# 2 Quantification of Spread Risk by Means of Historical Simulation

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## 2.1 Introduction

Modeling spread risk for interest rate products, i.e., changes of the yield difference between a yield curve characterizing a class of equally risky assets and a riskless benchmark curve, is a challenge for any financial institution seeking to estimate the amount of economic capital utilized by trading and treasury activities. With the help of standard tools this contribution investigates some of the characteristic features of yield spread time series available from commercial data providers. From the properties of these time series it becomes obvious that the application of the parametric variance-covariance-approach for estimating idiosyncratic interest rate risk should be called into question. Instead we apply the non-parametric technique of historical simulation to synthetic zero-bonds of different riskiness, in order to quantify general market risk and spread risk of the bond. The quality of value-at-risk predictions is checked by a backtesting procedure based on a mark-to-model profit/loss calculation for the zero-bond market values. From the backtesting results we derive conclusions for the implementation of internal risk models within financial institutions.

## 2.2 Risk Categories – a Definition of Terms

For the analysis of obligor-specific and market-sector-specific influence on bond price risk we make use of the following subdivision of “price risk”, Gaumert (1999), Bundesaufsichtsamt für das Kreditwesen (2001).

1. General market risk: This risk category comprises price changes of a financial instrument, which are caused by changes of the general

market situation. General market conditions in the interest rate sector are characterized by the shape and the moves of benchmark yield curves, which are usually constructed from several benchmark instruments. The benchmark instruments are chosen in such a way so that they allow for a representative view on present market conditions in a particular market sector.

2. Residual risk: Residual risk characterizes the fact that the actual price of a given financial instrument can change in a way different from the changes of the market benchmark (however, abrupt changes which are caused by events in the sphere of the obligor are excluded from this risk category). These price changes cannot be accounted for by the volatility of the market benchmark. Residual risk is contained in the day-to-day price variation of a given instrument relative to the market benchmark and, thus, can be observed continuously in time. Residual risk is also called *idiosyncratic risk*.
3. Event risk: Abrupt price changes of a given financial instrument relative to the benchmark, which significantly exceed the continuously observable price changes due to the latter two risk categories, are called event risk. Such price jumps are usually caused by events in the sphere of the obligor. They are observed infrequently and irregularly.

Residual risk and event risk form the two components of so-called specific price risk or *specific risk* — a term used in documents on banking regulation, Bank for International Settlements (1998a), Bank for International Settlements (1998b) — and characterize the contribution of the individual risk of a given financial instrument to its overall risk.

The distinction between general market risk and residual risk is not unique but depends on the choice of the benchmark curve, which is used in the analysis of general market risk: The market for interest rate products in a given currency has a substructure (market-sectors), which is reflected by product-specific (swaps, bonds, etc.), industry-specific (bank, financial institution, retail company, etc.) and rating-specific (AAA, AA, A, BBB, etc.) yield curves. For the most liquid markets (USD, EUR, JPY), data for these submarkets is available from commercial data providers like Bloomberg. Moreover, there are additional influencing factors like collateral, financial restrictions etc., which give rise to further variants of the yield curves mentioned above. Presently, however, hardly any standardized data on these factors is available from data providers.

The larger the universe of benchmark curves a bank uses for modeling its interest risk, the smaller is the residual risk. A bank, which e.g. only uses product-specific yield curves but neglects the influence of industry- and rating-specific effects in modelling its general market risk, can expect specific price risk to be significantly larger than in a bank which includes these influences in modeling general market risk. The difference is due to the consideration of product-, industry- and rating-specific spreads over the benchmark curve for (almost) riskless government bonds. This leads to the question, whether the risk of a spread change, the *spread risk*, should be interpreted as part of the general market risk or as part of the specific risk. The uncertainty is due to the fact that it is hard to define what a market-sector is. The definition of benchmark curves for the analysis of general market risk depends, however, critically on the market sectors identified.

