Preface

Of all the great innovations and intellectual achievements of mankind there is nothing that rivals the invention of counting and discovery of the number system. The way in which this discovery led to the development of abstract higher mathematics is the least of its merits, compared to the universal fascination that the natural numbers hold for all people. Numbers are at the roots of magic, superstition, religion and science. Numerologists can interpret great historical and cosmic events, predict the future and explain human nature. Better informed, sophisticated people may frown upon and ridicule such claims, but the number of incidents that link numbers to physical effects is simply too large to ignore as mere coincidence. It is in cases like these that the more respectable *number theory* is substituted for numerology.

Although it is recognized as the most fundamental branch of mathematics, the vocabulary of number theory includes concepts such as prime number, perfect number, amicable number, square number, triangular number, pyramidal number, and even magic number, none of which sounds too scientific and may suggest a different status for the subject. Not surprisingly, number theory remains the pastime of amateurs and professionals alike – all the way from the great Gauss down. It may be claimed that abstract number theory is more lofty than mundane science, never to be degraded into a servant of physical theory. Even so, a constant stream of books rolls from the printing presses of the world, extolling the wonderful synergy that exists between Fibonacci numbers, the golden ratio and self-similar symmetry on the one hand, with works of art (e.g. Da Vinci), architecture (Parthenon), biological growth, classical music and cosmic structure, on the other.

Despite claims to the contrary some of the profound insights into the understanding of the world were directly inspired by numbers. The best known example is the realization by Pythagoras that harmonious music, produced by a stringed instrument, is dictated by a sequence of rational fractions, defined in terms of natural numbers. Armed with this insight he noted a parallel numerical regularity in the motion of heavenly bodies, could hear the *music of the spheres*, and concluded: *all is number*. Quite remarkably, modern astronomy has confirmed, but cannot explain, the numerical sequence that regulates the orbital motion of the planets in the solar system. The well-known, but often ridiculed, Bode–Titius law correctly predicted all planetary orbits, leaving a single gap, where the asteroid belt was subsequently discovered, and also predicted the correct orbit for the unknown planet Pluto. The rings of Saturn are found to obey the same law, which, however, is still treated by the academic world as no more than entertaining coincidence.

Another entertaining coincidence was discovered by the high-school teacher Johann Balmer, in the form of a simple numerical formula, which accounted for the spectrum of light emitted by incandescent atomic matter, that continued to baffle the physicists of the world. Thirty years later Niels Bohr, on the basis of Balmer's formula, managed to construct the first convincing model of the atom and introduced the quantum theory into atomic physics. The quantum mechanics that developed from Bohr's model managed to extend the Balmer formula into a complete description of atomic structure on the basis of five sets of (integer and half-integer) quantum numbers.

The quantum numbers held out the immediate promise of accounting for the most fundamental concept of chemistry, known as the *Periodic Table of the Elements*, described by one of its discoverers, Alexandre-Émile de Chancourtois, in the statement: *the properties of the elements are the properties of numbers*. Although the quantum-mechanical explanation of elemental periodicity was only partially successful, the scientific world stopped looking (1926) for the numbers of de Chancourtois, until the chance discovery of these numbers by the present authors (2001).

In the interim, the development of atomic theory had been prodigious and impressive. It saw the identification of atomic species, called isotopes or nuclides, not included in the periodic classification, and of antimatter, the mirror image of ordinary matter. Once the proper numerical basis of the periodic classification had been spotted, all the new forms of atomic matter now find their proper place in the extended periodic classification. *Number theory and the Periodicity of Matter* deals with this discovery, its background, significance and predictions. The consequences are enormous. It shows why periodicity cannot be fully described by the quantum theory of electrons. The role of protons and neutrons, the other stable sub-atomic particles, are of equal importance. Only by taking the number of all these particles (called nucleons) into account is it possible to rationalize many aspects of atomic and nuclear physics. These aspects include nuclear synthesis, cosmic abundance of nuclides, nuclear stability, radioactive decay, nuclear spin and parity, nuclear size and shape, details of neutron scattering and superconductivity. At this stage the discovery is at the same level as that of Balmer, with all the science that it promises to produce still in the future.

New ideas on the theme have been communicated at several international conferences and as postgraduate lecture series at the University of Pretoria, South Africa, and of Heidelberg, Germany. The single idea received most enthusiastically was the decisive role of algebraic number theory, the Fibonacci series, the Farey sequence and the golden ratio in shaping the grand periodic system in a closed topological space, best described by the geometry of a Möbius strip. That explains why the packing of nucleons in an atomic nucleus follows the same pattern as the arrangement of florets in a flower head and why only special elements turn superconducting on cooling.

The haunting question, which is constantly insinuated but never answered conclusively, concerns the nature of the natural numbers. Have they been invented or discovered by humans? In other words, do numbers have an independent existence outside of the human mind? The answer to this question is non-trivial and probably of decisive importance for the future development of both science and mathematics. At this stage it is not even clear where to look for understanding – the abstract or the mundane, or where the twain shall never meet.

