
Preface

The *Theory of the Top* attained its great fame from both its monumental scope and its outstanding authors. In the early twentieth century, Felix Klein was known as a mathematician of world fame; Arnold Sommerfeld, Klein's disciple, had acquired his reputation as a rising star of theoretical physics. By 1910, when the final volume of this treatise was published, the names of Klein and Sommerfeld would signal to a student that a matter as complex as the top was presented in a most authoritative manner, from the perspective of both mathematics and physics. The work also stands out in other regards: by its sheer extent—four volumes comprising a total of almost a thousand pages—and by the time lag of about fifteen years between inception and completion. Klein himself regarded the final result as somewhat disjointed. Its “idiosyncratic disposition,” he reflected in 1922, may be understood only by taking into account the historic circumstances at its inception in 1895; the developments between the first and last parts derailed the project from its intended course, so that for the technical applications described in Volume IV “almost no use was made of the theoretical framework developed at the beginning” [Klein 1922, p. 659].

It seems appropriate, therefore, to recall the historical circumstances under which this treatise was conceived and pursued. Felix Klein was not only a renowned mathematician, but also an entrepreneurial and ambitious university professor striving for a broader acknowledgment of mathematics as a cultural asset. During the Wilhelmian Era, when Germany was struggling for recognition as a great power, cultural affairs were no longer innocent bystanders of national politics. Friedrich Althoff, a powerful reformer at the Prussian Ministry of Culture, attempted to form centers of excellence at certain universities [Brocke 1980]. Klein had become professor in Göttingen University in 1886.

After a few years of frustration and uncertain prospects, Klein persuaded Althoff that Göttingen would assume the desired rank only if rising stars like David Hilbert and Hermann Minkowski were called to the university as his colleagues. As a result of Klein's strategy, backed by the almighty Althoff, Göttingen became a mecca of mathematics [Rowe 1989].

But Klein's ambitions were not restricted to local affairs at his university. As a part of his attempts to gain widespread recognition for mathematics, he began to edit an *Encyclopedia of Mathematical Sciences*, an enterprise that lasted until the 1920s and encompassed, in addition to pure mathematics, a broad spectrum of mathematical applications to mechanics, physics, and astronomy. Klein also established contacts with the Association for the Advancement of Mathematical and Scientific Education, in order to gain influence on high school teaching of mathematics. Furthermore, he displayed considerable interest in the scientific training of engineers, which was traditionally the realm of technical universities, and therefore made Klein the enemy of engineering professors who regarded his tendencies as an unwelcome interference in their own affairs. In 1895, for example, Klein conceived a memorandum in which he suggested the foundation of a new institute in Göttingen University for the education of the "general staff" of technology, whereas the training of "front officers" could be left to the technical universities [Rowe 1989, p. 203].

Such was the broader context for the birth of the *Theory of the Top*. Under this title Klein announced a special lecture in the winter semester of 1895/96, addressed to high school teachers who wished to keep in touch with advanced mathematical subjects. In the preceding semester, Klein had held another special lecture for the same audience on *Elementary Geometry*. One of his assistants was charged with elaborating the manuscript of this lecture into a booklet, which Klein presented to the high school teachers association as a special gift by which he intended to prepare the ground for his further engagement in aiming at a general reform of high school teaching. By lecturing on the top in the winter of 1895/96, Klein attempted to demonstrate that his university teaching was not an ivory tower activity but had relations to technological as well as educational affairs. Like the *Elementary Geometry* of the preceding semester, the *Theory of the Top* was meant to be printed afterward as a small booklet and presented as a gift to his extramural clients. Klein remarked in an autobiographical note in 1913

that the *Top* was a tactic intended as a second dedicatory publication [Jacobs 1977, p. 18].

The course of subsequent events, however, precluded a smooth realization of these plans. Klein entrusted Arnold Sommerfeld, who had become his assistant in the autumn of 1894, with much more than the elaboration of this lecture. Sommerfeld had just accomplished his habilitation (the German ritual to acquire the right to lecture in a university) on the theory of diffraction [Sommerfeld 1896] and was busy with the elaboration of Klein's *Number Theory*, a lecture that Klein held in two parts in the winter of 1895/96 and summer of 1896 [Klein 1896a; Klein 1896b]. When Sommerfeld finally started to work on the theory of the top in the autumn of 1896, he did so without great enthusiasm. He worked on several projects at the same time, all of them related to one or another of Klein's activities, such as a register for the *Mathematische Annalen* (a journal edited by Klein) or a review article on partial differential equations for Klein's *Encyclopedia of Mathematical Sciences*. Furthermore, he discovered that the methods developed in his habilitation work proved to be more fertile than he had originally anticipated. Writing papers on his own research appeared more interesting to him than editing Klein's lecture on the theory of the top.

Although the first parts of the lecture advanced to the state of proof reading by the spring of 1897, its completion was dragging on. In March of 1897 Sommerfeld wrote to Klein that "the number of boundary value problems that I am able to solve by my extension of Thomson's mirror method is very considerable." He felt sure that Klein would appreciate the temporary neglect of the top, because his method for solving physical differential equations was completely in line with Klein's tendencies: "I hope you will enjoy it yourself. But I still have several days to do with it. If you could arrange for this work to be published soon in an English journal, such as the London Math. Soc. [*Proceedings of the London Mathematical Society*], I would be very happy." To please Klein he added some remarks about his elaboration of the theory of the top, but finally revealed that this had a rather low priority on his to-do list. "Unfortunately, I have to admit that in the meantime the top has been in the nonetheless very interesting 'sleeping top' state" [Sommerfeld 1897a].

