2. Model of Knowledge Conflict

The aim of this chapter is to introduce the subject of conflict analysis, its representation, and tools useful in its resolution. These tools consist of consistency functions which measure the degree of the coherence of elements being in a conflict. Conflict is something we often have to deal with in our everyday life and in many kinds of computer systems. Because of its very rich and varied nature, in this book we investigate only one class of conflicts related to knowledge management.

2.1. Introduction

Conflict can be considered in many cases as inconsistency. Conflict, however, seems to have more a specific meaning than inconsistency. Inconsistency most often refers to states of resources (such as knowledge base, database, etc.) of one or more integral systems, whereas with a conflict we have in mind some inconsistency between a few autonomous unities. Thus conflict refers to a set of unities and a concrete matter. It then is suitable for the kind of inconsistency on distributed aspect analyzed in Chapter 1. Thus conflict is very often considered in distributed environments, in which several autonomous systems function and have to co-operate in order to realize some common tasks.

Let's consider a distributed system which is understood as a collection of independent computers connected to each other by a network and equipped with distributed software and data [27]. In such a kind of system independence is one of the basic features of the sites, meaning that within the confines of available resources each of them does independent data processing, and takes care of consistency of data. Most often each system site is placed in a region of the real world and its task relies on storing and processing information about this region. Regions occupied by the sites often overlap. This redundancy is needed for the following reasons.

- Uncertain and incomplete information (data) of the sites referring to regions they occupy.

- Entrusting one region to several sites may cause a complement of sites' information and owing to this it can enlarge the credibility of information-processing results.

However, the autonomy feature of sites and their indeterminism in information processing may cause disagreement (conflict) between sites referring to a common subject. Independent sites may generate different versions of information about the same real world. This in turn may cause trouble in information processing of the whole system.

As shown in Figure 2.1, for the same event in the real world four agents may generate different scenarios. Thus there arises a conflict profile:

Conflict profile = {Scenario 1, Scenario 1, Scenario 2, Scenario 3},

where Scenario 1 appears two times and each of Scenarios 2 and 3 appears only once.

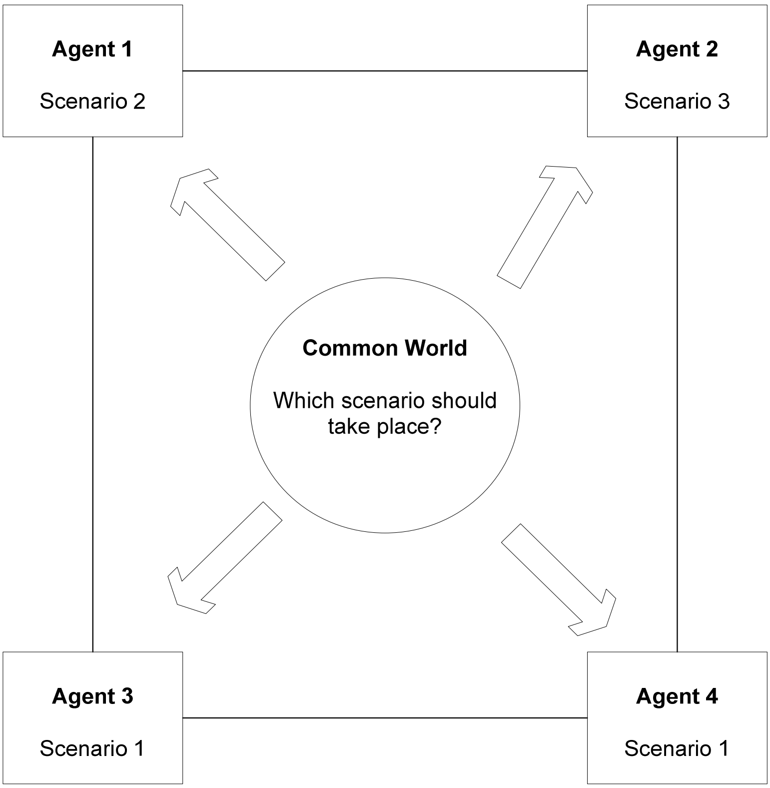

Figure 2.1. An example of conflict in an agent-based distributed environment.

Another, more practical example refers to a multiagent system serving intrusion detection in a network system [61, 72]. It is assumed that the network system consists of a set of sites. Two types of agents are designed: monitoring agents (MA) and managing agents (MaA). Monitoring agents observe the sites, process the captured information and draw conclusions that are necessary to evaluate the current state of system security. Managing agents are responsible for managing the work of monitoring agents (Figure 2.2). Each monitoring agent monitors its area consisting of several sites. If an attack appears, the agent must deduce the kind, source, and the path of propagation of this attack. It is assumed that the areas of the monitoring agents might mutually overlap. Owing to such an assumption the credibility of monitoring agents' results could be high, the sources of attacks might be more quickly established, and the risks of security breach of the main sites might be reduced. Each managing agent manages a set of monitoring agents.

The deduction results of monitoring agents referring to the same site may be inconsistent, meaning that for the same attack and the same site different agents may diagnose different sources of attack, propagation paths, and so on. In other words, they may be in conflict.

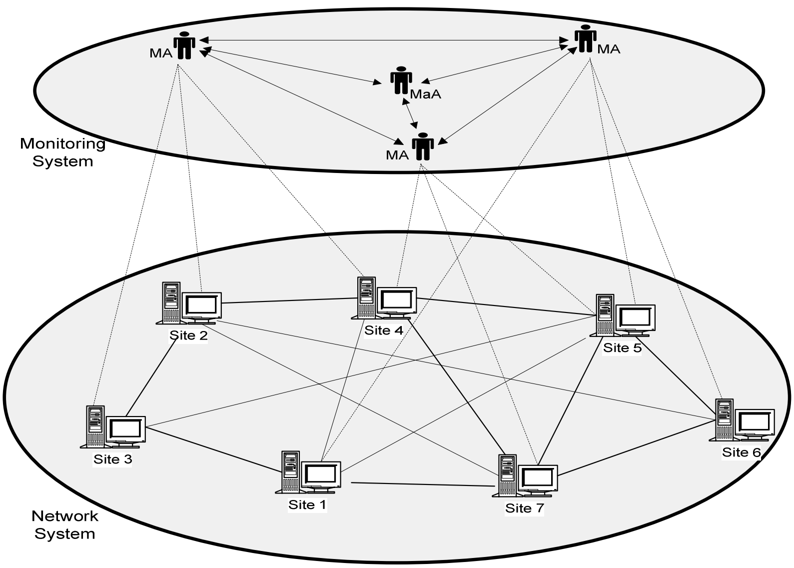

Figure 2.2. A multiagent system for intrusion detection.

The above examples show that conflict situations take place when for the same subject several sites cooperating in a distributed environment may generate different versions of data (e.g., different scenarios of a future event or different solutions of the same problem). In this chapter we deal with the definition of conflict and tools for its analysis. We restrict our consideration to conflict of knowledge and propose a formal model for its representation. We also define and analyze consistency measures of conflict profiles. Consistency is a very important parameter of conflict profiles, and its value could say some essential things about the conflict.

2.2. What is Conflict?

In the most general way one can say that a conflict takes place when at least two bodies have different opinions on the same subject. In a general context we can distinguish the following components of a conflict [104]:

- *Conflict body*: a set of participants of the conflict.
- *Conflict subject*: a set of matters which are occupied by the participants.
- *Conflict content*: a set of opinions of the participants on the conflict subject. These opinions represent the knowledge states of the participants on this subject.

