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(*****)
(* *)
(* Generalized Collocation Methods: Solutions to Nonlinear Problems *)
(* Bellomo, N., Lods, B., Revelli, R., Ridolfi, L. *)
(* A Birkhäuser book *)
(* ISBN: 978-0-8176-4525-0 *)
(* *)
(* Program TwoDLaSiInt *)
(* *)
(*****)

$TextStyle = {FontFamily -> "Times", FontSize -> 12};
Off[General::"spell", General::"spell1"]

TwoDLaSiInt[function_, NodesX_, NodesY_] := Module[{},
  (** INITIAL CONDITION **)
   $\phi[x_, y_] := \text{function};$ 
  {hhX, hhY} = { $\frac{1}{\text{NodesX} - 1}$ ,  $\frac{1}{\text{NodesY} - 1}$ };
  xi_ := (i - 1) * hhX;
  yi_ := (i - 1) * hhY;
  x1i_ :=  $-\frac{1}{2} \left( \cos \left[ (i - 1) * \frac{\pi}{\text{NodesX} - 1} \right] - 1 \right);$ 
  y1i_ :=  $-\frac{1}{2} \left( \cos \left[ (i - 1) * \frac{\pi}{\text{NodesY} - 1} \right] - 1 \right);$ 

  (** Lagrange polynomial definition **)

   $\text{Lagr1X}[j_, x_] := \prod_{p=1}^{\text{NodesX}} \left( \text{If}[p \neq j, \frac{x - x1_p}{x1_j - x1_p}, 1] \right);$ 

   $\text{Lagr1Y}[j_, y_] := \prod_{p=1}^{\text{NodesY}} \left( \text{If}[p \neq j, \frac{y - y1_p}{y1_j - y1_p}, 1] \right);$ 

  FunctionLagr1[x_, y_] :=
     $\sum_{m=1}^{\text{NodesY}} \sum_{k=1}^{\text{NodesX}} ((\phi[x, y] /. \{x \rightarrow x1_k, y \rightarrow y1_m\}) * \text{Lagr1X}[k, x] * \text{Lagr1Y}[m, y]);$ 

  (** Sinc function definition **)

   $\text{SincX}[j_, x_] := \text{Which}[0 \leq x \leq 1 \ \&\& \ x \neq (j - 1) * hhX,$ 
     $\sin \left[ \frac{\pi * (x - (j - 1) * hhX)}{hhX} \right] / \left( \frac{\pi * (x - (j - 1) * hhX)}{hhX} \right), x == (j - 1) * hhX, 1];$ 
   $\text{SincY}[j_, y_] := \text{Which}[0 \leq y \leq 1 \ \&\& \ y \neq (j - 1) * hhY,$ 
     $\sin \left[ \frac{\pi * (y - (j - 1) * hhY)}{hhY} \right] / \left( \frac{\pi * (y - (j - 1) * hhY)}{hhY} \right), y == (j - 1) * hhY, 1];$ 

  FunctionSinc[x_, y_] :=
     $\sum_{m=1}^{\text{NodesY}} \sum_{k=1}^{\text{NodesX}} (\phi[x, y] /. \{x \rightarrow x_k, y \rightarrow y_m\}) * \text{SincX}[k, x] * \text{SincY}[m, y];$ 

  Plot3D[Evaluate[FunctionSinc[x, y]], {x, 0, 1}, {y, 0, 1}, PlotRange -> All,
    AxesLabel -> TraditionalForm /@ {x, y, "w(x,y)"}, Ticks -> {{0, 1}, {0, 1}, {0, 1}},
    PlotLabel -> "Sinc Interpolation", PlotPoints -> 51];

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Plot3D[Evaluate[FunctionLagrl[x, y]], {x, 0, 1}, {y, 0, 1}, PlotRange → All,
  AxesLabel → TraditionalForm /@ {x, y, "w(x,y)"}, Ticks → {{0, 1}, {0, 1}, {0, 1}},
  PlotPoints → 51, PlotLabel -> "Lagrange Interpolation"];

Plot3D[Evaluate[100000 * Abs[φ[x, y] - FunctionSinc[x, y]]],
  {x, 0, 1}, {y, 0, 1}, PlotRange → {{0, 1}, {0, 1}, {0, 1}},
  AxesLabel → TraditionalForm /@ {x, y, " $\mathcal{E}^n * 10^5$ "}, Ticks → {{0, 1}, {0, 1}, {0, 1}},
  PlotPoints → 51, PlotLabel → "Error (Sinc interpolation)"];

Plot3D[Evaluate[1000 * Abs[φ[x, y] - FunctionLagrl[x, y]]], {x, 0, 1},
  {y, 0, 1}, PlotRange → All, AxesLabel → TraditionalForm /@ {x, y, " $\mathcal{E}^n * 10^3$ "},
  Ticks → {{0, 1}, {0, 1}, {0, 4}}, PlotPoints → 51,
  PlotLabel → "Error (Lagrange interpolation)"]
];

{function, NodesX, NodesY} = {Exp[-25 (2 x - 1)2 - 25 (2 y - 1)2], 15, 15};
TwoDLaSiInt[function, NodesX, NodesY]

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