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(*********************************************************************)
(*                                                                   *)
(* Generalized Collocation Methods:Solutions to Nonlinear Problems *)
(* Bellomo,N.,Lods,B.,Revelli,R.,Ridolfi,L.                          *)
(* A Birkhäuser book                                                 *)
(* ISBN:978-0-8176-4525-0                                            *)
(*                                                                   *)
(* Program OneDLaSiErr                                               *)
(*                                                                   *)
(*********************************************************************)

<< Graphics`MultipleListPlot`

$TextStyle = {FontFamily -> "Times", FontSize → 12};
Off[General::"spell", General::"spell1"]

OneDLaSiErr[function_, nodes_, delta_] :=
 Module[{},
  φ[x_] := function;

  (*** Definition of the equispaced and Chebyshev collocation ***)

  hh = 1/(nodes - 1);
  x_i_ := (i - 1) * hh;
  x1_i_ := -1/2 (Cos[(i - 1) * π/(nodes - 1)] - 1);

  (*** Lagrange polynomial with Chebyshev collocation ***)

      Lagr1[j_, x_] := ∏_(p=1)^nodes (If[p ≠ j, (x - x1_p)/(x1_j - x1_p), 1]);

      FunctionLagr1[x_] := ∑_(k=1)^nodes ((φ[x] /. x → x1_k) * Lagr1[k, x]);

  (*** Sinc function ***)
      Sinc[j_, x_] := Which[0 ≤ x ≤ 1 && x ≠ (j - 1) * hh,
    Sin[π * (x - (j - 1) * hh)/hh]/(π * (x - (j - 1) * hh)/hh), x == (j - 1) * hh, 1];

      FunctionSinc[x_] := ∑_(k=1)^nodes (φ[x] /. x -> x_k) * Sinc[k, x];

  (*** Error Evaluation ***)

  ErrSinc = Table[Evaluate[Abs[φ[x] - FunctionSinc[x]]], {x, 0, 1, delta}];
  ErrLagr = Table[Evaluate[Abs[φ[x] - FunctionLagr1[x]]], {x, 0, 1, delta}];

  {MaxSinc, MaxLagr} = {Max[ErrSinc], Max[ErrLagr]};
  {MeanSinc, MeanLagr} = {Mean[ErrSinc], Mean[ErrLagr]};

  Print["Nodes ", " MaxSinc ", " MaxLagr ", " MeanSinc ", " MeanLagr "];
  Print[nodes, " ", MaxSinc, " ", MaxLagr, " ", MeanSinc, " ", MeanLagr];

  ErrorMax[nodes] = {nodes, MaxSinc, MaxLagr};
  ErrorMean[nodes] = {nodes, MeanSinc, MeanLagr};];

(*****************************************************)
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{function, MinNodes, MaxNodes, delta} = {Exp[-2 (10 x - 5)^2], 2, 41, 0.001};
For[i = MinNodes, i < MaxNodes, OneDLaSiErr[function, i, delta]; i = i + 1];

SincMaxEven = Table[{ErrorMax[j][[1]], ErrorMax[j][[2]]}, {j, MinNodes, MaxNodes, 2}];
LagrMaxEven = Table[{ErrorMax[j][[1]], ErrorMax[j][[3]]}, {j, MinNodes, MaxNodes, 2}];
SincMeanEven =
  Table[{ErrorMean[j][[1]], ErrorMean[j][[2]]}, {j, MinNodes, MaxNodes, 2}];
LagrMeanEven = Table[{ErrorMean[j][[1]], ErrorMean[j][[3]]},
   {j, MinNodes, MaxNodes, 2}];

SincMaxOdd =
  Table[{ErrorMax[j][[1]], ErrorMax[j][[2]]}, {j, MinNodes + 1, MaxNodes - 1, 2}];
LagrMaxOdd = Table[{ErrorMax[j][[1]], ErrorMax[j][[3]]},
   {j, MinNodes + 1, MaxNodes - 1, 2}];
SincMeanOdd = Table[{ErrorMean[j][[1]], ErrorMean[j][[2]]},
   {j, MinNodes + 1, MaxNodes - 1, 2}];
LagrMeanOdd = Table[{ErrorMean[j][[1]], ErrorMean[j][[3]]},
   {j, MinNodes + 1, MaxNodes - 1, 2}];

(*** Max  Error for even number of nodes ***)

p1a = ListPlot[SincMaxEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 1.05}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None}, PlotStyle → {GrayLevel[0]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];

p1b = ListPlot[LagrMaxEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 1.05}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];

 p1 = Show[{p1a, p1b}, DisplayFunction → $DisplayFunction];