Conflict has been a popular subject in such sciences as psychology and sociology. In computer science conflict analysis has been more and more needed because of using autonomous programs and processing data knowledge originating from these sources. The first formal model for conflict has been proposed by Pawlak [133]. Referring to the above-mentioned parameters the Pawlak conflict can be specified as follows.

- *Conflict body*: a set of agents.
- *Conflict subject*: a set of issues.
- *Conflict content*: a set of tuples representing the opinions of these agents on these issues. Each agent referring to each issue has three possibilities for presenting his opinion: (+), yes; (–), no; and (0), neutral.

For example [133], if there are five agents (#1, #2, #3, #4, #5) and five issues (*a*, *b*, *c*, *d*, *e*) then the opinion of an agent on these issues may be represented by a row in the following information table.

Agent	*a*	*b*	*c*	*d*	*e*
#1	–	–	+	+	+
#2	+	0	+	–	–
#3	+	–	+	–	0
#4	0	–	–	0	–
#5	+	–	–	–	–

Pawlak has created a set of tools that enable conflict analysis. This set consists of such elements as relations of coalition, neutrality, and conflict. Pawlak has used neither distance functions for measuring the similarity of agents' opinions nor the method for determination of conflict solution. In addition, Pawlak's model is very simple and does not allow the agents to express more complex opinions. As stated above, referring to a matter an agent has only three possibilities for expressing her opinion: *approval*, *objection*, and *neutrality*. An enhancement of Pawlak's model has been proposed by Skowron and Deya [144]. In this work the authors define local states of agents which may be interpreted as the sources of their opinions expressed referring to the matters in contention. This model considers conflicts on several levels, among others on the level of reasons of conflicts. In this approach it is still assumed that attribute values are atomic.

In this chapter we define conflicts in distributed environments with the above-mentioned parameters. We build a formalism which has the following purposes:

- Conflict can be defined on a general level.
- It is possible to calculate the inconsistency degree of conflict, and within conflict values of the attributes representing participants' opinions should more precisely describe these opinions. We realize this aim with the assumption that values of attributes representing conflict contents are not atomic as in Pawlak's approach, but sets of elementary values where an elementary value is not necessarily an atomic one. Thus we accept the assumption that attributes are multivalued, similar to Lipskis's [79] and Pawlak's [132] concepts of multivalued information systems.
- We introduce three kinds of conflict of participants' knowledge: positive, negative, and ignorance. Positive knowledge serves to express such types of opinion as "In my opinion it should be," negative knowledge, "In my opinion it should not be," and uncertain knowledge, "I have no basis to state if it should be." For example, an expert is asked to forecast the increase of GDP of a country. She can

give her three kinds of opinions: an interval to which the increase should most likely belong, an interval (intervals) to which the increase should not belong, and an interval (intervals) to which she does not know if the increase may belong or not. An example of the expert's opinion is presented as follows.

Should be	Should not be	Uncertain
[3, 5]	$(-\infty, 2], (10, +\infty)$	(5,10]

- Owing to this assumption a conflict participant functioning in some real world and having limited possibilities does not have to know "everything". In Pawlak's approach positive knowledge is represented by value "+", and negative knowledge by value "–". Some difference occurs between the semantics of Pawlak's "neutrality" and the semantics of "uncertainty" presented in this work. Namely, most often neutrality appears in voting processes and does not mean uncertainty, whereas uncertainty means that an agent or expert is not competent to express its opinions on some matter.
- We define the inconsistency level for conflict.
- Weights of conflict participants are taken into account in determining the inconsistency level.
- Criteria for susceptibility to consensus are defined for conflict.

In the next section we present a general model for conflict.

2.3. Conflict Representation

2.3.1. Basic Notions

In this section we present some notions needed for use in the next sections. In the context of the components of conflict defined in Section 2.2 (conflict body, subject, and content), we assume in this model that the conflict subject is given and for this subject the set of all potential opinions which may be included in the conflict content may be determined. We investigate the properties of behaviour of different cases of conflict content.

By U we denote a finite set of objects representing the potential opinions for the conflict subject. Symbol 2^U denotes the powerset of U, that is, the set of all subsets of U.

By $\Pi_k(U)$ we denote the set of all k-element subsets (with repetitions) of set U for $k \in \aleph$ ($\aleph$ is the set of natural numbers), and let

$$\Pi(U) = \bigcup_{k\in\aleph} \Pi_k(U).$$

Thus $\Pi(U)$ is the set of all nonempty subsets with repetitions of set U.

Each element of set $\Pi(U)$ is called a *conflict profile*. A conflict profile is then a subset with repetitions of set U and should represent a conflict content.[1]

In this work we do not use the formalism often used in consensus theory [13] in which the domain of consensus is defined as $U^* = \bigcup_{k>0} U^k$, where U^k is the k-fold Cartesian product of U. In this way we can specify how many times an object occurs in a profile and we can ensure that the order of objects belonging to a profile is not important. We also use in this book the algebra of sets with repetitions (multisets) defined by Lipski and Marek [80]. Some of its elements are presented by the following examples.

An expression

$$X = \{x, x, y, y, y, z\}$$

is called a set with repetitions with cardinality equal to 6. In this set element x appears two times, y three times, and z one time. Set X can also be written as

$$X = \{2 * x, 3 * y, 1 * z\}.$$

The sum of sets with repetitions is denoted by the symbol $\dot{\cup}$ and is defined in the following way. If element x appears in set X n times and in Y n' times, then in their sum $X \dot{\cup} Y$ this element should appear $n + n'$ times. For example, if $X = \{2 * x, 3 * y, 1 * z\}$ and $Y = \{4 * x, 2 * y\}$, then

$$X \dot{\cup} Y = \{6 * x, 5 * y, 1 * z\}.$$

The difference of sets with repetitions is denoted by the symbol "–" and is defined in the following way. If element x appears in set X n times and in Y n' times, then the number of occurrences of x in their difference $X - Y$ should be equal to $n - n'$ if $n \geq n'$, and 0 otherwise. For example:

$$\{6 * x, 5 * y, 1 * z\} - \{2 * x, 3 * y, 1 * z\} = \{4 * x, 2 * y\}.$$

[1] In this work sets with repetitions refer only to conflict profiles, and any symbol representing a conflict profile also represents a set with repetitions.

A set X with repetitions is a subset of a set Y with repetitions ($X \subseteq Y$) if each element from X does not have a greater number of occurrences than it has in set Y. For example,

$$\{2 * x, 3 * y, 1 * z\} \subseteq \{2 * x, 4 * y, 1 * z\}.$$

Set U can have two kinds of structure: the *macrostructure* and the *microstructure*. By a macrostructure of U we understand some relationship between its elements, for example, a binary relation on U, or some relationship between its elements and elements of another set, for example, some function from $U \times U$ to another set. In this work as the macrostructure of the set U we assume a distance function:

$$d: U \times U \to [0, 1],$$

which is

1. *Nonnegative*:

$$\forall x,y \in U: d(x,y) \geq 0,$$

2. *Reflexive*:

$$\forall x,y \in U: d(x,y) = 0 \text{ iff } x = y, \text{ and}$$

3. *Symmetrical*:

$$\forall x,y \in U: d(x,y) = d(y,x),$$

where [0, 1] is the closed interval of real numbers between 0 and 1.