Working under Klein must have been quite demanding. "I really cannot write to you each day," Sommerfeld once apologized to his fiancée Johanna Höpfner. "Klein's bullwhip is rather close behind me" [Sommerfeld 1897b]. At some point in 1897, Klein must have decided

to split the publication into several parts. Klein did not leave the theory of the top in the state in which he had presented it in his lecture of 1895/96. In the summer semester of 1896 he lectured on technical mechanics. In October and November of 1896, he chose the theory of the top as a theme for guest lectures at Princeton University [Klein 1897]. In view of Klein's goal of demonstrating the uses of mathematics to engineers at technical universities, he must have regarded it expedient to include more applied matters and charged Sommerfeld to work out the details.

Under these premises, the mathematical foundations as laid out in earlier lectures were published in 1897 as Volume I. As Sommerfeld prepared the subsequent volume, progress became slow because he struggled with problems that were the subject of controversial debates. "With regard to the equilibrium stability H.[adamard] does not go one step further than Lyapunov"; such remarks in the correspondence between Sommerfeld and Klein [Sommerfeld 1898a] illustrate that subjects like stability, dealt with in a chapter of the second volume, could easily give rise to new debates and prevent rapid publication. Nevertheless, Sommerfeld completed the second volume without much delay, so that it appeared just a year after the first volume in 1898. As the first reactions made evident, the more subtle parts of the book such as the chapter on stability provoked criticism: "I would have a number of remarks about your definition of stability," Heinrich Burkhardt commented after the appearance of Volume II. "But I would need a day or two to formulate them clearly and precisely, which I do not now have. It seems to me that in your definition stability is the rule, instability the exception . . . I tend to guess that all motions of the top are stable according to this definition, except those whose instability you have proven" [Burkhardt 1898].

Such reactions cautioned against rushing to publication—all the more because the plan for the remaining parts addressed subjects beyond the realm of mathematics proper: gyroscopic phenomena in geophysics, astronomy, and technology. Employing mathematical virtuosity in these fields was easier proclaimed than done. But Sommerfeld was not afraid to meet this challenge. For example, he corresponded extensively with the naval engineer Carl Diegel in Kiel, the German naval base, about the application of the theory to the gyroscopic guidance of torpedos [Diegel 1898]. In December of 1898 he wrote to Klein that "My letters to D.[iegel] have the extent of treatises. I may travel from Göttingen back to Cl.[austhal] via Kiel. In any case

this correspondence gives rise to a nice paragraph about ‘applications of theory in technology’” [Sommerfeld 1898b].

Since the autumn of 1897, Sommerfeld had been professor of mathematics at the mining academy in Clausthal, so that his communication with Klein had to occur via the exchange of letters and at occasional meetings, which further retarded the project. In addition, Klein persuaded Sommerfeld in 1898 to join his *Encyclopedia* project as an editor for the planned volumes on physics, a capacity that contributed to the derailment of Sommerfeld’s career as a mathematician; he transformed more and more from a mathematician into a theoretical physicist, a metamorphosis that is also reflected by the choice of his research papers at the turn of the century. “Unfortunately I had no time for the top,” he apologized in a letter to Klein in November 1899. “I have to get rid of my [paper on] X-rays before I can deal with something else” [Sommerfeld 1899]. Klein responded that he was “thinking of our top with silent sorrow” [Klein 1899]. In 1900, Sommerfeld exchanged his position as a professor of mathematics in Clausthal for a professorship of mechanics at the technical university in Aachen. Although this brought him into closer contact with technological applications of gyroscopic theory, it did not accelerate the publication of the pending volumes. “When I will have time to resume the top?” he responded to Klein’s urging in November 1900. “The entire next week there are examinations without interruption. I will hope for the best but promise nothing” [Sommerfeld 1900].

To cut a long story short, it took five years after the appearance of Volume II in 1898 before the third volume of the *Theory of the Top* was published, and seven more years before the fourth and final volume appeared in 1910. In the meantime, gyroscopic theory itself had advanced or was made the subject of other reviews. In 1907, for example, Klein admonished Sommerfeld to pay attention to a recent article of Paul Stäckel, who was writing on the top for the *Encyclopedia* [Klein 1907]. In the foreword to the fourth volume, dated April 1910, Klein and Sommerfeld had to admit that during this long time span “the unity of substance and manner of presentation was lost.” The loss of unity and coherence was caused not just by a turn from mathematical foundations to technological applications. Despite Sommerfeld’s close contacts with technology, his presentation of applied subjects in Volumes III and IV was written from the perspective of a mathematician and theoretical physicist, so that it did not really address engineering concerns. With regard to the technology of the gyrocompass, for

example, Sommerfeld admitted later that even in the part about the technical applications the text “nowhere addresses technical details” [Broelmann 2002, p. 138].

The *Theory of the Top*, therefore, is a strange monument of scientific literature from the turn of the nineteenth to the twentieth century: too heterogeneous to please one or another orientation, and yet outstanding in its scope and detail. Klein and Sommerfeld hoped that its versatility would be considered as a compensation for its “lack of systematicness.” In the end, they confessed that the top was for them what it had been already for many natural philosophers in the nineteenth century: a target of opportunity for “awakening the sense for true mechanics,” a “philosophical instrument.”

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The Theory of the Top. Volume I

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