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(*********************************************************************)
(*                                                                   *)
(* Generalized Collocation Methods:Solutions to Nonlinear Problems *)
(* Bellomo,N.,Lods,B.,Revelli,R.,Ridolfi,L.                          *)
(* A Birkhäuser book                                                 *)
(* ISBN:978-0-8176-4525-0                                            *)
(*                                                                   *)
(* Program DiffNeu                                                   *)
(*                                                                   *)
(*********************************************************************)

$TextStyle = {FontFamily -> "Times", FontSize → 12};
Off[General::"spell", General::"spell1"]

DiffNeu[InitBoundData_, Nodes_, Type_] := Module[{},

  (*** INITIAL CONDITION ***)

  φ[x_] := InitBoundData[[2]];

  hh = 1/(Nodes - 1);

  Which[Type === Sinc, x1_i_ := (i - 1) * hh,
   Type === Lagrange, x1_i_ := -1/2. (Cos[(i - 1) * π/(Nodes - 1)] - 1)];

  If[Type === Lagrange, Lagr[j_, x_] := ∏_{p=1}^{Nodes} (If[p ≠ j, (x - x1_p)/(x1_j - x1_p), 1]);

   InitValue[x_] := ∑_{k=1}^{Nodes} (φ[x] /. x → x1_k) * Lagr[k, x]];

  If[Type === Sinc, Sinc[j_, x_] := Which[0 ≤ x ≤ 1 && x ≠ (j - 1) * hh,
     Sin[π * (x - (j - 1) * hh)/hh]/(π * (x - (j - 1) * hh)/hh), x == (j - 1) * hh, 1];

   InitValue[x_] := ∑_{k=1}^{Nodes} (φ[x] /. x -> x1_k) * Sinc[k, x]];

  Print["Initial condition interpolation:"];
  Plot[Evaluate[{φ[x], InitValue[x]}], {x, 0, 1},
   PlotStyle → {GrayLevel[0], Dashing[{0.15, 0.05}]}, AxesLabel → {x, u[x, 0]}];

  Print["Don't worry! I'm working"];
  (*** DERIVATIVE MATRICES ***)

  If[Type === Lagrange,

   FDM[j_, i_] := Which[i == j, ∑_{k=1}^{Nodes} If[k == i, 0, 1/(x1_i - x1_k)], i != j,

     (∏_{p=1}^{Nodes} (If[p == i, 1, x1_i - x1_p]))/((x1_i - x1_j) ∏_{p=1}^{Nodes} (If[p == j, 1, x1_j - x1_p]))];

   FirstDer = Table[FDM[j, i], {j, 1, Nodes}, {i, 1, Nodes}];
   SecondDer =
    Table[If[i ≠ j, 2 (FirstDer[[j, i]] * FirstDer[[i, i]] - FirstDer[[j, i]]/(x1_i - x1_j)), 0],
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   {j, 1, Nodes}, {i, 1, Nodes}];
 Do[SecondDer[[i, i]] = -∑_{k=1}^{Nodes} SecondDer[[k, i]], {i, 1, Nodes}]];
If[Type === Sinc,
 FDM[j_, i_] := Which[i != j, (-1)^(i-j) / (hh * (i - j)), i == j, 0];
 SDM[j_, i_] := Which[i != j, 2 * (-1)^(i-j+1) / (hh^2 * (i - j)^2), i == j, -π^2 / (3 * hh^2)];
 FirstDer = Table[FDM[j, i], {j, 1, Nodes}, {i, 1, Nodes}];
 SecondDer = Table[SDM[j, i], {j, 1, Nodes}, {i, 1, Nodes}]];

(*** Nodal Equations ***)
NdEq[i_] := C_i'[t] == C_i[t] (1 - C_i[t]) (∑_{k=1}^{Nodes} SecondDer[[k, i]] * C_k[t]) +
    (1 - 2 C_i[t]) (∑_{k=1}^{Nodes} FirstDer[[k, i]] * C_k[t])^2 /. x -> x1_i;
γ = InitBoundData[[1]];
δ = InitBoundData[[3]];

(******* Determination of u_1 e u_n *******)
aa = FirstDer[[1, 1]];
bb = FirstDer[[Nodes, 1]];
dd = FirstDer[[1, Nodes]];
ee = FirstDer[[Nodes, Nodes]];
cc = γ - ∑_{k=2}^{Nodes-1} FirstDer[[k, 1]] * C_k[t];
ff = δ - ∑_{k=2}^{Nodes-1} FirstDer[[k, Nodes]] * C_k[t];

{u_1, u_Nodes} = {(cc ee - bb ff) / (-bb dd + aa ee), (cc dd - aa ff) / (bb dd - aa ee)};

(******* End of determination of u_1 e u_n *******)

EqsSys = Flatten[Table[{NdEq[i] /. {C_1[t] -> u_1, C_Nodes[t] -> u_Nodes},
    C_i[0] == (φ[x] /. x → x1_i)}, {i, 2, Nodes - 1}]];
NdEqsSys = Join[EqsSys];
UnKnown = Join[Table[C_i, {i, 2, Nodes - 1}]];
NumSol = NDSolve[NdEqsSys, UnKnown, {t, 0, 1}, MaxSteps → 10000000] // Flatten;

(*** Solution ***)
C1 = Evaluate[u_1 /. NumSol] * Lagr[1, x];
CNodes = Evaluate[u_Nodes /. NumSol] * Lagr[Nodes, x];
Which[Type === Sinc, TotSol[x_, t_] :=
  C1 + CNodes + ∑_{k=2}^{Nodes-1} NumSol[[k - 1, 2]][t] * Sinc[k, x], Type === Lagrange,
 TotSol[x_, t_] := C1 + CNodes + ∑_{k=2}^{Nodes-1} NumSol[[k - 1, 2]][t] * Lagr[k, x]];
Print["Solution:"];

Plot3D[Evaluate[TotSol[x, t]], {x, 0, 1}, {t, 0, 1}, PlotRange →
  {{0, 1}, {0, 1}, {-0.05, 0.4}}, AxesLabel → TraditionalForm /@ {x, t, "u(x,t)"},
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  Ticks → {{0, 1}, {0, 1}, {0.0, 0.4}}, PlotPoints → 101];
 ];
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Nodes = 21;
InitCond = 4 x^2 (x - 1)^2 ;
Bound0Cond = 0;
Bound1Cond = 0;
IBData = {Bound0Cond, InitCond, Bound1Cond};

DiffNeu[IBData, Nodes, Lagrange];
```