We will not further pursue this question in the following but will instead investigate some properties of this spread risk and draw conclusions for modeling spread risk within internal risk models. We restrict ourselves to the continuous changes of the yield curves and the spreads, respectively, and do not discuss event risk. In this contribution different methods for the quantification of the risk of a fictive USD zero bond are analyzed. Our investigation is based on time series of daily market yields of US treasury bonds and US bonds (banks and industry) of different credit quality (rating) and time to maturity.

## 2.3 Yield Spread Time Series

Before we start modeling the interest rate and spread risk we will investigate some of the descriptive statistics of the spread time series. Our investigations are based on commercially available yield curve histories. The Bloomberg dataset we use in this investigation consists of daily yield data for US treasury bonds as well as for bonds issued by banks and financial institutions with ratings AAA, AA+/AA, A+, A, A− (we use the Standard & Poor‘s naming convention) and for corporate/industry bonds with ratings AAA, AA, AA−, A+, A, A−, BBB+, BBB, BBB−, BB+, BB, BB−, B+, B, B−. The data we use for the industry sector covers the time interval from March 09 1992 to June 08 2000 and corresponds to 2147 observations. The data for banks/financial institutions covers the interval from March 09 1992 to September 14 1999 and corresponds to 1955 observations. We use yields for 3 and 6 month (3M, 6M) as well as 1, 2, 3, 4, 5, 7, and 10 year maturities (1Y, 2Y, 3Y, 4Y, 5Y, 7Y, 10Y). Each yield curve is based on information on the prices of a set of representative bonds with different maturities. The yield curve, of course, depends on the choice of bonds. Yields are option-adjusted but not corrected for coupon payments. The yields for the chosen maturities are constructed by Bloomberg's interpolation algorithm for yield curves. We use the USD treasury curve as a benchmark for riskless rates and calculate yield spreads relative to the benchmark curve for the different rating categories and the two industries. We correct the data history for obvious flaws using complementary information from other data sources. Some parts of our analysis in this section can be compared with the results given in Kiesel, Perraudin and Taylor (1999).

### 2.3.1 Data Analysis

We store the time series of the different yield curves in individual files. The file names, the corresponding industries and ratings and the names of the matrices used in the XploRe code are listed in Table 2.2. Each file contains data for the maturities 3M to 10Y in columns 4 to 12. XploRe creates matrices from the data listed in column 4 of Table 2.2 and produces summary statistics for the different yield curves. As example files the data sets for US treasury and industry bonds with rating AAA are provided. The output of the `summarize` command for the `INAAA` curve is given in Table 2.1.

| | **Minimum** | **Maximum** | **Mean** | **Median** | **Std.Error** |
|---|---|---|---|---|---|
| 3M | 3.13 | 6.93 | 5.0952 | 5.44 | 0.95896 |
| 6M | 3.28 | 7.16 | 5.2646 | 5.58 | 0.98476 |
| 1Y | 3.59 | 7.79 | 5.5148 | 5.75 | 0.95457 |
| 2Y | 4.03 | 8.05 | 5.8175 | 5.95 | 0.86897 |
| 3Y | 4.4 | 8.14 | 6.0431 | 6.1 | 0.79523 |
| 4Y | 4.65 | 8.21 | 6.2141 | 6.23 | 0.74613 |
| 5Y | 4.61 | 8.26 | 6.3466 | 6.36 | 0.72282 |
| 7Y | 4.75 | 8.3 | 6.5246 | 6.52 | 0.69877 |
| 10Y | 4.87 | 8.36 | 6.6962 | 6.7 | 0.69854 |

Table 2.1. Output of `summarize` for the `INAAA` curve. `XFGsummary`

The long term means are of particular interest. Therefore, we summarize them in Table 2.3. In order to get an impression of the development of the treasury yields in time, we plot the time series for the `USTF` 3M, 1Y, 2Y, 5Y, and 10Y yields. The results are displayed in Figure 2.1, `XFGtreasury` . The averaged yields within the observation period are displayed in Figure 2.2 for `USTF`, `INAAA`, `INBBB2`, `INBB2` and `INB2`, `XFGyields` .