Thus function d is a half-metric because the above conditions include only a part of the metric conditions. Here there is the lack of a transitive condition because it is too strong for many practical situations [20].

By a microstructure of U we understand the structure of the elements of U. For example, if U is the set of all partitions of some set X then the microstructure of U is a partition of X, and as the macrostructure of U we can define a distance function between partitions of X.

Let us notice that a space (U, d) defined in the above way does not need to be a metric space. Therefore, we call it a *distance space* [103].

We distinguish the following classes of conflict profiles.

Definition 2.1.
A conflict profile $X \in \Pi(U)$ is called:

(a) Homogeneous, if all its elements are identical;, that is, $X = \{n * x\}$ for some $x \in U$ and n being a natural number

(b) Heterogeneous, if it is not homogeneous

(c) Distinguishable, if all its elements are different from each other

(d) Multiple, referring to a profile Y (or X is a multiple of Y, written as $X = n * Y$) if

$$Y = \{x_1, x_2, \ldots, x_k\}$$

and

$$X = \{n * x_1,\ n * x_2, \ldots, n * x_k\}$$

for k and n being natural numbers and $n > 1$

(e) Regular, if it is distinguishable or a multiple of some distinguishable profile

As defined above, a conflict profile $X \in \Pi(U)$ contains opinions of conflict participants on the given matter of contention. It should be the main subject for conflict analysis in order to solve it. In fact, if the profile is homogeneous then there is no conflict inasmuch as all the opinions are identical. However, for completeness of the theory, we assume that it is a special case of conflict.

2.3.2. Definition of Knowledge Conflict

We assume that there is given a finite set A of agents which work in a distributed environment. The term "agent" is used here in a very general sense: as an agent we may understand an expert or an intelligent and autonomous computer program. We assume that these agents have their own knowledge bases in which knowledge states can be distinguished. In general, by a state of agent knowledge we understand these elements of an agent knowledge base which reflect the state of the real world occupied by the agent referring to a given timestamp. Such a state may be treated as a view or an opinion of the agent on some matter. We realize that the structures of agent knowledge bases may be differentiated from each other. However, in this chapter we do not deal with them; this is the subject of the next chapters where we define concrete structures (logical or relational) of agent knowledge. We assume that there is a common platform for presenting the knowledge states of all agents.

We assume that the agents from set A work on a finite set of common subjects (matters) of their interest. This set is denoted by S.

Now by U we denote the set of all possible states of agent knowledge presented in the platform. Owing to this assumption two different states belonging to U should have different "content".

An agent $a \in A$ referring to subject (matter) $s \in S$ can generate the following kinds of knowledge:

- *Positive knowledge*: a state $u \in U$ is called the positive knowledge of agent a referring to subject s if in the opinion of the agent state u is the most proper description (alternative, scenario, etc.) related to subject s.
- *Negative knowledge*: a state $u \in U$ is called the negative knowledge of agent a referring to subject s if in the opinion of the agent state u cannot be the proper description (alternative, scenario, etc.) related to subject s.
- *Uncertain knowledge*: a state $u \in U$ is called the uncertain knowledge of agent a referring to subject s if it does not know if state u can be the proper description (alternative, scenario, etc.) related to subject s.

Thus positive knowledge represents the kind of agent opinions that something should take place, whereas by means of negative knowledge an agent can express its contrary opinion. Notice that for the same agent the state representing positive knowledge must be different from the state representing its negative knowledge.

In this way for a subject $s \in S$ we can define the following profiles:

- *Positive profile*: $X^+(s)$ – as the set of knowledge states from U representing positive knowledge of the agents referring to subject
- *Negative profile*: $X^-(s)$ – as the set of knowledge states from U representing negative knowledge of the agents referring to subject s
- *Uncertain profile*: $X^\pm(s)$ – as the set of knowledge states from U representing uncertain knowledge of the agents referring to subject s

Positive, negative, and uncertain profiles referring to subject s should satisfy the following conditions:

- They are pairwise disjoint.
- They are sets with repetitions because of the fact that some agents may generate the same knowledge state.

Now we can present the definition of knowledge conflict.

Definition 2.2.
A knowledge conflict referring to subject s appears if at least one of profiles $X^{+}(s)$ *and* $X^{-}(s)$ *is heterogeneous.*

From Definition 2.1 it follows that a conflict takes place if at least two agents generate different (positive or negative) knowledge states referring to the same subject. Notice that in this definition there is no reference to the uncertain profile $X^{\pm}(s)$. The reason is that the role of uncertain knowledge is not as important as the role of the two remaining kinds of knowledge. If referring to a subject the agents have identical positive and negative knowledge then although their states of uncertainty are different, then there is no conflict.

Sets $X^{+}(s)$ and $X^{-}(s)$ are also called *positive conflict profile* and *negative conflict profile*, respectively.

Below we present an example.

Example 2.1. Consider a group of experts who have to analyze the economic situation of a country and forecast its increase of GDP in a given year. An expert may generate an interval to which in his opinion the increase of GDP is most likely to belong. He may also give some other intervals to which in his opinion the increase of GDP should not belong. As a knowledge state we define a subset of the set of real numbers. In the following table the opinions of five experts are presented.

Expert	X^{+}	X^{-}	$X^{\pm}$
E_1	[3, 5]	(–∞, 3), (5, +∞)	∅
E_2	[2, 6]	(–∞, 2), (6, 8)	[8, +∞)
E_3	4	[1, 3], (7, +∞)	(–∞, 1), (3,4), (4,7]
E_4	[3, 5]	(–∞, 3), (5, +∞)	∅
E_5	[3, 5]	(–∞, 3), (10, +∞)	(5,10]

In this way we have a conflict because the profiles X^{+} and X^{-} are not homogeneous. Notice that the opinions of some experts (E_2, E_3, E_5) do not exhaust the set of all real numbers. This means they can have some ignorance. For example, expert E_5 does not know if the increase of GDP may belong to interval (5, 10] or not. ♦

2.3.3. Credibility Degree of Conflict Participants

In a conflict the roles of its participations (i.e., the agents) do not have to be the same. The need for differentiating agent roles follows from the fact that one agent may be more credible than another. Credibility of an agent or an expert in many cases should be a multidimensional value. For simplicity here we assume it to be a one-dimensional value. This value should play the following roles.

- It represents the competence of the agent.
- It represents the credibility of the agent.

We define this value by means of the following function,

$$w: A \times S \to [0,1],$$

where for a given agent a and a subject s value $w(a, s)$ is called the weight of agent a referring to subject s.

For simplicity, if the subject of conflict is known and if the elements of a conflict profile are well identified, then we may denote the weight of an element in a profile by $w(x)$ where x is an element of the profile. For example,

$$X = \{x_1, x_2, x_3, x_4\},$$

where x_1 = "*Yes*", x_2 = "*Yes*", x_3 = "*No*", and x_4 = "*Yes*". The weights can be written as

$$w(x_1) = 0.4; \quad w(x_2) = 0.1; \quad w(x_3) = 0.9; \quad w(x_4) = 1.$$

2.4. Consistency Measure for Conflict Profiles

2.4.1. Notion of Conflict Profile Consistency

Referring to conflict profiles one can say that the conflict represented by one profile is larger than that by another. Let's consider an example.

