Preface

Back in 1954, a paper [2] by Bondi and Gold was to pick up on a much older question and raise a new one that would trigger another long debate. The old question had been around since the beginning of the twentieth century, when Born first raised it [1] and others followed suit. This was the question of whether a uniformly accelerated charge (in flat spacetime) would radiate electromagnetic energy. The new question arose from the claim by Bondi and Gold that (in the context of general relativity now) a static charge in a static gravitational field cannot radiate energy. If this were the case, then a particular version of the equivalence principle would thereby be contradicted.

This book reviews the problem discovered by Bondi and Gold and discusses the ensuing debate as carried on by Fulton and Rohrlich [3], DeWitt and Brehme [4], Mould [5], Boulware [6], and Parrott [7]. Various solutions have been proposed by the above (and others who are not discussed here). One of the aims here will be to put forward a rather different solution to Bondi and Gold's radiation problem. So even though the papers cited are discussed to a large extent in chronological order, the reason for writing this is not just to produce an historical reference. And even though the version of general relativity applied here is entirely consensual, every one of these papers is criticised on at least one important count, so I suspect that the result as a whole should not be described as consensual.

Chronologically speaking, I began to read the above papers somewhere in the middle and moved randomly through them, ending with the first! But, of course, Bondi and Gold's paper is not the first in this debate. Here we have one of those situations where we expect the latest papers to reveal the state of the art, so that all previous ones are redundant, but in fact we find that there was something important in the paper just before the one we are reading, that requires more attention, and we end up delving further and further into the history of the problem. This was why I eventually got hold of Bondi and Gold's 1954 paper, and I am glad I did so. I think it is here that we find out why the equivalence principle came into the problem.

This is the kind of debate that one gets drawn into almost against one's better judgement. As I understood the notion of equivalence principle, it was one of the lynchpins of general relativity, and my whole understanding of this theory would have been in trouble without it. This is why I thought I had better have a look. Several years and hundreds of pages of calculations later, it seems to me that I have a clearer view of some of the pitfalls awaiting those who try to understand general relativity more deeply than the account in a standard introductory textbook.

Here is a well-known principle of learning: if you want to really understand something, you have to get involved with it, asking difficult questions and trying to answer them. So another aim here, in fact the main aim, is to look closely at what general relativity is telling us with regard to some difficult questions, and to emphasise those pitfalls. It is true that the specific question of electromagnetic radiation from charged particles dealt with in this book is very narrow, but it provides a stage on which one can exercise the art and become stronger.

So my idea was to take my own voluminous notes on this issue and extract a story with the chronology of the above sequence of papers, illustrating along the way this or that theoretical problem and forging my own conclusions about the notion of equivalence principle, the question of whether uniformly accelerating charges do or do not radiate electromagnetic energy, and the validity or otherwise of the Lorentz–Dirac equation. The logical structure of the result is not simpler for having taken this storyline as backbone, rather than a textbook approach where logic, or rather economy of logic, comes first. However, there is a detailed table of contents and an index, and there are many cross-references. And the reader interested in a non-linear theory is assumed to be thoroughly capable of reading a non-linear book.

But who is the reader? It is certainly someone who has read several of those standard textbooks, and on reading the second was a little disappointed to find that it recounted in the same way more or less the same problems, and with the same solutions as the first. It is also someone who is never happy with scientific theories and has a lot of determination. In brief, a scientist, I suppose. The calculations here are not always entirely self-contained, as happens when one leaves the world of textbooks and ventures into the journal literature. Even the notation is not always the same, as happens when one moves from one publication to the next. In any case, the mathematics is not the most important thing, it is only equally important, and the text is definitely intended to be user-friendly. This explains the frequent recycling and development of ideas. There is a deliberate redundancy in the text which means that you may find yourself reading almost the same thing several times, something I always find comforting in a book.

So who is the reader? An undergraduate who likes a challenge, or a graduate who finds general relativity more difficult than the textbooks suggest with their glib solutions to Einstein's equations. Anybody who wants to know more about the way electromagnetism can be done in a curved spacetime, or the way the notion of equivalence principle can subtly change from one author to another. And here is another problem when one turns to the literature. Often abbreviated to get it into limited journal space, there is not always enough room to state all one's principles carefully. The potential reader is invited to get hold of the papers cited above and scan them for statements of the equivalence principle, bearing in mind the question: is that useable in practice? In the present book, it is hoped that things have been stated clearly. For example, the easy reference in the first paragraph of this preface to a static charge in a static spacetime is spelt out and given due attention. The potential reader is therefore someone who baulks at theory-laden statements when they are not treated with care, and someone who does not mind a slightly bulkier paraphrase that reminds us of the full content of a claim when that is useful. Some parts of the book look rather pedantic as a result, and I apologise for this in advance, although I suspect that it will not be disagreeable to the veterans of this debate.