In the next step we calculate spreads relative to the treasury curve by subtracting the treasury curve from the rating-specific yield curves and store

| Industry | Rating | File Name | Matrix Name |
|---|---|---|---|
| Government | riskless | USTF | USTF |
| Industry | AAA | INAAA | INAAA |
| Industry | AA | INAA2.DAT | INAA2 |
| Industry | AA- | INAA3.DAT | INAA3 |
| Industry | A+ | INA1.DAT | INA1 |
| Industry | A | INA2.DAT | INA2 |
| Industry | A- | INA3.DAT | INA3 |
| Industry | BBB+ | INBBB1.DAT | INBBB1 |
| Industry | BBB | INBBB2.DAT | INBBB2 |
| Industry | BBB- | INBBB3.DAT | INBBB3 |
| Industry | BB+ | INBB1.DAT | INBB1 |
| Industry | BB | INBB2.DAT | INBB2 |
| Industry | BB- | INBB3.DAT | INBB3 |
| Industry | B+ | INB1.DAT | INB1 |
| Industry | B | INB2.DAT | INB2 |
| Industry | B- | INB3.DAT | INB3 |
| Bank | AAA | BNAAA.DAT | BNAAA |
| Bank | AA+/AA | BNAA12.DAT | BNAA12 |
| Bank | A+ | BNA1.DAT | BNA1 |
| Bank | A | BNA2.DAT | BNA2 |
| Bank | A- | BNA3.DAT | BNA3 |

Table 2.2. Data variables

them to variables SINAAA, SINAA2, etc. For illustrative purposes we display time series of the 1Y, 2Y, 3Y, 5Y, 7Y, and 10Y spreads for the curves INAAA, INA2, INBBB2, INBB2, INB2 in Figure 2.3, XFGseries .

We run the summary statistics to obtain information on the mean spreads. Our results, which can also be obtained with the mean command, are collected in Table 2.4, XFGmeans .

| **Curve** | **3M** | **6M** | **1Y** | **2Y** | **3Y** | **4Y** | **5Y** | **7Y** | **10Y** |
|---|---|---|---|---|---|---|---|---|---|
| USTF | 4.73 | 4.92 | 5.16 | 5.50 | 5.71 | 5.89 | 6.00 | 6.19 | 6.33 |
| INAAA | 5.10 | 5.26 | 5.51 | 5.82 | 6.04 | 6.21 | 6.35 | 6.52 | 6.70 |
| INAA2 | 5.19 | 5.37 | 5.59 | 5.87 | 6.08 | 6.26 | 6.39 | 6.59 | 6.76 |
| INAA3 | 5.25 | - | 5.64 | 5.92 | 6.13 | 6.30 | 6.43 | 6.63 | 6.81 |
| INA1 | 5.32 | 5.50 | 5.71 | 5.99 | 6.20 | 6.38 | 6.51 | 6.73 | 6.90 |
| INA2 | 5.37 | 5.55 | 5.76 | 6.03 | 6.27 | 6.47 | 6.61 | 6.83 | 7.00 |
| INA3 | - | - | 5.84 | 6.12 | 6.34 | 6.54 | 6.69 | 6.91 | 7.09 |
| INBBB1 | 5.54 | 5.73 | 5.94 | 6.21 | 6.44 | 6.63 | 6.78 | 7.02 | 7.19 |
| INBBB2 | 5.65 | 5.83 | 6.03 | 6.31 | 6.54 | 6.72 | 6.86 | 7.10 | 7.27 |
| INBBB3 | 5.83 | 5.98 | 6.19 | 6.45 | 6.69 | 6.88 | 7.03 | 7.29 | 7.52 |
| INBB1 | 6.33 | 6.48 | 6.67 | 6.92 | 7.13 | 7.29 | 7.44 | 7.71 | 7.97 |
| INBB2 | 6.56 | 6.74 | 6.95 | 7.24 | 7.50 | 7.74 | 7.97 | 8.34 | 8.69 |
| INBB3 | 6.98 | 7.17 | 7.41 | 7.71 | 7.99 | 8.23 | 8.46 | 8.79 | 9.06 |
| INB1 | 7.32 | 7.53 | 7.79 | 8.09 | 8.35 | 8.61 | 8.82 | 9.13 | 9.39 |
| INB2 | 7.80 | 7.96 | 8.21 | 8.54 | 8.83 | 9.12 | 9.37 | 9.68 | 9.96 |
| INB3 | 8.47 | 8.69 | 8.97 | 9.33 | 9.60 | 9.89 | 10.13 | 10.45 | 10.74 |
| BNAAA | 5.05 | 5.22 | 5.45 | 5.76 | 5.99 | 6.20 | 6.36 | 6.60 | 6.79 |
| BNAA12 | 5.14 | 5.30 | 5.52 | 5.83 | 6.06 | 6.27 | 6.45 | 6.68 | 6.87 |
| BNA1 | 5.22 | 5.41 | 5.63 | 5.94 | 6.19 | 6.39 | 6.55 | 6.80 | 7.00 |
| BNA2 | 5.28 | 5.47 | 5.68 | 5.99 | 6.24 | 6.45 | 6.61 | 6.88 | 7.07 |
| BNA3 | 5.36 | 5.54 | 5.76 | 6.07 | 6.32 | 6.52 | 6.68 | 6.94 | 7.13 |