Example 2.2. Let agents participating in a conflict generate answers "*Yes*," "*No*," or "*Neutral*;" then it seems that profile

$$X = \{Yes, No, Neutral\}$$

represents a more serious conflict than profile

$$X' = \{Yes, Yes, No\},$$

which in turn seems to be worse than profile

$$X'' = \{Yes, Yes, Neutral\}.$$

Assume that our aim is to make a decision on the basis of participants' opinions. The basis of our observation relies on the fact that profile X contains all three potential opinions, each of which appears exactly one time. In this case we often say "*Nothing is known*;" that is, it is not possible to make any sensible decision. The second profile contains two opinions "*Yes*" and one opinion "*No.*" Here using the rule of majority we could suggest accepting "*Yes*" as the decision. However, not taking into account the opinion "*No*" in this case may cause doubt inasmuch as the relationship between the number of "*Yesses*" to the number of "*Nos*" is only 2 to 1. The third profile is in the smallest degree embarrassing because two of participants say "*Yes*" and the third is neutral. ♦

For this reason an idea for measuring the degree of conflict has arisen. Referring to opinions belonging to conflict profiles one can observe that elements of a certain profile are more similar to each other (i.e., they are more convergent or more consistent) than elements of some other profile. In Example 2.2 opinions included in profile X' seem to be more consistent than opinions included in profile X, and opinions from profile X'' seem to be more consistent than opinions from profile X'. This shows that for a given profile it is needed to determine a value which would represent the degree (or level) of its consistency (or inconsistency). This value should be very useful in evaluating whether a conflict is "solvable." We propose to define this parameter for conflict profiles by means of consistency functions. For this aim we define a set of several postulates representing the intuitive conditions for consistency degree which should be satisfied by these functions. We also define several consistency functions and show which postulates they fulfil.

By symbol c we denote the consistency function for a conflict profile. This function has the signature:

$$c: \Pi(U) \to [0,1].$$

The idea of this function relies on measuring the consistency degree of profile elements. The consistency degree of a profile mentions the notion of indiscernibility defined by Pawlak for an information system [132, 145]. However, they are different conceptions. The difference is based on the fact that the consistency degree represents the coherence level of the profile elements and to measure it one should first define the distances between these elements. Indiscernibility, in turn, reflects the possibility for differentiating two tuples, and the distance function is not used.

By $\boldsymbol{C}(U)$ we denote the set of all consistency functions for $\Pi(U)$.

The notion of inconsistency measure has been referred to in paraconsistent logics [18, 59, 69, 70]. The general idea of these works is based on measuring the inconsistency value for a set of formulae which are not consistent. In our approach the inconsistency is expressed by means of the distance function, and the consistency value is determined on its basis.

2.4.2. Postulates for Consistency Functions

The requirements for consistency functions are expressed in the following postulates. We assume that the universe U consists at least of two elements. Let $c \in \boldsymbol{C}(U)$. The postulates are presented as follows.

P1a. Postulate for maximal consistency:
If X is a homogeneous profile then

$$c(X) = 1.$$

P1b. Extended postulate for maximal consistency:
For $Y, Z \in \Pi(U)$ *where* $Y = \{x\}$ *and*

$$X^{(n)} = (n * Y) \dot{\cup} Z$$

being such a profile that element x occurs at least n times, and the numbers of occurrences of other elements from Z are constant, then the following equation should be true,

$$\lim_{n \to +\infty} c(X^{(n)}) = 1.$$

P2a. Postulate for minimal consistency:
If $X = \{a, b\}$ *and* $d(a,b) = \max_{x,y \in U} d(x,y)$ *then*

$$c(X) = 0.$$

P2b. Extended postulate for minimal consistency:
For $Y, Z \in \Pi(U)$, $Y = \{a, b\}$ *and*

$$X^{(n)} = (n * Y) \dot{\cup} Z$$

being a profile such that elements a and b occur at least n times, and the numbers of occurrences of other elements from Z are constant, and

$$d(a,b) = \max_{x,y \in U} d(x,y),$$

then the following equation should take place,

$$\lim_{n \to +\infty} c(X^{(n)}) = 0.$$

P2c. Alternative postulate for minimal consistency:

If $X = U$ *then* $c(X) = 0$.

P3. Postulate for nonzero consistency:

If there exist elements $a, b \in X$ *such that*

$$d(a,b) < \max_{x,y \in U} d(x,y)$$

and $X \neq n * U$ *for all* $n = 1, 2, \ldots,$ *then*

$$c(X) > 0.$$

P4. Postulate for heterogeneous profiles:

If X is a he terogeneous profile then

$$c(X) < 1.$$

P5. Postulate for multiple profiles:

If profile X is a multiple of profile Y then

$$c(X) = c(Y).$$

P6. Postulate for greater consistency:

Let

$$d(x,X) = \Sigma_{y \in X} d(x,y)$$

denote the sum of distances between an element x of universe U and the elements of profile X. Let

$$D(X) = \{d(x,X): x \in U\}$$

be the set of all such sums. For any profiles $X, Y \in \Pi(U)$ *the following dependency should be true,*

$$\left(\frac{\min(D(X))}{card(X)} \leq \frac{\min(D(Y))}{card(Y)}\right) \Rightarrow (c(X) \geq c(Y)).$$

P7a. Postulate for consistency improvement:
Let a and a′ be such elements of universe U that

$$d(a,X) = \min\{d(x,X): x \in X\}$$

and

$$d(a',X) = \min\{d(x,X): x \in U\};$$

then

$$c(X-\{a\}) \leq c(X) \leq c(X \mathbin{\dot\cup} \{a'\}).$$

P7b. Second postulate for consistency improvement:
Let b and b′ be such elements of universe U that

$$d(b,X) = \max\{d(x,X): x \in X\}$$

and

$$d(b',X) = \max\{d(x,X): x \in U\};$$

then

$$c(X \mathbin{\dot\cup} \{b'\}) \leq c(X) \leq c(X - \{b\}).$$

P8. Postulate for simplification:
For Y, $Z \in \Pi(U)$ and

$$X^{(n)} = (n * Y) \mathbin{\dot\cup} Z$$

then the following equation should be true,

$$\lim_{n \to +\infty} c(X^{(n)}) = c(Y).$$

The above-mentioned postulates illustrate intuitive conditions for consistency functions. However, some commentaries should be given.

- *Postulate for maximal consistency* (P1a): This postulate seems to be very natural because it requires that if in a profile only one element occurs then the consistency should be maximal. As stated earlier, a homogeneous profile in fact does not represent any conflict, thus for this situation the consistency should be maximal. In the example of the experts generating opinions about the GDP level for a country, if their opinions are the same then the consistency of their knowledge on this matter should be maximal.