The theme of this book is to explore the consequences of the serendipitous discovery that stable nuclides obey the same periodic law as the chemical elements; both laws are rooted in elementary number theory. The nature of the discovery is such that, from the related periodic structures that occur in the natural numbers, as well as atomic matter, fundamental details about the electronic configuration of atoms and the baryonic arrangement in atomic nuclei can be derived, without the use of higher mathematics. A high degree of self-consistency substantiates the basic thesis from internal evidence, without assumption. This self-consistency includes convergence of nucleon distribution to the golden ratio and a natural limit on the number of elements and nuclides which are stable against radioactive decay. On this basis all observed properties of atoms and atomic nuclei can be understood as characteristic of a number system defined on a closed interval.

The key to understanding of atomic matter through number theory exists therein that atoms consist of whole numbers of protons, neutrons and electrons. The ratio of protons to neutrons in any nuclide therefore is a simple rational fraction, and this quantity, in relation to mass number, is the important factor that determines the stability of nuclides against radioactive decay. For light nuclei the ratio is $Z/N = 1$ and, for heavy nuclides, it converges to the golden mean ($\tau = 1/\Phi$), as shown by geometrical construction.

The point of convergence is established by plotting a modular(4) set of rational fractions, ordered in Farey sequence. This infinite set contains a modular subset of points that represent stable nuclides of allowed nuclear composition along a set of 11 regularly spaced festoons. A second Farey sequence, defined by Fibonacci fractions on the interval $(1, \tau)$, defines the set of 11 curves. This procedure shows that the limiting golden ratio is approached as $Z \to 102$, $N \to 102\Phi$ (165) and $A \to 102\Phi^2$ (267), which matches experimental observation. The analysis is valid for all mod(4) sets of nuclides, totalling 264, which decomposes into 11 periods of 24.

Periodic laws in terms of atomic and neutron numbers are readily projected out from the general law. In the case of atomic number, four periodic laws that reflect different cosmic environments are obtained, and these are interpreted to define a mechanism of nuclear synthesis by α-particle addition. The neutron-based periodic law, for the first time, rationalizes the empirically derived magic numbers of nuclear physics and provides a rational basis for the analysis of nuclear properties, including spin and parity.

To understand the full impact of the discovery the reader should have a working knowledge of elementary number theory, the periodic table of the elements, introductory atomic physics and elementary cosmography. The layout of the book has been planned in accordance with these needs. The first chapter is an introductory summary of the main thesis, followed by a primer on number theory, and similar chapters on the periodic table and the distribution of matter in the universe. With all background material in place, subsequent chapters re-examine the main arguments in more detail and with more emphasis on the wider implications of the results.

The work is presented without any pretence to expose inadequacies in existing science or provide an alternative, more fundamental model description of any aspect of chemistry, physics or cosmography. It only seeks to highlight an amazing facility of number theory to throw new light, particularly on old chemistry and nuclear physics. This descrial is all the more remarkable when read with the following quotation from Michio Kaku [1]:

> [...]some mathematical structures, such as number theory (which some mathematicians claim to be the purest branch of mathematics), have never been incorporated into any physical theory. Some argue that this situation may always exist: Perhaps the human mind will always be able to conceive of logically consistent structures that cannot be expressed through any physical principle.

Our conclusions indicate a definite link between natural numbers and atomic structure, supported by irrefutable internal evidence. The parallel with conclusions reached by W.D. Harkins, almost a century ago, is of interest.

He displayed the stable nuclides, known at the time, before discovery of the neutron, as a function of proton/neutron ratio, converging to $Z/N \rightarrow 0.62$, apparently without recognizing this limit as the golden ratio. This golden ratio turns out to be pivotal for the understanding of emerging self-similar relationships between different forms of matter, ranging from the sub-atomic to the cosmic.

The basis of atomic periodicity in number theory was explored by extended discussions with Demetrius Levendis during a period of sabbatical leave that he spent with me at the University of Pretoria. This interaction led to the interpretation in terms of Farey sequences and demonstration of the equivalent roles of mass number and nuclear binding energy. Without his insight this work would not have been possible.

Many of the arguments reached maturity at the Ruprecht-Karls-University of Heidelberg where, as a visiting professor, I had the opportunity to explore these ideas with members of the Inorganic Chemistry Institute of Professor Peter Comba in a seminar series. Critical comments and technical assistance from many others, over several conferences, seminars and informal discussions are gratefully acknowledged. In this respect I must single out Sonke Adlung, Pari Antalis, Aloysio Janner, Tibor Koritsanszky, Gert Krynauw, Richard Lemmer, John Ogilvie, Zorka Papadopolos and Casper Schutte. All inaccuracies remain my sole responsibility.

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