Table 2.3. Long term mean for different USD yield curves

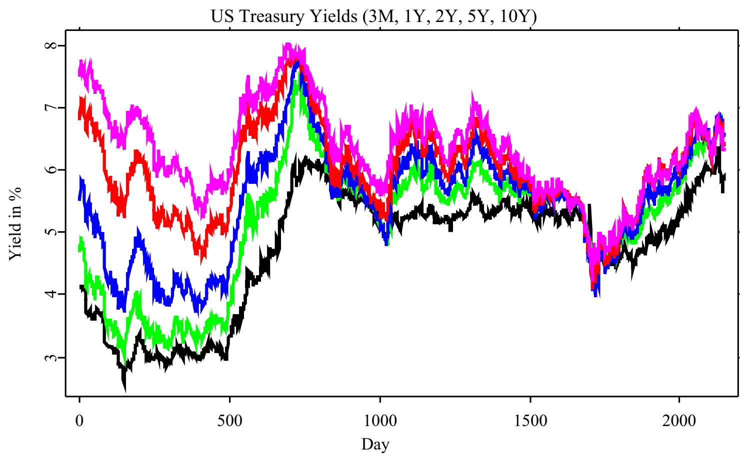


Figure 2.1. US Treasury Yields. `XFGtreasury`

Now we calculate the 1-day spread changes from the observed yields and store them to variables `DASIN01AAA`, etc. We run the `descriptive` routine to

