

# Preface

In December 2006 I posted my manuscript *Vector Analysis for Computer Graphics* to Springer and looked forward to a short rest before embarking upon another book. But whilst surfing the Internet, and probably before my manuscript had reached its destination, I discovered a strange topic called *geometric algebra*. Advocates of geometric algebra (GA) were claiming that a revolution was coming and that the cross product was dead. I couldn't believe my eyes. I had just written a book about vectors extolling the power and benefits of the cross product, and now moves were afoot to have it banished! I continued to investigate GA and was amazed that a Google search revealed over 2 million entries. I started to read up the subject and discovered that GA was a Clifford algebra which had a natural affinity with geometry. It appeared that Prof. David Hestenes [14] had invented *geometric calculus* and successfully applied it to classical and quantum mechanics, electrodynamics, projective and conformal geometry. Chris Doran, Anthony and Joan Lasenby at Cambridge University had continued this research and were a driving force behind its understanding, dissemination and application to computer graphics. It seems that if I had been attending SIGGRAPH regularly, I would have been aware of these developments, but alas that was not the case, and I had a lot of catching up to do.

As I started reading various technical papers, especially by Hestenes, Doran and the Lasenbys, I realized the importance of the subject and the need to understand it. Slowly I was drawn into a world of complex numbers, antisymmetric operators, non-commutative products, conformal space, null vectors and the promise of elegance in CGI algorithms. I would be able to divide, rotate and reflect vectors with an ease never before known.

As I was finding it so difficult to understand GA, probably other people would also be finding it difficult, and then I realized the title of my next book: *Geometric Algebra for Computer Graphics*. But how could I write about a subject of which I knew nothing? This was a real challenge and became the driving force that has kept me working day and night for the past year. I took every opportunity to read about the subject: in bed, on planes, trains and boats; whilst waiting at the dentist and even waiting whilst my car was being serviced!

Before embarking on my summer vacation this year (2007) I bought a copy of Doran & Lasenby's excellent book *Geometric Algebra for Physicists* and took it, and my embryonic manuscript, with me to the south of France. My wife and I stayed at the Hotel Horizon in Cabris, overlooking Grasse and Cannes on the Côte d'Azur. Previous guests have included authors, philosophers and musicians such as Leonard Bernstein, Jean-Paul Sartre, Simone de Beauvoir, Gregory Peck and Antoine de St. Exupéry whose names have been carved into table tops in the

bar. Now that I have spent a few days at Hotel Horizon studying bivectors, trivectors and multi-vector products, I am looking forward to seeing my name cut into a table top when I return next year!

This book is a linear narrative of how I came to understand geometric algebra. For example, when I started writing the manuscript, *conformal geometry* were no more than two words, about which, I knew I would eventually have to master and write a chapter. The conformal model has been the most challenging topic I have ever had to describe. To say that I understand conformal geometry would be an overstatement. I understand the action of the algebra but I do not have a complete picture in my mind of 5D Minkowski space which is the backdrop for the conformal model. I admire the authors who have written so confidently about the conformal model, not only for their mathematical skills but their visual skills to visualize what is happening at a geometric level.

When I first started to read about GA I was aware of the complex features of the algebra, in that certain elements had imaginary qualities. Initially, I thought that this would be a major stumbling block, but having now completed the manuscript, the imaginary side of GA is a red herring. If one accepts that some algebraic elements square to  $-1$ , that is all there is to it. Consequently, do not be put off by this aspect of the algebra.

Another, stumbling block that retarded my progress in the early days was the representation at a programming level of bivectors, trivectors, quadvectors, etc. I recall spending many days walking my dog Monty trying to resolve this problem. Monty, a Westie, whose knowledge of Clifford algebra was only slightly less than my own, made no contribution whatsoever, but this daily mental and physical exercise eventually made the penny drop and I realized that bivectors, trivectors, quadvectors, etc., were just names recording a numerical value within the algebra. Why had I found it so difficult? Why had this not been explicitly described by other authors? If only someone had told me, I could have avoided this unnecessary mental anguish. But, in retrospect, the mental pain of learning about GA single-handed, has provided me with some degree of confidence when talking about the subject. In fact, in September 2007, I organized a one-day Workshop on GA in London where Dr. Hugh Vincent, Dr. Chris Doran, Dr. Joan Lasenby and me gave presentations to an audience from the computer animation and computer games sectors. It was extremely successful.

I have structured this book such that the first six chapters provide the reader with some essential background material covering complex algebra, vector algebra, quaternion algebra and geometric conventions. These can be skipped if you are already familiar with these topics. Chapter 7 goes into the history of geometric algebra, but I was already prepared for this as I had read Michael Crowe's fantastic book *A History of Vector Analysis*. In fact, this book is so good I have read it at least four times! Chapter 8 describes the geometric product, which was introduced by Clifford and is central to GA. Chapter 9 explores how GA handles reflections and rotations. Chapter 10 shows how GA is used to solve various problems in 2D and 3D geometry. Chapter 11 describes the conformal model. Chapter 12 is a short review of some typical applications of GA and Chapter 13 identifies important programming tools for GA. Finally, chapter 14 draws the book to a conclusion.

I am not a mathematician, just a humble consumer of mathematics, and whenever I read a book about mathematics I need to see examples, which is why I have included so many in this book. It is so tempting to write:

“It is obvious that Eq. (12.56) is the required rotor”;

for very often it is not obvious that this equation is a rotor, or even how it is used in practice. Therefore, whenever I have introduced an equation, I have shown its derivation and its application.

I would like to thank Dr. Hugh Vincent for reading through an early manuscript and offering some constructive feedback. I would also like to thank Dr. Chris Doran for taking the time to read the manuscript and advising me on numerous inconsistencies, and Dr. Joan Lasenby for her responsive, supportive emails when I had lost my way in untangling conformal null vectors. Once again I would like to acknowledge Chris Doran and Anthony Lasenby's excellent book *Geometric Algebra for Physicists*. I could not have written this book without their book. I also must not forget to thank Helen Desmond and Beverley Ford, General Manager of Springer, UK, for their continuous support, memorable lunches and transforming my manuscript into such a beautiful book.

Although I have done my best to ensure that the book is error free, if there are any inconsistencies, I apologize, as they are entirely my fault.

Finally, I must remind the reader that this book is intended only as a gentle introduction to GA. Hopefully, it will provide a bridge that will ease the understanding of technical papers and books about GA, where the subject is covered at a more formal and rigorous level.

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John Vince



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Vince, J.

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