of a spring is the ratio between force and deformation, $k = -F_e/x$. Express the elastic force k of the hydrogen molecule in the SI unit, newton/meter.

The frequency of oscillation is $\nu = (k/\mu)^{1/2}/2\pi$, where $\mu = 2m$ is the reduced mass of the molecule, and m is the mass of the hydrogen atom. Taking into account that a hydrogen atom contains one proton and one electron, calculate its mass m (using Table C.2 of Appendix C.2), and verify that the frequency of oscillation is $\nu = 1.3 \times 10^{14}\,\text{Hz}$.

2.5. *Dimensional analysis.* The gravitational interaction between two masses m_1 and m_2 gives rise to an attractive force, proportional to the product of the masses and inversely proportional to the square of their distance: $F \propto m_1 m_2/r^2$. Verify that the equation $F = m_1 m_2/r^2$ is dimensionally wrong.

The right expression is $F = G m_1 m_2/r^2$, where G is the gravitational constant; determine the dimensions of G.

Springer
http://www.springer.com/978-0-387-78649-0

The Uncertainty in Physical Measurements
An Introduction to Data Analysis in the Physics Laboratory
Fornasini, P.
2008, XII, 289 p., Hardcover
ISBN: 978-0-387-78649-0