3.6 Multiplicity formula

The space L^2_ψ will be the sum of the elements of A_ψ with some multiplicities. More precisely, Arthur attached to ψ a quadratic character ϵ_ψ of S_ψ; we will not give the definition here, but will say what it is when we discuss the Saito–Kurokawa representations in Section 4. Now if η is an irreducible character of $S_{\psi,\mathbb{A}}$, we set

$$m_\eta = \langle \Delta^*(\eta), \epsilon_\psi \rangle_{S_\psi} = \frac{1}{\# S_\psi} \cdot \left(\sum_{s \in S_\psi} \epsilon_\psi(s) \cdot \eta(s) \right).$$

Then Arthur conjectures that

$$L^2_\psi = \bigoplus_\eta m_\eta \pi_\eta.$$

3.7 Inner forms

We should state that the above description of Arthur's conjecture is only accurate for split groups (though it can be extended to quasi-split groups naturally). For inner forms of a split group, some modifications are necessary; we indicate these briefly.

A global A-parameter ψ for a split group G is also an A-parameter for an inner form G' provided that ψ is relevant, i.e., its image is not contained in the Levi of an irrelevant parabolic subgroup. We saw above that the representations in the local packet for $G(F_v)$ are indexed by irreducible characters η_v of $Z_{\psi_v}/Z^0_{\psi_v} Z_{\hat{G}}$. We should think of η_v as a character of $Z_{\psi_v}/Z^0_{\psi_v}$ which is trivial on $Z_{\hat{G}}$.

Now the main modification for G' is that the representations in the local packet of $G'(F_v)$ should be indexed by (some of) the characters of $Z_{\psi_v}/Z^0_{\psi_v}$ which are *not trivial* on $Z_{\hat{G}}$, at least when G is an adjoint group.

The definition of the quadratic character ϵ_ψ does not change; thus ϵ_ψ is a quadratic character of Z_ψ which is trivial on $Z_{\hat{G}}$. For a representation π_η in the global A-packet, where η is an irreducible character of $Z_{\psi,\mathbb{A}}$, the multiplicity $m(\pi_\eta)$ is given by $\langle \epsilon_\psi, \Delta^*(\eta) \rangle_{Z_\psi}$. Note that this is non-zero only if $\Delta^*(\eta)$ is trivial on $Z_{\hat{G}}$. We do not know if this condition is automatic (though for the case of interest in these notes, it is).

3.8 The A-parameters of $PGSp_4$

Now we want to see more concretely what Arthur's conjecture says for $PGSp_4$. We first describe the A-parameters of $PGSp_4$.

We can first partition the set of A-parameters ψ of $PGSp_4$ according to the restriction of ψ to $SL_2(\mathbb{C})$. Recall that the Jacobson–Morozov theorem

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