Naturally, it is hoped that anyone who reads this book will come out stronger. It is hoped that they will find the weak points and improve things or even reject something where necessary. My intention is certainly not to consolidate some widely accepted theory or other, only to put it to the test, to see whether it can hold its ground on these particular issues. There are many other conflicts between general relativity and the rest of theoretical physics which should make us continue to be very tough on *all* our theories, as befits scientific endeavour.

Chapter 1 is a brief introduction to the question raised by Bondi and Gold, which sets the scene as it were. Chapter 2 then goes straight to the heart of the matter, because it discusses semi-Euclidean coordinate systems, sometimes advocated for accelerating observers in Minkowski spacetime, and their interpretation after taking the step to general relativity as describing static and homogeneous gravitational fields. This chapter launches the discussion about the notion of equivalence principle, a notion which turns out to look upon the world in many disparate ways. It is one of the key chapters. Chapter 3 then recalls how one might treat gravity as a force in the framework of special relativity, drawing attention to an amusing fact, viz., it is easy to get uniform acceleration in this scenario.

Chapter 4 introduces what has become known as the strong equivalence principle, which allows one to do electromagnetism in curved spacetimes, and indeed to transpose other bits of non-gravitational physics to this context. But in fact this principle does far more than that. It is a *sine qua non* for relating coordinates on curved spacetimes to length and time measurements one might make in the real world. From the historical angle, this chapter returns to Bondi and Gold's paper and spells out their problem.

Chapter 5 looks at the way Fulton and Rohrlich took up the debate, discussing the problem of electromagnetic radiation from an eternally uniformly accelerating charged particle in some detail. Chapter 6 then takes a first glance at Parrott's excellent review of this question, which is much more recent, so that the chronology of the story is broken (a kind of flash-forward). Chapters 7 and 8 return to the main time line and get to grips with the the problem of defining radiation in the context of Fulton and Rohrlich's paper. They also reveal some of the subtleties of the fields due to an eternally uniformly accelerating charged particle. Chapter 9 analyses Fulton and Rohrlich's view on the threat to the equivalence principle. This illustrates just how difficult it can be to interpret an author when the main claims have not been unambiguously stated. One possible interpretation of Fulton and Rohrlich's view is that they confuse some features of special and general relativity, and Chap. 10 aims to spell out the differences between the two theories with regard to radiation predictions.

Chapter 11 comes back to the Lorentz–Dirac equation, discussing the seminal Dirac paper rather briefly and exposing the much more recent account by Parrott, which is taken as definitive here. A brief mention is made of self-force calculations for spatially extended charge sources and their relevance to the Lorentz–Dirac equation. This is one of the key chapters for anyone wishing to find out about this much-discussed equation of motion. Chapter 12 moves on to DeWitt and Brehme's famous extension of the Lorentz–Dirac equation to curved spacetimes, a notoriously difficult paper which is essential reading for anyone who needs to sharpen their teeth on relativity theory. I mention some of the problems with this calculation and also with the later paper by Hobbs, which attempts to transpose the method to a more modern notation. It then reviews what DeWitt and Brehme have to say concerning the equivalence principle and the possibility of radiation from static charges in static spacetimes. Chapter 13 focuses more closely on the latter question, in a highly critical way, and is perhaps the key chapter for understanding the solution to the radiation problem that I am advocating here.

Chapter 14 discusses an entirely theoretical radiation detector proposed by Mould in 1964. The discussion here mainly concerns how the theoretical predictions for what this detector will detect affect or do not affect conclusions about the equivalence principle. But there is some analysis of the theory itself since it does illustrate the difficulties involved in interpreting results expressed relative to general coordinate systems, and it raises the question of how one should define radiation.

Chapter 15 is a detailed criticism of the paper by Boulware that is now often taken as definitive on these issues (although not by Parrott). Indeed one might describe it as the definitive mathematical analysis of the electromagnetic fields due to an eternally uniformly accelerating charged particle. However, there is a great deal of criticism here of the way the mathematics is interpreted, and especially the conclusions regarding equivalence principles. Parrott's arguments against Boulware are exposed in some detail in Chap. 16. The aim once again is to highlight the dangers of naive interpretation of tensor quantities when coordinates are not (locally) inertial. Parrott's neat analysis of the charged rocket, so useful for revealing the strangeness of the Lorentz–Dirac equation that it seems essential reading at some point, is recounted in detail in Chap. 17.

Chapter 18 beginning on p. 307 is effectively an abstract and could be read right at the beginning. It is long for a summary. I felt it was useful to recycle and repeat some of the main themes, given the logical complexity of the layout.

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