- *Extended postulate for maximal consistency* (P1b): If in a given conflict profile some element is dominant (by its large number of occurrences) in such degree that the numbers of occurrences of other elements are insignificant, then the consistency should be near to the maximal value. This is consistent with the rule of majority.
- *Postulate for minimal consistency* (P2a): If a profile X consists of two opinions and their distance is maximal, that is,

$$X = \{a,\ b\} \text{ and } d(a,b) = \max_{x,y \in U} d(x,y),$$

  then it represents the "worst conflict," so the consistency should be minimal.
- *Extended postulate for minimal consistency* (P2b): This postulate characterizes also a "very bad" conflict in which two maximally differing opinions dominate because their numbers of occurrences aim to infinity. For this situation the consistency should aim to 0. This postulate is also consistent with the rule of majority.
- *Alternative postulate for minimal consistency* (P2c): This postulate presents another aspect of "the worst conflicts," meaning that the profile is distinguishable and moreover, it consists of all possible opinions which may be generated. Referring to such kind of profiles we can say that everything is possible, and that is it is very hard to infer something from the situation. Therefore, the consistency degree should be minimal.
- *Postulate for nonzero consistency* (P3): This postulate defines a set of profiles with consistency degree larger than 0. In some sense it is complementary to postulates P2b and P2c which give consistency value 0 for those profiles nonreflected by postulate P3. In addition, if a profile contains at least two such elements that their distance is smaller than the maximal distance in the universe, and it is neither the universe nor its multiple, then this means that there is some coherence in it. For such profiles the consistency should be greater than 0.
- *Postulate for heterogeneous profiles* (P4): The consistency of "true" conflict may not be maximal. The maximal value of consistency is then reserved for profiles representing nonconflict situations. Thus if a profile is heterogeneous then its consistency may not be maximal, because the opinions represented by its elements are not identical.

- *Postulate for multiple profiles* (P5): If profile X is a multiple of profile Y then it means that the proportions of the numbers of occurrences of any two elements are identical in profiles X and Y. In other words, the quantity relationships between element x and element y ($x, y \in U$) are the same in profiles X and Y. Thus these profiles should have the same consistency degree.
- *Postulate for greater consistency* (P6): The fact that

$$\frac{\min(D(X))}{card(X)} \leq \frac{\min(D(Y))}{card(Y)}$$

  means that the elements of profile X are more concentrated than the elements of profile Y. In other words, profile X is denser than profile Y. Thus there should be $c(X) \geq c(Y)$.
- *Postulate for consistency improvement* (P7a): This postulate shows when it is possible to improve the consistency. According to this postulate removing the best element of a profile (i.e., the sum of distances between it and other elements is minimal) should worsen the inconsistency, whereas adding the best element of the universe referring to the profile (i.e., the sum of distances between it and elements of the profile is minimal) should improve the consistency.
- *Postulate for consistency improvement* (P7b): This postulate is dual to postulate P7a. It states that removing the worst element of a profile (i.e., the sum of distances between it and other elements is maximal) should improve the inconsistency, whereas adding the worst element of the universe referring to the profile (i.e., the sum of distances between it and elements of the profile is maximal) should worsen the consistency.

Postulates P7a and P7b are very useful in consistency improvement. According to them the consistency value should be larger if we add to a profile this element of universe U which generates the minimal sum of distances to the profile elements, or if we remove from the profile this element which generates the maximal sum of distances. For example, let

$$U = \{x, y, z, s, t, u, w\}$$

be such a universe that the distances between element w and other elements are much greater than the distances among elements x, y, z, t and u (see Figure 2.3). Let

$$X = \{x, y, z, s, t, w\},$$

$$d(u,X) = \min\{d(x,X): x \in U\}$$

and

$$d(w,X) = \max\{d(x,X): x \in X\}.$$

According to postulates P7a and P7b we can improve the consistency by moving element w or adding element u; that is

$$c(X) \leq c(X - \{w\}) \quad \text{and} \quad c(X) \leq c(X \dot{\cup} \{u\}).$$

So we have

$$c(X) \leq c(X - \{w\}) \leq c(X - \{w\} \dot{\cup} \{u\}) = c(\{x, y, z, s, t, u\})$$

and

$$c(X) \leq c(X \dot{\cup} \{u\}) \leq c(X \dot{\cup} \{u\} - \{w\}) = c(\{x, y, z, s, t, u\}).$$

- *Postulate for simplification* (P8): This postulate in a certain sense is a general form of postulates P1b and P2b regarding postulate P5. It requires accepting the consistency value of a profile as the consistency value of this part of the profile which dominates. In this case the dominating subprofile is $n * Y$, and because according to postulate P5 we have $c(n * Y) = c(Y)$, then the consistency is equal to the consistency of Y.

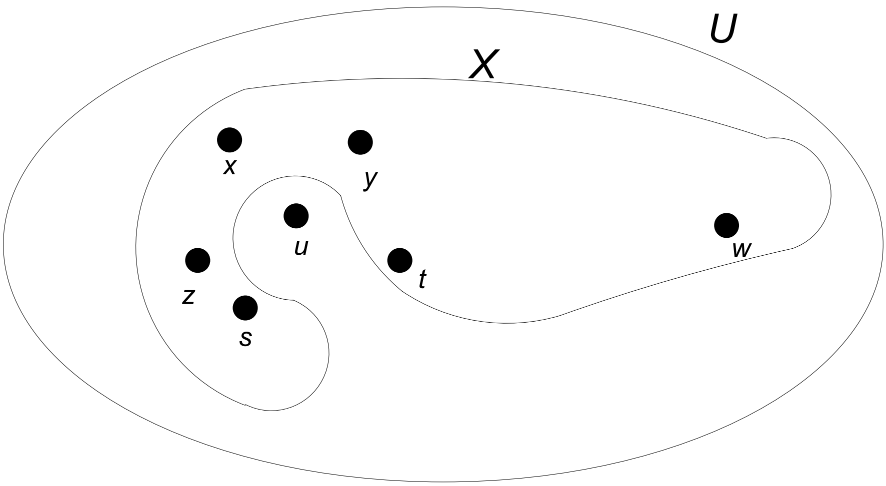

Figure 2.3. An example of profile X.

2.4.3. Analysis of Postulates

Let

$$\boldsymbol{P} = \{\text{P1a, P1b, P2a, P2b, P2c, P3, P4, P5, P6, P7a, P7b, P8}\}$$

be the set of all postulates. A postulate $p \in \boldsymbol{P}$ may be treated as an atomic logical formula for which the interpretation should be the set of pairs

$$<U, \boldsymbol{C}(U)>$$

where U is a universe and $\boldsymbol{C}(U)$ is the set of all consistency functions for conflict profiles from $\Pi(U)$.

We say that a postulate $p \in \boldsymbol{P}$

- is *satisfied* if there exist a universe U and a function in $\boldsymbol{C}(U)$ which satisfies postulate p
- is *u-true* if there exists a universe U for which p is satisfied by all functions from $\boldsymbol{C}(U)$
- is *true* if it is *u*-true referring to all universes
- is *false* if it is not satisfied

We can build complex formulae on the basis of the atomic formulae using logic quantifiers and such logical connectives as $\vee$, $\wedge$, $\neg$, $\Rightarrow$. For these formulae we accept the same semantics as defined for atomic formulae and the semantic rules of classical logic.

We prove the following.

Theorem 2.1.

(a) All postulates are independent, meaning that for each pair $p, p' \in \boldsymbol{P}$ and $p \neq p'$ formula

$$p \Rightarrow p'$$

is not true.

(b) The set of all postulates is not contradictory, meaning that there exist a universe and a function which satisfies all the postulates; that is, formula

$$\text{P1a} \wedge \text{P1b} \wedge \text{P2a} \wedge \text{P2b} \wedge \text{P2c} \wedge \text{P3} \wedge \text{P4} \wedge \text{P5} \wedge \text{P6} \wedge \text{P7a} \wedge \text{P7b} \wedge \text{P8}$$

is satisfied.

Proof.

