

---

# Contents

|  |     |
|--|-----|
| <b>Preface</b> .....   | vii |
| <b>1 Introduction to Derivative Instruments</b> .....                        | 1   |
| 1.1 Financial Options and Their Trading Strategies .....                     | 2   |
| 1.1.1 Trading Strategies Involving Options .....                             | 5   |
| 1.2 Rational Boundaries for Option Values .....                              | 10  |
| 1.2.1 Effects of Dividend Payments .....                                     | 16  |
| 1.2.2 Put-Call Parity Relations .....  | 18  |
| 1.2.3 Foreign Currency Options .....   | 19  |
| 1.3 Forward and Futures Contracts .....                                      | 21  |
| 1.3.1 Values and Prices of Forward Contracts .....                           | 21  |
| 1.3.2 Relation between Forward and Futures Prices .....                      | 24  |
| 1.4 Swap Contracts .....   | 25  |
| 1.4.1 Interest Rate Swaps .....  | 26  |
| 1.4.2 Currency Swaps .....   | 28  |
| 1.5 Problems .....   | 29  |
| <b>2 Financial Economics and Stochastic Calculus</b> .....                   | 35  |
| 2.1 Single Period Securities Models .....                                    | 36  |
| 2.1.1 Dominant Trading Strategies and Linear Pricing Measures ..             | 37  |
| 2.1.2 Arbitrage Opportunities and Risk Neutral Probability<br>Measures ..... | 43  |
| 2.1.3 Valuation of Contingent Claims .....                                   | 48  |
| 2.1.4 Principles of Binomial Option Pricing Model .....                      | 52  |
| 2.2 Filtrations, Martingales and Multiperiod Models .....                    | 55  |
| 2.2.1 Information Structures and Filtrations .....                           | 56  |
| 2.2.2 Conditional Expectations and Martingales .....                         | 58  |
| 2.2.3 Stopping Times and Stopped Processes .....                             | 62  |
| 2.2.4 Multiperiod Securities Models .....                                    | 64  |
| 2.2.5 Multiperiod Binomial Models .....                                      | 69  |

|          |  |            |
|----------|--|------------|
| 2.3      | Asset Price Dynamics and Stochastic Processes                      | 72         |
| 2.3.1    | Random Walk Models   | 73         |
| 2.3.2    | Brownian Processes   | 76         |
| 2.4      | Stochastic Calculus: Ito's Lemma and Girsanov's Theorem            | 79         |
| 2.4.1    | Stochastic Integrals   | 79         |
| 2.4.2    | Ito's Lemma and Stochastic Differentials                           | 82         |
| 2.4.3    | Ito's Processes and Feynman–Kac Representation Formula             | 85         |
| 2.4.4    | Change of Measure: Radon–Nikodym Derivative and Girsanov's Theorem | 87         |
| 2.5      | Problems   | 89         |
| <b>3</b> | <b>Option Pricing Models: Black–Scholes–Merton Formulation</b>     | <b>99</b>  |
| 3.1      | Black–Scholes–Merton Formulation                                   | 101        |
| 3.1.1    | Riskless Hedging Principle   | 101        |
| 3.1.2    | Dynamic Replication Strategy                                       | 104        |
| 3.1.3    | Risk Neutrality Argument   | 106        |
| 3.2      | Martingale Pricing Theory  | 108        |
| 3.2.1    | Equivalent Martingale Measure and Risk Neutral Valuation           | 109        |
| 3.2.2    | Black–Scholes Model Revisited                                      | 112        |
| 3.3      | Black–Scholes Pricing Formulas and Their Properties                | 114        |
| 3.3.1    | Pricing Formulas for European Options                              | 115        |
| 3.3.2    | Comparative Statics  | 121        |
| 3.4      | Extended Option Pricing Models                                     | 127        |
| 3.4.1    | Options on a Dividend-Paying Asset                                 | 127        |
| 3.4.2    | Futures Options  | 132        |
| 3.4.3    | Chooser Options  | 135        |
| 3.4.4    | Compound Options   | 136        |
| 3.4.5    | Merton's Model of Risky Debts                                      | 139        |
| 3.4.6    | Exchange Options   | 142        |
| 3.4.7    | Equity Options with Exchange Rate Risk Exposure                    | 144        |
| 3.5      | Beyond the Black–Scholes Pricing Framework                         | 147        |
| 3.5.1    | Transaction Costs Models   | 149        |
| 3.5.2    | Jump-Diffusion Models  | 151        |
| 3.5.3    | Implied and Local Volatilities                                     | 153        |
| 3.5.4    | Stochastic Volatility Models                                       | 159        |
| 3.6      | Problems   | 164        |
| <b>4</b> | <b>Path Dependent Options</b>                                      | <b>181</b> |
| 4.1      | Barrier Options  | 182        |
| 4.1.1    | European Down-and-Out Call Options                                 | 183        |
| 4.1.2    | Transition Density Function and First Passage Time Density         | 188        |
| 4.1.3    | Options with Double Barriers                                       | 195        |
| 4.1.4    | Discretely Monitored Barrier Options                               | 201        |

