

# Contents

<b>Preface</b>	vii
<b>1 Constructive Elements and Approaches in Approximation Theory</b>	<b>1</b>
1.1 Introduction to Approximation Theory	1
1.1.1 Basic Notions	1
1.1.2 Algebraic and Trigonometric Polynomials	4
1.1.3 Best Approximation by Polynomials	7
1.1.4 Chebyshev Polynomials	9
1.1.5 Chebyshev Extremal Problems	14
1.1.6 Chebyshev Alternation Theorem	17
1.1.7 Numerical Methods	20
1.2 Basic Facts on Trigonometric Approximation	24
1.2.1 Trigonometric Kernels	24
1.2.2 Fourier Series and Sums	30
1.2.3 Moduli of Smoothness, Best Approximation and Besov Spaces	32
1.3 Chebyshev Systems and Interpolation	38
1.3.1 Chebyshev Systems and Spaces	38
1.3.2 Algebraic Lagrange Interpolation	39
1.3.3 Trigonometric Interpolation	40
1.3.4 Riesz Interpolation Formula	44
1.3.5 A General Interpolation Problem	46
1.4 Interpolation by Algebraic Polynomials	48
1.4.1 Representations and Computation of Interpolation Polynomials	48
1.4.2 Interpolation Array and Lagrange Operators	51
1.4.3 Interpolation Error for Some Classes of Functions	54
1.4.4 Uniform Convergence in the Class of Analytic Functions	56
1.4.5 Bernstein's Example of Pointwise Divergence	61
1.4.6 Lebesgue Function and Some Estimates for the Lebesgue Constant	63
1.4.7 Algorithm for Finding Optimal Nodes	68
<b>2 Orthogonal Polynomials and Weighted Polynomial Approximation</b>	<b>75</b>
2.1 Orthogonal Systems and Polynomials	75
2.1.1 Inner Product Space and Orthogonal Systems	75
2.1.2 Fourier Expansion and Best Approximation	77
2.1.3 Examples of Orthogonal Systems	79

2.1.4	Basic Facts on Orthogonal Polynomials and Extremal Problems . . . . .	89
2.1.5	Zeros of Orthogonal Polynomials . . . . .	93
2.2	Orthogonal Polynomials on the Real Line . . . . .	95
2.2.1	Basic Properties . . . . .	95
2.2.2	Asymptotic Properties of Orthogonal Polynomials . . . . .	103
2.2.3	Associated Polynomials and Christoffel Numbers . . . . .	111
2.2.4	Functions of the Second Kind and Stieltjes Polynomials . . . . .	117
2.3	Classical Orthogonal Polynomials . . . . .	121
2.3.1	Definition of the Classical Orthogonal Polynomials . . . . .	121
2.3.2	General Properties of the Classical Orthogonal Polynomials . . . . .	124
2.3.3	Generating Function . . . . .	128
2.3.4	Jacobi Polynomials . . . . .	131
2.3.5	Generalized Laguerre Polynomials . . . . .	140
2.3.6	Hermite Polynomials . . . . .	145
2.4	Nonclassical Orthogonal Polynomials . . . . .	146
2.4.1	Semi-classical Orthogonal Polynomials . . . . .	146
2.4.2	Generalized Gegenbauer Polynomials . . . . .	147
2.4.3	Generalized Jacobi Polynomials . . . . .	148
2.4.4	Sonin-Markov Orthogonal Polynomials . . . . .	152
2.4.5	Freud Orthogonal Polynomials . . . . .	154
2.4.6	Orthogonal Polynomials with Respect to Abel, Lindelöf, and Logistic Weights . . . . .	159
2.4.7	Strong Non-classical Orthogonal Polynomials . . . . .	159
2.4.8	Numerical Construction of Orthogonal Polynomials . . . . .	160
2.5	Weighted Polynomial Approximation . . . . .	166
2.5.1	Weighted Functional Spaces, Moduli of Smoothness and $K$ -functionals . . . . .	166
2.5.2	Weighted Best Polynomial Approximation on $[-1, 1]$ . . . . .	170
2.5.3	Weighted Approximation on the Semi-axis . . . . .	174
2.5.4	Weighted Approximation on the Real Line . . . . .	178
2.5.5	Weighted Polynomial Approximation of Functions Having Isolated Interior Singularities . . . . .	182
<b>3</b>	<b>Trigonometric Approximation . . . . .</b>	<b>193</b>
3.1	Approximating Properties of Operators . . . . .	193
3.1.1	Approximation by Fourier Sums . . . . .	193
3.1.2	Approximation by Fejér and de la Vallée Poussin Means . . . . .	195
3.2	Discrete Operators . . . . .	197
3.2.1	A Quadrature Formula . . . . .	197
3.2.2	Discrete Versions of Fourier and de la Vallée Poussin Sums . . . . .	202
3.2.3	Marcinkiewicz Inequalities . . . . .	205