(a) It is easy to check the lack of truth of all formulae $p \Rightarrow p'$ where p, $p' \in \boldsymbol{P}$ *and* $p \neq p'$. The reason is that each postulate occupies a region in the set $\boldsymbol{C}$ of all consistency functions, and although these regions may overlap they are not included one in another. For example, consider postulates P1a and P1b. Postulate P1a deals with only heterogeneous profiles and postulate P1b deals with nonheterogeneous profiles; if a consistency function satisfies one of these postulates it does not have to satisfy the second.

(b) Here we should define a function which satisfies all postulates. Let

$$U = \{x, y\},$$

and consistency function c be defined as follows,

$$c(X) = \begin{cases} 1 & \text{if } X \text{ is heterogeneous} \\ 0 & \text{if } X = n*U \text{ for } n = 1,2,... \\ \dfrac{k}{k+l} & \text{if } X = \{k*x, l*y\} \text{ and } k > l \\ \dfrac{l}{k+l} & \text{if } X = \{k*x, l*y\} \text{ and } k < l \end{cases} \cdot$$

It is not hard to check that function c satisfies all postulates. ♦

Below we present some other properties of the classes of consistency functions.

Theorem 2.2.

Let $X \in \Pi(U)$, $card(X) > 1$ *and let* $c \in \boldsymbol{C}(U)$ *be a consistency function satisfying postulate* P6. *Let x be such an element of universe U that*

$$d(x, X) = \min\{d(t, X) \colon t \in U\}$$

and y be such an element of profile X that

$$d(x, y) = \max_{z \in X} d(x, y).$$

The following dependence is true

$$c(X) \leq c(X - \{y\}).$$

Proof.

Let $Y = X - \{y\}$ and $card(X) = n > 1$, then $card(Y) = n - 1$. Let x' be such an element of universe U that

$$d(x',Y) = \min\ \{d(t,Y) \colon t \in U\}.$$

We have then $d(x',Y) \leq d(x,Y)$. Also, from

$$d(x,y) = \max_{z \in X} d(x,z)$$

it follows that

$$(n-1) \cdot d(x,y) \geq d(x,Y).$$

Thus

$$\frac{\min\{d(t,X) : t \in U\}}{card(X)} = \frac{d(x,X)}{n} = \frac{d(x,Y) + d(x,y)}{n}$$

$$\geq \frac{d(x,Y)}{n-1} \geq \frac{d(x',Y)}{n-1} = \frac{\min\{d(t,Y) : t \in U\}}{card(Y)}.$$

Because function c satisfies postulate P6, then there should be

$$c(X) \leq c(Y).$$ ♦

Owing to this property one can improve the consistency value for a profile by removing from it this element which is farthest from the element of U with the minimal sum of distances to the profile's elements.

This property also shows that if a consistency function satisfies postulate P6 then it should also partially satisfy postulate P7a.

Theorem 2.3.
Let $X \in \Pi(U)$ and let $c \in \boldsymbol{C}(U)$ be a consistency function satisfying postulate P6. *Let x be such an element of universe U that*

$$d(x,X) = \min\{d(t,X) \colon t \in U\};$$

then the following dependence is true

$$c(X) \leq c(X \dot{\cup} \{x\}).$$

Proof.
Let $Y = X \dot{\cup} \{x\}$. Because

$$d(x,X) = \min\{d(t,X) \colon t \in U\},$$

it implies that

$$d(x,Y) = \min\{d(t,Y) \colon t \in U\}.$$

In addition we have $d(x, X) = d(x, Y)$. Taking into account the fact that

$$card\,(Y) = card(X) + 1$$

it follows

$$\frac{\min\{d(t,X) : t \in U\}}{card(X)} \geq \frac{\min\{d(t,Y) : t \in U\}}{card(Y)}.$$

Because function c satisfies postulate P6 then we have

$$c(X) \leq c(X \dot{\cup} \{x\}).$$ ♦

This property allows improving the consistency value by adding to the profile an element which generates the minimal sum of distances to the profile's elements. It also shows that if a consistency function satisfies postulate P6 then it should also partially satisfy postulate P7b.

Theorem 2.4.
Postulates P1a *and* P2a *in general are inconsistent with postulate* P7a; *that is, formula*

$$(\text{P1a} \wedge \text{P2a}) \Rightarrow \neg\text{P7a}$$

is u-true.

Proof.
We show that there exists a universe U for which any function c from $\boldsymbol{C}(U)$ if satisfying postulates P1a and P2a cannot satisfy postulate P7a. Let then $U = \{a,b\}$ and $a \neq b$, then

$$d(a,b) = \max_{x,y \in U} d(x,y) > 0.$$

Let $X = U$; according to postulate P2a we have $c(X) = 0$. Because function c satisfies postulate P1a we have

$$c(X - \{a\}) = c(\{b\}) = 1.$$

In addition, we have

$$d(a,X) = \min\{d(t,X) \colon t \in X\}$$

and

$$c(X - \{a\}) = 1 > c(X) = 0,$$

so function c cannot satisfy postulate P7a. ♦

Theorem 2.5.
Postulates P1a *and* P4 *in general are inconsistent with postulate* P7a; *that is, formula*

$$(\text{P1a} \wedge \text{P4}) \Rightarrow \neg\text{P7a}$$

is u-true.

Proof.
We show that there exists a universe U for which any function c from $\boldsymbol{C}(U)$ if satisfying postulates P1a and P4 cannot satisfy postulate P7a. Let then $U = \{a,b\}$ and $a \neq b$; then

$$d(a,b) = \max_{x,y \in U} d(x,y) > 0.$$

Let $X = U$; according to postulate P4 we have $c(X) < 1$. Because c satisfies postulate P1a we have

$$c(X - \{a\}) = c(\{b\}) = 1.$$

Also, we have

$$d(a,X) = \min\{d(t,X): t \in X\}$$

and

$$c(X - \{a\}) > c(X),$$

so function c cannot satisfy postulate P7a. ♦

Theorem 2.6.
Postulates P1a *and* P4 *in general are inconsistent with postulate* P7a; *that is, formula*

$$(\text{P2a} \wedge \text{P3}) \Rightarrow \neg\text{P7b}$$

is u-true.

Proof.
We show that there exists a universe U for which any function c from $\boldsymbol{C}(U)$ if satisfying postulates P2a and P3 cannot satisfy postulate P7b. Let then $U = \{a,b\}$ and $a \neq b$; then

$$d(a,b) = \max_{x,y \in U} d(x,y) > 0.$$

Let $X = U$; we have

$$d(a,X) = \max\{d(t, X): t \in X\}$$

and

$$d(b,X) = \max\{d(t, X): t \in U\}.$$

According to postulate P2a we have $c(X) = 0$. In addition,

$$c(X \dot{\cup} \{b\}) = c(\{a, b, b\}) > 0$$

according to postulate P3, so function c cannot satisfy postulate P7b. ♦

Theorems 2.4 through 2.6 show some inconsistency between postulates P7a and P7b and other postulates. As stated above, postulates P7a and P7b play an important role in consistency improvement, and it has turned out that it is not always possible.

Postulate P8 in some sense is more general than postulates P1b and P2b. The following theorem proves this fact.

Theorem 2.7.
The following formulae

$$(\text{P8} \wedge \text{P1a}) \Rightarrow \text{P1b}$$

and

$$(\text{P8} \wedge \text{P2a}) \Rightarrow \text{P2b}$$

are true.