|          |  |            |
|----------|--|------------|
| 4.2      | Lookback Options   | 201        |
| 4.2.1    | European Fixed Strike Lookback Options                   | 203        |
| 4.2.2    | European Floating Strike Lookback Options                | 205        |
| 4.2.3    | More Exotic Forms of European Lookback Options           | 207        |
| 4.2.4    | Differential Equation Formulation                        | 209        |
| 4.2.5    | Discretely Monitored Lookback Options                    | 211        |
| 4.3      | Asian Options  | 212        |
| 4.3.1    | Partial Differential Equation Formulation                | 213        |
| 4.3.2    | Continuously Monitored Geometric Averaging Options       | 214        |
| 4.3.3    | Continuously Monitored Arithmetic Averaging Options      | 217        |
| 4.3.4    | Put-Call Parity and Fixed-Floating Symmetry Relations    | 219        |
| 4.3.5    | Fixed Strike Options with Discrete Geometric Averaging   | 222        |
| 4.3.6    | Fixed Strike Options with Discrete Arithmetic Averaging  | 225        |
| 4.4      | Problems   | 230        |
| <b>5</b> | <b>American Options</b>                                  | <b>251</b> |
| 5.1      | Characterization of the Optimal Exercise Boundaries      | 253        |
| 5.1.1    | American Options on an Asset Paying Dividend Yield       | 253        |
| 5.1.2    | Smooth Pasting Condition                                 | 255        |
| 5.1.3    | Optimal Exercise Boundary for an American Call           | 256        |
| 5.1.4    | Put-Call Symmetry Relations                              | 260        |
| 5.1.5    | American Call Options on an Asset Paying Single Dividend | 263        |
| 5.1.6    | One-Dividend and Multidividend American Put Options      | 267        |
| 5.2      | Pricing Formulations of American Option Pricing Models   | 270        |
| 5.2.1    | Linear Complementarity Formulation                       | 270        |
| 5.2.2    | Optimal Stopping Problem                                 | 272        |
| 5.2.3    | Integral Representation of the Early Exercise Premium    | 274        |
| 5.2.4    | American Barrier Options                                 | 278        |
| 5.2.5    | American Lookback Options                                | 280        |
| 5.3      | Analytic Approximation Methods                           | 282        |
| 5.3.1    | Compound Option Approximation Method                     | 283        |
| 5.3.2    | Numerical Solution of the Integral Equation              | 284        |
| 5.3.3    | Quadratic Approximation Method                           | 287        |
| 5.4      | Options with Voluntary Reset Rights                      | 289        |
| 5.4.1    | Valuation of the Shout Floor                             | 290        |
| 5.4.2    | Reset-Strike Put Options                                 | 292        |
| 5.5      | Problems   | 297        |
| <b>6</b> | <b>Numerical Schemes for Pricing Options</b>             | <b>313</b> |
| 6.1      | Lattice Tree Methods                                     | 315        |
| 6.1.1    | Binomial Model Revisited                                 | 315        |
| 6.1.2    | Continuous Limits of the Binomial Model                  | 316        |
| 6.1.3    | Discrete Dividend Models                                 | 320        |
| 6.1.4    | Early Exercise Feature and Callable Feature              | 322        |

