

# PREFACE

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As a number of good books covering the boundary element method have appeared relatively recently, we are perhaps obligated to justify the appearance of this volume. In this preface we would therefore like to delineate how this text differs from its predecessors and what we hope that it can add to the field.

The first distinguishing feature is that, as indicated by the title, this book is concerned solely with a Galerkin approximation of boundary integral equations, and more specifically, symmetric Galerkin. Most books on boundary elements deal primarily with more traditional collocation methods, so we hope that this volume will complement existing material. The symmetric Galerkin approximation is an accurate and versatile numerical analysis method, possessing the attractive feature of producing a symmetric coefficient matrix. Moreover, it is based upon the ability - that Galerkin provides - to handle the hypersingular (as well as the standard singular) boundary integral equation by means of standard continuous elements (*i.e.* no special elements are needed). Thus, a second noteworthy aspect of this book is that singular and hypersingular equations are introduced together and are treated numerically by means of a unified approach, as elaborated below.

A primary reason that this can be accomplished, and as well an important theme of this book, is that all singular integrals - weakly singular, singular (Cauchy principal value), and hypersingular - can be handled using the same basic concept and algorithms. These algorithms are based upon a mathematically rigorous definition of the integrals as *limits to the boundary*, certainly a unique feature of this book. We hope that our readers, especially students, will find this direct approach more intuitive than (in our prejudiced view) the somewhat sleight-of-hand removal of divergences in the (more or less) standard principal value and Hadamard finite

part treatments of the singular and hypersingular integrals. Moreover, as the limit based singular integration methods are largely independent of the particular Green's function, it suffices to separately discuss the analyses for two and three-dimensional problems. While most texts have separate chapters devoted to specific applications (potential theory, elasticity, etc.), here we take a different approach to emphasize that all equations can be treated by fundamentally the same methods leading to a variety of applications.

The consolidation of the basic analysis procedures into two chapters also leaves us room to include some important topics not usually covered in other texts. These include surface gradient evaluation (a key advantage of limit based singular integration methods), error estimation (once again based upon the ability to easily treat hypersingular equations), and non-homogeneous (*e.g.*, graded) materials. Finally, a symmetric Galerkin Matlab<sup>®</sup> educational program called BEAN, which stands for Boundary Element ANalysis, is available for download, and our hope is that this will assist in using the book as classroom material and also in learning how to program the symmetric Galerkin boundary element method.

The book is arranged into 11 Chapters. A brief summary of the organization is as follows.

**Chapter 1** provides an introduction to the boundary element method with special emphasis on the symmetric Galerkin formulation. The basic aspects of the integral equation formulation are introduced, and the advantages of this technique are discussed in the context of a specific industrial application, the electrodeposition of paint in automotive manufacturing. This application is integrated with advanced visualization techniques, including virtual reality. Other integral equation methods, such as meshless and mesh-reduction techniques, are briefly discussed, and the chapter concludes with a succinct history of the Galerkin approximation.

**Chapter 2** introduces boundary integral equations and their numerical approximations. For the Laplace equation, the integral equations for surface potential and for surface flux are derived, which involve the Green's function and its first and second derivatives. These functions are divergent when the source and field points coincide, the singularity becoming progressively stronger with higher derivatives, and thus the evaluation of (highly) singular integrals is of paramount importance. As noted above, the fundamental approach adopted in this book is to define and evaluate all singular integrals as '*limits to the boundary*'. This approach unifies the numerical analysis of the integral equations and affects almost every aspect of the book, most especially the evaluation of gradients examined in Chapter 5.

**Chapters 3 & 4** are the core of the book, presenting the numerical implementation of a Galerkin boundary integral analysis in two and three dimensions, respectively. The analysis techniques presented in these Chapters represent well the book philosophy outlined above. The primary task is the evaluation of singular integrals, and for the hypersingular integral it is necessary to isolate the divergent terms and to prove that they cancel. The methods are first described in the simplest possible setting, a piecewise linear solution of the Laplace equation. Subsequently, higher order curved interpolation and more complicated Green's functions are considered. As noted above, the limit to the boundary approach provides a consistent scheme for defining all singular integrals, and moreover results in direct semi-analytical evaluation algorithms. Symbolic computation is utilized to simplify the work involved in carrying out the limit process and related analytical integration, and example Maple<sup>®</sup> codes are provided.