Proof.
For the first formula assume that for any universe U and any function $c \in \boldsymbol{C}(U)$ let c satisfy postulates P8 and P1a. Let $Y, Z \in \Pi(U)$ where $Y = \{x\}$ and

$$X^{(n)} = (n * Y) \dot{\cup} Z.$$

According to postulate P8 we have

$$\lim_{n \to +\infty} c(X^{(n)}) = c(Y),$$

and according to postulate P1a we have:

$$c(Y) = c(\{x\}) = 1.$$

Thus function c satisfies postulate P1b.
A similar proof may be done for the second formula. ♦

2.4.4. Consistency Functions

In this section we present the definitions of five consistency functions and analyze their satisfaction referring to the defined postulates. These functions are defined as follows.

Let $X = \{x_1, \dots, x_M\}$ be a conflict profile. We introduce the following parameters.

- The matrix of distances between the elements of profile X:

$$D^X = \left[d_{ij}^X\right] = \begin{bmatrix} d(x_1, x_1) & \dots & d(x_1, x_M) \\ \vdots & \ddots & \vdots \\ d(x_M, x_1) & \cdots & d(x_M, x_M) \end{bmatrix}.$$

- The vector of average distances between an element to the rest (for $M > 1$) :

$$W^X = (w_1^X, w_2^X, \dots, w_M^X),$$

where

$$w_i^X = \frac{1}{M-1} \sum_{j=1}^{M} d_{ji}^X$$

for $i = 1, 2, \dots, M$. Notice that although in the above sum there are M components, the average is calculated only for $M - 1$. This is because for each i value $d_{ii}^X = 0$.

- Diameters of set X:

$$Diam(X) = \max_{x,y \in X} d(x, y),$$

and the maximal element of vector W^X:

$$Diam(W^X) = \max_{1 \le i \le M} w_i^X,$$

representing this element of profile X which generates the maximal sum of distances to other elements. Because the values of distance function d are normalized we assume that the diameter of universe U is equal to 1; that is,

$$Diam(U) = 1.$$

- The average distance in profile X:

$$d_{\text{mean}}(X) = \begin{cases} \dfrac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1}^{M} d_{ij}^{X} = \dfrac{1}{M} \sum_{i=1}^{M} w_i^X & \text{for } M > 1 \\ 0 & \text{for } M = 1 \end{cases} .$$

Value $d_{\text{mean}}(X)$ represents the average of all distances between different elements of the profile. Notice that the word "different" refers only to the indexes of the elements. In fact some of them may be identical because of repetitions.

- The total average distance in profile X:

$$d_{\text{t\_mean}}(X) = \frac{\sum_{x,y \in X} d(x,y)}{M(M+1)} .$$

This value serves to represent the average distance of all distances between elements of profile X. The sum of these distances is expressed by the numerator of the quotient. However, one can ask a question: why is k^2 not in the denominator, but $k(k + 1)$ is? The answer is: in the numerator each distance $d(x,y)$, where $x \neq y$, occurs exactly twice, whereas each distance $d(x,y)$, where $x = y$, occurs exactly only once. Because $d(x, y) = 0$ for $x = y$, then adding such distance does not change the value of the numerator. However, in determining the average each distance should be taken into account twice. Thus the denominator should be $k(k + 1)$, but not k^2. For example, see Figure 2.4 where we have profile $X = \{a, b\}$ and $M = 2$. In sum $\sum_{x,y \in X} d(x,y)$ distance $d(a, b)$ appears twice because $d(a,b) = d(b,a)$, therefore for calculating the total average of distances each of the distances $d(a,a)$ and $d(b,b)$ should be taken into account twice. Thus the number of distances should be $2 \cdot 3 = 6$.

In fact, the value $d_{\text{t\_mean}}(X)$ of the total distance average and value $d_{\text{mean}}(X)$ of the distance average are dependent on each other referring to value M:

$$d_{\text{t\_mean}}(X) = \frac{M-1}{M+1}\, d_{\text{mean}}(X) .$$

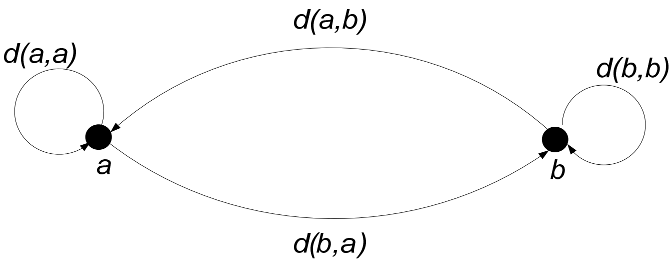

Figure 2.4. Profile $X = \{a, b\}$.

- The sum of distances between an element x of universe U and the elements of set X:

$$d(x, X) = \Sigma_{y \in X} d(x, y).$$

- The set of all sums of distances:

$$D(X) = \{d(x, X): x \in U\}.$$

- The minimal average distance from an element to the elements of profile X:

$$d_{\min}(X) = \frac{1}{M} \min (D(X)).$$

These parameters are now applied for defining the following consistency functions,

$$c_1(X) = 1 - Diam(X),$$

$$c_2(X) = 1 - Diam(W^X),$$

$$c_3(X) = 1 - d_{\text{mean}}(X),$$

$$c_4(X) = 1 - d_{\text{t\_mean}}(X),$$

$$c_5(X) = 1 - d_{\min}(X).$$

Some comments for the defined functions:

- $c_1(X)$: This function reflects the maximal distance between two elements of profile X. The intuitive sense of this function is based on the fact that if this maximal distance is equal to 0 (i.e., profile X is homogeneous) then the consistency is maximal (equal to 1). However, the density and coherence of profiles are not taken into account.

- $c_2(X)$: This function refers to the maximal average distance between an element of profile X and other elements of this profile. If the value of this maximal average distance is small, that is, the elements of profile X are near each other, then the consistency should be high. The density and coherence of profiles are in some degree taken into account by this function.
- $c_3(X)$: This function takes into account the average distance between elements of X. This parameter seems to be very representative for consistency. The larger this value is, the smaller is the consistency and vice versa. This function reflects to a large degree the density of profiles.
- $c_4(X)$: This function takes into account the total average distance between elements of X. As noted above, this function is dependent on function $c_3(X)$ and vice versa. However, there is an essential difference between these functions. Consider a distinguishable profile X in which all elements are maximally distanced from each other; that is,

$$\forall(x, y \in X, x \neq y): d(x, y) = 1.$$

For $card(X) = 2$ we have:

$$d_{\text{t\_mean}}(X) = 1/3 \quad \text{and} \quad d_{\text{mean}}(X) = 1.$$

For $card(X) = M > 2$ we can prove that $d_{\text{mean}}(X)$ is constant and $d_{\text{mean}}(X) = 1$, whereas

$$d_{\text{t\_mean}}(X) = \frac{M-1}{M+1}.$$

It seems that in this case function c_4 better reflects the consistency than function c_3. This is because the larger M is, the smaller is the inconsistency. However, for each value of M the value of function c_3 is constant and equals 0, whereas

$$c_4(X) = 1 - \frac{M-1}{M+1} = \frac{2}{M+1}.$$

Values of c_4 then better reflect this situation than those of c_3.

- $c_5(X)$: The minimal average distance between an element of universe U and elements of X. The elements of universe U which generate the minimal average distance to elements of profile X may be treated as representatives for this profile. The average distance between a representative to the profile elements also reflects the coherence of the profile. In clustering theory, this criterion is used for determining the centre of a group, called a *centroid*.