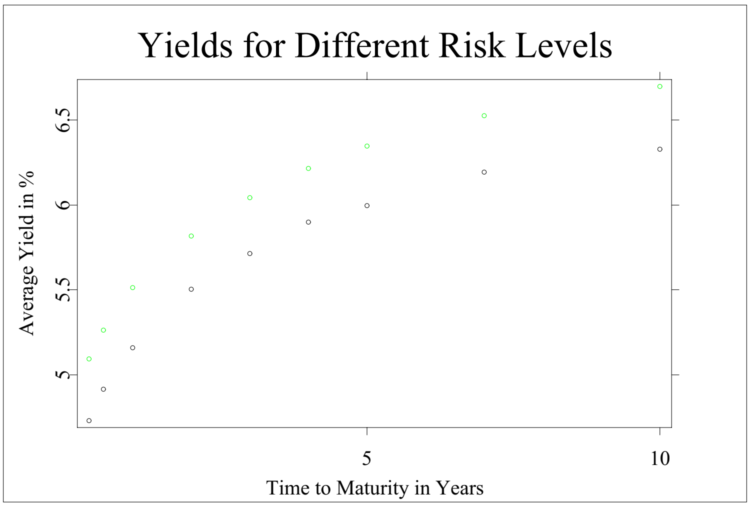


Figure 2.2. Averaged Yields. XFGyields

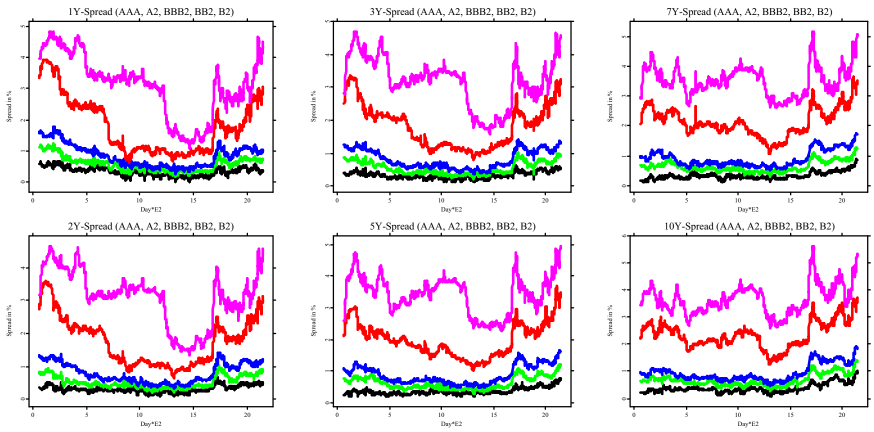


Figure 2.3. Credit Spreads. XFGseries

calculate the first four moments of the distribution of absolute spread changes. Volatility as well as skewness and kurtosis for selected curves are displayed in Tables 2.5, 2.6 and 2.7.

XFGchange

| Curve | 3M | 6M | 1Y | 2Y | 3Y | 4Y | 5Y | 7Y | 10Y |
|---|---|---|---|---|---|---|---|---|---|
| INAAA | 36 | 35 | 35 | 31 | 33 | 31 | 35 | 33 | 37 |
| INAA2 | 45 | 45 | 43 | 37 | 37 | 36 | 40 | 39 | 44 |
| INAA3 | 52 | - | 48 | 42 | 42 | 40 | 44 | 44 | 49 |
| INA1 | 58 | 58 | 55 | 49 | 49 | 49 | 52 | 53 | 57 |
| INA2 | 63 | 63 | 60 | 53 | 56 | 57 | 62 | 64 | 68 |
| INA3 | - | - | 68 | 62 | 63 | 64 | 69 | 72 | 76 |
| INBBB1 | 81 | 82 | 78 | 71 | 72 | 74 | 79 | 83 | 86 |
| INBBB2 | 91 | 91 | 87 | 80 | 82 | 82 | 87 | 90 | 94 |
| INBBB3 | 110 | 106 | 103 | 95 | 98 | 98 | 104 | 110 | 119 |
| INBB1 | 160 | 156 | 151 | 142 | 141 | 140 | 145 | 151 | 164 |
| INBB2 | 183 | 182 | 179 | 173 | 179 | 185 | 197 | 215 | 236 |
| INBB3 | 225 | 225 | 225 | 221 | 228 | 233 | 247 | 259 | 273 |
| INB1 | 259 | 261 | 263 | 259 | 264 | 271 | 282 | 294 | 306 |
| INB2 | 306 | 304 | 305 | 304 | 311 | 322 | 336 | 348 | 363 |
| INB3 | 373 | 377 | 380 | 382 | 389 | 400 | 413 | 425 | 441 |
| BNAAA | 41 | 39 | 38 | 33 | 35 | 35 | 41 | 43 | 47 |
| BNAA12 | 50 | 47 | 45 | 40 | 42 | 42 | 49 | 52 | 56 |
| BNA1 | 57 | 59 | 57 | 52 | 54 | 54 | 59 | 64 | 68 |
| BNA2 | 64 | 65 | 62 | 57 | 59 | 60 | 65 | 71 | 75 |
| BNA3 | 72 | 72 | 70 | 65 | 67 | 67 | 72 | 76 | 81 |

Table 2.4. Mean spread in basis points p.a.

For the variable `DASIN01AAA[,12]` (the 10 year AAA spreads) we demonstrate the output of the `descriptive` command in Table 2.8.