**Chapter 5** addresses the evaluation of surface gradients. A significant advantage of the boundary limit approach is that it leads to a highly accurate and efficient scheme for computing surface derivatives: only local singular integrals need to be evaluated, *i.e.* a complete boundary integration is not required. The key is to exploit the limit definition, writing the gradient equation as a difference of interior and exterior limits. In many applications, most notably moving boundary problems, the knowledge of boundary derivatives is necessary, *e.g.*, the potential gradient for Laplace problems or the complete stress tensor in elasticity. A specific, and in many ways typical, example of a free boundary problem is discussed, a coupled level set-boundary element analysis for modeling two-dimensional breaking waves over sloping beaches. This example demonstrates the advantage of the gradient techniques for the general class of moving front problems.

**Chapter 6** considers three-dimensional axisymmetric problems. As an example, the boundary integral equation for the axisymmetric Laplace equation is solved by employing modified Galerkin weight functions. The alternative weights smooth out the singularity of the Green's function at the symmetry axis, and restore symmetry to the formulation. The modified weight functions, together with a boundary limit definition, also result in a relatively simple algorithm for the post-processing of the surface gradient.

**Chapter 7** presents a symmetric Galerkin boundary integral method for interface and multizone problems. This type of problem arises, for example, in applications such as composite materials (bi-material interfaces) and geophysical simulations (internal boundaries). In the present formulation, the physical quantities are known to satisfy continuity conditions across the interface, but no boundary conditions are specified. The algorithm described herein achieves a symmetric matrix of reduced size.

**Chapter 8** addresses error estimation and adaptivity from a practical viewpoint. The so-called *a-posteriori* error estimators are used to guide the associated adaptive mesh refinement procedure. The estimators make use of the “hypersingular residuals”, originally developed for error estimation in a standard collocation approximation, and later extended to the symmetric Galerkin setting. This leads to the formulation of Galerkin residuals, which are natural to the symmetric Galerkin boundary integral approach, and forms the basis of the present error estimation scheme. The error estimation and adaptive procedure are implemented in the educational, user-friendly, symmetric Galerkin Matlab<sup>®</sup> code BEAN mentioned above, which solves problems governed by the Laplace equation.

**Chapter 9** is dedicated to one of the most important and successful application areas for boundary integral methods: fracture analysis. Problems involving fracture and failure arise in many critical engineering areas, and boundary integral methods have inherent advantages for these calculations. It is therefore essential that an efficient and effective symmetric Galerkin approximation be developed for this class of problems, and this chapter demonstrates that this is indeed the case.

**Chapter 10** focuses on the development of symmetric Galerkin formulations for nonhomogeneous problems of potential theory. Specifically, a formulation and corresponding implementation for heat conduction in three dimensional functionally graded materials are presented. The Green's function of the actual problem is used to develop a boundary-only formulation without any domain discretization, in which the thermal conductivity varies exponentially in one coordinate. A transient

implementation using the Laplace transform Galerkin boundary element method is also provided.

**Chapter 11** is dedicated to the educational computer code Boundary Element ANalysis, including its graphical user interface. BEAN is a user-friendly adaptive symmetric Galerkin BEM code to solve the two-dimensional Laplace Equation. The Chapter outlines the specific procedures to set up the problems, and the steps to utilize BEAN's post-processing capabilities. The book website contains additional related material, as discussed below. In an effort to make the book as self-contained as possible, three appendices are provided, covering mathematical preliminaries, some Gaussian integration tables, and the symbolic Maple<sup>®</sup> codes. As discussed above, the Maple<sup>®</sup> codes are an integral part of the book and should be used in conjunction with Chapters 2-4.

A web site at [http://www.ghpaulino.com/SGBEM\\_book](http://www.ghpaulino.com/SGBEM_book) associated with this book will be maintained, primarily as a means of allowing access to the codes employed in the text. It contains the BEAN code, including the graphical user interface. The complete source-code is provided, together with a library of practical examples, and a video tutorial which demonstrates step-by-step how to use the software. The source code can be used for instructional purposes, as well as the building block for new applications. In addition, the symbolic Maple<sup>®</sup> codes, for singular and hypersingular integrations, are also provided. These codes supplement the explanations of Chapters 2 through 4. We hope that readers will also use the web site to report misprints, errors, and other comments/suggestions about the text.

Finally, this book would not have been possible without the patience of our wives and children. We also owe a tremendous debt to the many colleagues and students, most of whom are now good friends, with whom we have collaborated over the past twenty years. They are too numerous to mention, but they know who they are, and we thank them for many enjoyable hours of struggling to get on the right path, hunting down insidious bugs, and on occasion, finding gold.

We hope that you enjoy the book.

ALOK SUTRADHAR, G. H. PAULINO, L. J. GRAY

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Sutradhar, A.; Paulino, G.; Gray, L.J.

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