3.2.4	Uniform Approximation . . . . .	210
3.2.5	Lagrange Interpolation Error in $L^p$ . . . . .	212
3.2.6	Some Estimates of the Interpolation Errors in $L^1$ -Sobolev Spaces . . . . .	221
3.2.7	The Weighted Case . . . . .	224
<b>4</b>	<b>Algebraic Interpolation in Uniform Norm . . . . .</b>	<b>235</b>
4.1	Introduction and Preliminaries . . . . .	235
4.1.1	Interpolation at Zeros of Orthogonal Polynomials . . . . .	235
4.1.2	Some Auxiliary Results . . . . .	239
4.2	Optimal Systems of Nodes . . . . .	248
4.2.1	Optimal Systems of Knots on $[-1, 1]$ . . . . .	248
4.2.2	Additional Nodes Method with Jacobi Zeros . . . . .	252
4.2.3	Other “Optimal” Interpolation Processes . . . . .	264
4.2.4	Some Simultaneous Interpolation Processes . . . . .	268
4.3	Weighted Interpolation . . . . .	271
4.3.1	Weighted Interpolation at Jacobi Zeros . . . . .	271
4.3.2	Lagrange Interpolation in Sobolev Spaces . . . . .	276
4.3.3	Interpolation at Laguerre Zeros . . . . .	278
4.3.4	Interpolation at Hermite Zeros . . . . .	287
4.3.5	Interpolation of Functions with Internal Isolated Singularities . . . . .	292
<b>5</b>	<b>Applications . . . . .</b>	<b>319</b>
5.1	Quadrature Formulae . . . . .	319
5.1.1	Introduction . . . . .	319
5.1.2	Some Remarks on Newton-Cotes Rules with Jacobi Weights . . . . .	322
5.1.3	Gauss-Christoffel Quadrature Rules . . . . .	324
5.1.4	Gauss-Radau and Gauss-Lobatto Quadrature Rules . . . . .	328
5.1.5	Error Estimates of Gaussian Rules for Some Classes of Functions . . . . .	332
5.1.6	Product Integration Rules . . . . .	345
5.1.7	Integration of Periodic Functions on the Real Line with Rational Weight . . . . .	350
5.2	Integral Equations . . . . .	362
5.2.1	Some Basic Facts . . . . .	362
5.2.2	Fredholm Integral Equations of the Second Kind . . . . .	369
5.2.3	Nyström Method . . . . .	382
5.3	Moment-Preserving Approximation . . . . .	385
5.3.1	The Standard $L^2$ -Approximation . . . . .	385
5.3.2	The Constrained $L^2$ -Polynomial Approximation . . . . .	388
5.3.3	Moment-Preserving Spline Approximation . . . . .	389
5.4	Summation of Slowly Convergent Series . . . . .	397
5.4.1	Laplace Transform Method . . . . .	398

5.4.2	Contour Integration Over a Rectangle . . . . .	401
5.4.3	Remarks on Some Slowly Convergent Power Series . . . .	411
<b>References</b>	. . . . .	<b>415</b>
<b>Index</b>	. . . . .	<b>437</b>



<http://www.springer.com/978-3-540-68346-9>

Interpolation Processes

Basic Theory and Applications

Mastroianni, G.; Milovanovic, G.

2008, XIV, 446 p. 42 illus., Hardcover

ISBN: 978-3-540-68346-9