|          |   |            |
|----------|---|------------|
| 6.1.5    | Trinomial Schemes . . . . .   | 323        |
| 6.1.6    | Forward Shooting Grid Methods . . . . .   | 327        |
| 6.2      | Finite Difference Algorithms . . . . .  | 332        |
| 6.2.1    | Construction of Explicit Schemes . . . . .  | 333        |
| 6.2.2    | Implicit Schemes and Their Implementation Issues . . . . .                        | 337        |
| 6.2.3    | Front Fixing Method and Point Relaxation Technique . . . . .                      | 340        |
| 6.2.4    | Truncation Errors and Order of Convergence . . . . .                              | 344        |
| 6.2.5    | Numerical Stability and Oscillation Phenomena . . . . .                           | 346        |
| 6.2.6    | Numerical Approximation of Auxiliary Conditions . . . . .                         | 349        |
| 6.3      | Monte Carlo Simulation . . . . .  | 352        |
| 6.3.1    | Variance Reduction Techniques . . . . .   | 355        |
| 6.3.2    | Low Discrepancy Sequences . . . . .   | 358        |
| 6.3.3    | Valuation of American Options . . . . .   | 359        |
| 6.4      | Problems . . . . .  | 369        |
| <b>7</b> | <b>Interest Rate Models and Bond Pricing . . . . .</b>                            | <b>381</b> |
| 7.1      | Bond Prices and Interest Rates . . . . .  | 382        |
| 7.1.1    | Bond Prices and Yield Curves . . . . .  | 383        |
| 7.1.2    | Forward Rate Agreement, Bond Forward and Vanilla Swap . . . . .                   | 384        |
| 7.1.3    | Forward Rates and Short Rates . . . . .   | 387        |
| 7.1.4    | Bond Prices under Deterministic Interest Rates . . . . .                          | 389        |
| 7.2      | One-Factor Short Rate Models . . . . .  | 390        |
| 7.2.1    | Short Rate Models and Bond Prices . . . . .                                       | 391        |
| 7.2.2    | Vasicek Mean Reversion Model . . . . .  | 396        |
| 7.2.3    | Cox–Ingersoll–Ross Square Root Diffusion Model . . . . .                          | 397        |
| 7.2.4    | Generalized One-Factor Short Rate Models . . . . .                                | 399        |
| 7.2.5    | Calibration to Current Term Structures of Bond Prices . . . . .                   | 400        |
| 7.3      | Multifactor Interest Rate Models . . . . .  | 403        |
| 7.3.1    | Short Rate/Long Rate Models . . . . .   | 404        |
| 7.3.2    | Stochastic Volatility Models . . . . .  | 407        |
| 7.3.3    | Affine Term Structure Models . . . . .  | 408        |
| 7.4      | Heath–Jarrow–Morton Framework . . . . .   | 411        |
| 7.4.1    | Forward Rate Drift Condition . . . . .  | 413        |
| 7.4.2    | Short Rate Processes and Their Markovian Characterization . . . . .               | 414        |
| 7.4.3    | Forward LIBOR Processes under Gaussian HJM Framework . . . . .                    | 418        |
| 7.5      | Problems . . . . .  | 420        |
| <b>8</b> | <b>Interest Rate Derivatives: Bond Options, LIBOR and Swap Products . . . . .</b> | <b>441</b> |
| 8.1      | Forward Measure and Dynamics of Forward Prices . . . . .                          | 443        |
| 8.1.1    | Forward Measure . . . . .   | 443        |
| 8.1.2    | Pricing of Equity Options under Stochastic Interest Rates . . . . .               | 446        |
| 8.1.3    | Futures Process and Futures-Forward Price Spread . . . . .                        | 448        |

|                                |   |            |
|--------------------------------|---|------------|
| 8.2                            | Bond Options and Range Notes . . . . .  | 450        |
| 8.2.1                          | Options on Discount Bonds and Coupon-Bearing Bonds . . .                        | 450        |
| 8.2.2                          | Range Notes . . . . .   | 457        |
| 8.3                            | Caps and LIBOR Market Models . . . . .  | 460        |
| 8.3.1                          | Pricing of Caps under Gaussian HJM Framework . . . . .                          | 461        |
| 8.3.2                          | Black Formulas and LIBOR Market Models . . . . .                                | 462        |
| 8.4                            | Swap Products and Swaptions . . . . .   | 468        |
| 8.4.1                          | Forward Swap Rates and Swap Measure . . . . .                                   | 469        |
| 8.4.2                          | Approximate Pricing of Swaption under Lognormal<br>LIBOR Market Model . . . . . | 473        |
| 8.4.3                          | Cross-Currency Swaps . . . . .  | 477        |
| 8.5                            | Problems . . . . .  | 485        |
| <b>References . . . . .</b>    |   | <b>507</b> |
| <b>Author Index . . . . .</b>  |   | <b>517</b> |
| <b>Subject Index . . . . .</b> |   | <b>521</b> |



<http://www.springer.com/978-3-540-42288-4>

Mathematical Models of Financial Derivatives

Kwok, Y.-K.

2008, XV, 530 p., Hardcover

ISBN: 978-3-540-42288-4