(*** Max  Error for odd number of nodes ***)

p2a = ListPlot[SincMaxOdd, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 1.05}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None}, PlotStyle → {GrayLevel[0]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];
p2b = ListPlot[LagrMaxOdd, PlotJoined → True, PlotRange → {{-0.01, 40}, {-0.02, 1.05}},
   Frame → True, FrameTicks → {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02},
      {40, 40, 0.02}}, {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];
 p2 = Show[{p2a, p2b}, DisplayFunction → $DisplayFunction];
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(*** Mean  Error for even number of nodes ***)

p3a = ListPlot[SincMeanEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0]}, FrameLabel → TraditionalForm /@ {n, ℰ1^n},
   DisplayFunction → Identity];
 p3b = ListPlot[LagrMeanEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ1^n}, DisplayFunction → Identity];
 p3 = Show[{p3a, p3b}, DisplayFunction → $DisplayFunction];

(*** Mean  Error for odd number of nodes ***)

p4a = ListPlot[SincMeanOdd, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0]}, FrameLabel → TraditionalForm /@ {n, ℰ1^n},
   DisplayFunction → Identity];
 p4b = ListPlot[LagrMeanOdd, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ1^n}, DisplayFunction → Identity];
 p4 = Show[{p4a, p4b}, DisplayFunction → $DisplayFunction];

{function, MinNodes, MaxNodes, delta} =
  {Tanh[10 x - 5.] + 0.2 Sin[π (10 x - 5.)], 2, 41, 0.001};
For[i = MinNodes, i < MaxNodes, OneDLaSiErr[function, i, delta]; i = i + 1]

SincMaxEven = Table[{ErrorMax[j][[1]], ErrorMax[j][[2]]}, {j, MinNodes, MaxNodes, 2}];
LagrMaxEven = Table[{ErrorMax[j][[1]], ErrorMax[j][[3]]}, {j, MinNodes, MaxNodes, 2}];
SincMeanEven =
  Table[{ErrorMean[j][[1]], ErrorMean[j][[2]]}, {j, MinNodes, MaxNodes, 2}];
LagrMeanEven = Table[{ErrorMean[j][[1]], ErrorMean[j][[3]]},
   {j, MinNodes, MaxNodes, 2}];

SincMaxOdd =
  Table[{ErrorMax[j][[1]], ErrorMax[j][[2]]}, {j, MinNodes + 1, MaxNodes - 1, 2}];
LagrMaxOdd = Table[{ErrorMax[j][[1]], ErrorMax[j][[3]]},
   {j, MinNodes + 1, MaxNodes - 1, 2}];
SincMeanOdd = Table[{ErrorMean[j][[1]], ErrorMean[j][[2]]},
   {j, MinNodes + 1, MaxNodes - 1, 2}];
LagrMeanOdd = Table[{ErrorMean[j][[1]], ErrorMean[j][[3]]},
   {j, MinNodes + 1, MaxNodes - 1, 2}];
```

```
(*** Max  Error for Sinc ***)

p1a = ListPlot[SincMaxEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 1.05}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None}, PlotStyle → {GrayLevel[0]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];
 p1b = ListPlot[SincMaxOdd, PlotJoined → True, PlotRange → {{-0.01, 40}, {-0.02, 1.05}},
   Frame → True, FrameTicks → {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02},
      {40, 40, 0.02}}, {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];
 p1 = Show[{p1a, p1b}, DisplayFunction → $DisplayFunction];

(*** Max  Error for Lagrange ***)

p2a = ListPlot[LagrMaxEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 1.05}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None}, PlotStyle → {GrayLevel[0]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];
 p2b = ListPlot[LagrMaxOdd, PlotJoined → True, PlotRange → {{-0.01, 40}, {-0.02, 1.05}},
   Frame → True, FrameTicks → {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02},
      {40, 40, 0.02}}, {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ^n}, DisplayFunction → Identity];
p2 = Show[{p2a, p2b}, DisplayFunction → $DisplayFunction];

(*** Mean  Error for Sinc ***)

p3a = ListPlot[SincMeanEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0]}, FrameLabel → TraditionalForm /@ {n, ℰ₁^n},
   DisplayFunction → Identity];
 p3b = ListPlot[SincMeanOdd, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ₁^n}, DisplayFunction → Identity];
 p3 = Show[{p3a, p3b}, DisplayFunction → $DisplayFunction];

(*** Mean  Error for Lagrange ***)

p4a = ListPlot[LagrMeanEven, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0]}, FrameLabel → TraditionalForm /@ {n, ℰ₁^n},
   DisplayFunction → Identity];
 p4b = ListPlot[LagrMeanOdd, PlotJoined → True,
   PlotRange → {{-0.01, 40}, {-0.02, 0.65}}, Frame → True, FrameTicks →
    {{{0, 0, .02}, {10, 10, .02}, {20, 20, .02}, {30, 30, .02}, {40, 40, 0.02}},
     {{-1, -1, .02}, {0, 0, .02}, {0.6, 0.6, 0.02}}, None, None},
   PlotStyle → {GrayLevel[0], Dashing[{Dash, Dash}]},
   FrameLabel → TraditionalForm /@ {n, ℰ₁^n}, DisplayFunction → Identity];
 p4 = Show[{p4a, p4b}, DisplayFunction → $DisplayFunction];
```