

```

(*****)
(* *)
(* Generalized Collocation Methods: Solutions to Nonlinear Problems *)
(* Bellomo, N., Lods, B., Revelli, R., Ridolfi, L. *)
(* A Birkhäuser book *)
(* ISBN: 978-0-8176-4525-0 *)
(* *)
(* Program OneDLaSiInt *)
(* *)
(*****)

OneDLaSiInt[function_, nodes_] :=
Module[{},
  φ[x_] := function;

  (** Definition of the equispaced and Chebyshev collocation **)

  hh =  $\frac{1}{nodes - 1}$ ;
  xi_ := (i - 1) * hh;
  x1i_ :=  $-\frac{1}{2} \left( \cos \left[ (i - 1) * \frac{\pi}{nodes - 1} \right] - 1 \right)$ ;

  (** Lagrange polynomial with Chebyshev collocation **)

  Lagr1[j_, x_] :=  $\prod_{p=1}^{nodes} \left( \text{If}[p \neq j, \frac{x - x1_p}{x1_j - x1_p}, 1] \right)$ ;

  FunctionLagr1[x_] :=  $\sum_{k=1}^{nodes} ((\varphi[x] /. x \rightarrow x1_k) * \text{Lagr1}[k, x])$ ;

  (** Sinc function **)
  Sinc[j_, x_] := Which[0 ≤ x ≤ 1 && x ≠ (j - 1) * hh,
    Sin[ $\frac{\pi * (x - (j - 1) * hh)}{hh}$ ] /  $\left( \frac{\pi * (x - (j - 1) * hh)}{hh} \right)$ , x == (j - 1) * hh, 1];

  FunctionSinc[x_] :=  $\sum_{k=1}^{nodes} (\varphi[x] /. x \rightarrow x_k) * \text{Sinc}[k, x]$ ;

  (** Derivative Values **)

  Initder1Lagr1[x_] :=  $\left( \sum_{k=1}^{nodes} ((\varphi[x] /. x \rightarrow x1_k) * D[\text{Lagr1}[k, x], x]) \right)$ ;

  Initder2Lagr1[x_] :=  $\left( \sum_{k=1}^{nodes} ((\varphi[x] /. x \rightarrow x1_k) * D[\text{Lagr1}[k, x], \{x, 2\}]) \right)$ ;

  Initder1Sinc[x_] :=  $\left( \sum_{k=1}^{nodes} ((\varphi[x] /. x \rightarrow x_k) * D[\text{Sinc}[k, x], x]) \right)$ ;

  Initder2Sinc[x_] :=  $\left( \sum_{k=1}^{nodes} ((\varphi[x] /. x \rightarrow x_k) * D[\text{Sinc}[k, x], \{x, 2\}]) \right)$ ;

  Plot[Evaluate[{φ[x], FunctionLagr1[x], FunctionSinc[x]}], {x, 0, 1}, PlotRange → All,
    PlotStyle → {Dashing[{0.0}], {Dashing[{0.001, 0.005}]}, {Dashing[{0.02, 0.008}]}}},
    FrameLabel → {"x", "v(x)"}, AxesOrigin → {0, -1.12}, Frame → True,
    FrameTicks → {{0, 0, .02}, {0.5, 0.5, .02}, {1, 1, 0.02}},

```

```

    {{-1, -1, .02}, {0, 0, .02}, {1, 1, 0.02}}, None, None}}];

Plot[Evaluate[{{ $\partial_x \varphi[x]$ }, Initder1Lagrl[x], Initder1Sinc[x]}], {x, 0, 1}, PlotRange →
  All, PlotStyle → {Dashing[{0.0}], Dashing[{0.001, 0.005}], Dashing[{0.02, 0.008}]},
  FrameLabel → {"x", " $v_x(x)$ "}, Frame → True,
  FrameTicks → {{0, 0, .02}, {0.5, 0.5, .02}, {1, 1, 0.02}},
    {{-10, -10, .02}, {0, 0, .02}, {10, 10, 0.02}}, None, None}}];

Plot[Evaluate[{{ $\partial_{x,x} \varphi[x]$ }, Initder2Lagrl[x], Initder2Sinc[x]}],
  {x, 0, 1}, PlotRange → {{-0.01, 1}, {-410, 410}},
  PlotStyle → {Dashing[{0.0}], Dashing[{0.001, 0.005}], Dashing[{0.02, 0.008}]},
  FrameLabel → {"x", " $v_{xx}(x)$ "}, Frame → True,
  FrameTicks → {{0, 0, .02}, {0.5, 0.5, .02}, {1, 1, 0.02}},
    {{-400, -400, .02}, {0, 0, .02}, {400, 400, 0.02}}, None, None}}];

];

{function, nodes} = {Exp[-2 (10 x - 5)2], 21};
OneDLaSiInt[function, nodes]

{function, nodes} = {Tanh[10 x - 5.] + 0.2 Sin[ $\pi$  (10 x - 5.)], 21};
OneDLaSiInt[function, nodes]

```