Table 2.1 presented below shows the results of the defined functions. The columns represent postulates and the rows represent the defined functions. Satisfying a postulate by a consistency function means that the function satisfies the postulate for all universes. The symbol + means that the presented function satisfies the postulate, the symbol – means that the presented function does not satisfy the postulate, and the symbol ± means partially satisfying the given postulate. From these results it implies that function c_5 partially satisfies postulates P7a and P7b. As follows from Theorems 2.2 and 2.3, if a function satisfies postulate P6 then it should partially satisfy postulates P7a and P7b. As we can see, function c_5 satisfies postulate P6.

From Table 2.1 it follows that function c_4 satisfies most postulates (9 over 12), and the next is function c_3 (8 postulates). However, neither function c_3 nor function c_4 satisfies postulate P5, which seems to be very natural. Function c_5 also is good in referring to the number of satisfied postulates. Function c_1 is very simple to calculate and it satisfies 6 postulates, although it does not satisfy any of postulates P6, P7a, P7b, or P8. Referring to functions c_2, c_3, and c_4 we know that they satisfy postulates P7a, P7b, that is, the postulates for consistency improvement. Below we present some other property of these functions referring to another kind of consistency improvement.

Table 2.1. Results of analysis of consistency functions regarding postulates

	P1a	P1b	P2a	P2b	P2c	P3	P4	P5	P6	P7a	P7b	P8
c_1	+	–	+	+	+	–	+	+	–	–	–	–
c_2	+	–	+	–	–	–	+	+	–	+	+	–
c_3	+	+	+	–	–	+	+	–	–	+	+	+
c_4	+	+	–	+	–	+	+	–	+	+	+	+
c_5	+	+	–	–	–	+	+	+	+	±	±	+

Theorem 2.8.
Let $X \in \Pi(U)$, $X = \{x_1, x_2, \ldots, x_M\}$, *and let*

$$X' = \{x_i \in X\colon\ w_i^X = Diam(W^X) \text{ and } 1 \le i \le M\}.$$

Then profile $X \backslash X'$ *should not have a smaller consistency than* X; *that is,*

$$c(X \backslash X') \ge c(X)\ ,$$

where $c \in \{c_2, c_3, c_4\}$.

Proof.
(a) For function c_2 the proof follows immediately from the notice that

$$Diam(W^X) \le Diam(W^{X \backslash X'}).$$

(b) For function c_3 we have

$$\overline{d}(X) = \frac{1}{M}\sum_{i=1}^{M} w_i^X\ .$$

Because for each $x_i \in X'$ there is

$$w_i^X = Diam(W^X),$$

therefore there should be

$$\overline{d}(X) \ge \overline{d}(X \setminus X')\ ;$$

that is,

$$c_3(X \backslash X') \ge c_3(X).$$

(c) For function c_4 the proof is similar as for function c_3 ♦

This property shows another way for improving consistency referring to functions c_2, c_3, and c_4, which is based on removing from a profile those elements generating the maximal average distance.

2.4.5. Reflecting Weights in Consistency Measure

In Section 2.3.3 we have defined weights for conflict participants which have to reflect their competency and credibility. It is natural that these weights should be taken into account in the consistency value. First of all, these weights should be reflected in the distances between the opinions of the conflict participants. Formally, for reflecting the weights we introduce the set of participants; generally, we call them agents.

Let then X be a conflict profile which consists of opinions of agents from set A with such assumption that each agent has exactly one opinion in the profile. Of course, some agents may have the same opinion, therefore, X is a set of repetitions. For simplicity, we distinguish the elements of profile X, and for each $x \in X$ by w_x we denote the weight of the agent which is the author of opinion x. Notice that the way of weight representation is different from that presented in Section 2.3.3.

For two opinions $x, y \in X$ we define the weighted distance $d'(x,y)$ between them on the basis of distance function d taking weights of x and y into account, as follows,

$$d'(x, y) = w(x) \cdot w(y) \cdot d(x, y).$$

Using defined function d' in consistency functions c_1, c_2, c_3, c_4, and c_5 we give new consistency functions as c'_1, c'_2, c'_3, c'_4, and c'_5, respectively. Postulates P1–P8 can also be redefined using weighted distance function d'. We can notice that the satisfaction of postulates using distance function d can also be transformed to the satisfaction of postulates using weighted distance function d'.

In previous sections we have considered consistency improvement by removing those elements which spoilt the consistency, or adding new elements which enlarge this value. Here we also consider the problem of consistency improvement, but by modification of weights of the conflict participations. In some sense it is a banal problem because one can always modify the weights so that only one element has weight larger than zero. Owing to this there will arise a homogeneous profile which should have maximal value of consistency.

2.4.6. Practical Aspect of Consistency Measures

The aim of conflict solution most often consists in determining a version of data (or an alternative, or a scenario) which best represents the versions belonging to the conflict content (i.e., conflict profile). As stated above, the consistency value of a conflict profile delivers some image about the profile: it is dense (high value of consistency) or rare (low value of consistency). In corresponding to conflict a rare profile may provide that the conflict content is not “good” enough for the conflict to be solved. On the other hand, if the consistency value is high, one can say that the opinions of conflict participants are coherent enough, and such conflict may be solved. Let's consider an example, five witnesses of a crime describe the hair color of the criminal as follows.

$$Hair\_color = \{blond, black, redhead, green, bald\}.$$

As can be seen, the opinions of the witnesses are very different, and having to deal with such information it is very hard to infer what is probably the color of the criminal's hair. However, if the conflict profile is presented as follows,

$$Hair\_color = \{blond, blond, redhead, fair\text{-}haired, blond\},$$

then one can state that the probable hair color of the criminal is *blond*, and it is even more probable that he is *fair-haired*.

Another aspect of consistency refers to expert knowledge analysis. It very often happens that the same task is entrusted to several experts for solving. This is because experts as specialists of different domains can have different approaches to the problem. Owing to this the solutions given by experts should reflect different sides of the problem. In this situation if the consistency of expert opinions is high then of course the credibility of the final solution (determined on the basis of the solutions given by the experts) is also high. However, if the consistency is low, it is not a reason for concern.

As we show in Chapter 8, in expert knowledge analysis, the consistency of solutions given by experts for a problem does not have such essential significance. It is proved that if we can assume that the experts solve the problem with the same degree of credibility then a conflict profile with low consistency value can be better than a conflict profile with high consistency in the sense that the consensus determined on the basis of the first profile may be nearer to the proper solution than the consensus determined on the basis of the second profile. Consistency value then has sense for such conflict profiles which are sets of opinions of participants, referring to whom we cannot assume any degrees of credibility, as with the set of criminal witnesses presented above.

As a matter of fact there are many practical aspects of consistency measures. We can use the consistency degrees in multiagent systems and in all kinds of information systems where knowledge is processed by autonomous programs: in distributed database systems where data consistency is one of the key factors, and also in reasoning systems and many others.

2.5. Conclusions

The material on the subject of inconsistency measures for conflict profiles presented in this chapter is partially based on the material included in [83–85, 118–120]. However, in this chapter many postulates have been modified and postulate P8 has been introduced. Modification of postulates also causes new results of their analysis to arise. In addition, new consistency function c_5 has been defined and analyzed.

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