Finally we calculate 1-day relative spread changes and run the `descriptive` command. The results for the estimates of volatility, skewness and kurtosis are summarized in Tables 2.9, 2.10 and 2.11. `XFGrelchange`

## 2.3.2 Discussion of Results

*Time Development of Yields and Spreads:* The time development of US treasury yields displayed in Figure 2.1 indicates that the yield curve was steeper at the beginning of the observation period and flattened in the second half. However, an inverse shape of the yield curve occurred hardly ever. The long term average of the US treasury yield curve, the lowest curve in Figure 2.2, also has an upward sloping shape.

The time development of the spreads over US treasury yields displayed in Figure 2.3 is different for different credit qualities. While there is a large variation of spreads for the speculative grades, the variation in the investment grade sector is much smaller. A remarkable feature is the significant spread

| Curve | 3M | 6M | 1Y | 2Y | 3Y | 4Y | 5Y | 7Y | 10Y |
|---|---|---|---|---|---|---|---|---|---|
| INAAA | 4.1 | 3.5 | 3.3 | 2.3 | 2.4 | 2.2 | 2.1 | 2.2 | 2.5 |
| INAA2 | 4.0 | 3.5 | 3.3 | 2.3 | 2.4 | 2.2 | 2.2 | 2.2 | 2.5 |
| INAA3 | 4.0 | - | 3.3 | 2.2 | 2.3 | 2.2 | 2.2 | 2.2 | 2.5 |
| INA1 | 4.0 | 3.7 | 3.3 | 2.3 | 2.4 | 2.2 | 2.2 | 2.2 | 2.6 |
| INA2 | 4.1 | 3.7 | 3.3 | 2.4 | 2.4 | 2.1 | 2.2 | 2.3 | 2.5 |
| INA3 | - | - | 3.4 | 2.4 | 2.4 | 2.2 | 2.2 | 2.3 | 2.6 |
| INBBB1 | 4.2 | 3.6 | 3.2 | 2.3 | 2.3 | 2.2 | 2.1 | 2.3 | 2.6 |
| INBBB2 | 4.0 | 3.5 | 3.4 | 2.3 | 2.4 | 2.1 | 2.2 | 2.3 | 2.6 |
| INBBB3 | 4.2 | 3.6 | 3.5 | 2.4 | 2.5 | 2.2 | 2.3 | 2.5 | 2.9 |
| INBB1 | 4.8 | 4.4 | 4.1 | 3.3 | 3.3 | 3.1 | 3.1 | 3.9 | 3.4 |
| INBB2 | 4.9 | 4.6 | 4.5 | 3.8 | 3.8 | 3.8 | 3.7 | 4.3 | 4.0 |
| INBB3 | 5.5 | 5.1 | 4.9 | 4.3 | 4.4 | 4.2 | 4.1 | 4.7 | 4.3 |
| INB1 | 6.0 | 5.2 | 4.9 | 4.5 | 4.5 | 4.4 | 4.4 | 4.9 | 4.6 |
| INB2 | 5.6 | 5.2 | 5.2 | 4.8 | 4.9 | 4.8 | 4.8 | 5.3 | 4.9 |
| INB3 | 5.8 | 6.1 | 6.4 | 5.1 | 5.2 | 5.1 | 5.1 | 5.7 | 5.3 |
| BNAAA | 3.9 | 3.5 | 3.3 | 2.5 | 2.5 | 2.3 | 2.2 | 2.3 | 2.6 |
| BNAA12 | 5.4 | 3.6 | 3.3 | 2.4 | 2.3 | 2.2 | 2.1 | 2.3 | 2.6 |
| BNA1 | 4.1 | 3.7 | 3.2 | 2.1 | 2.2 | 2.1 | 2.0 | 2.2 | 2.6 |
| BNA2 | 3.8 | 3.5 | 3.1 | 2.3 | 2.2 | 2.0 | 2.1 | 2.2 | 2.5 |
| BNA3 | 3.8 | 3.5 | 3.2 | 2.2 | 2.2 | 2.1 | 2.1 | 2.2 | 2.5 |

Table 2.5. volatility for absolute spread changes in basis points p.a.

| Curve | 3M | 6M | 1Y | 2Y | 3Y | 4Y | 5Y | 10Y |
|---|---|---|---|---|---|---|---|---|
| INAAA | 0.1 | 0.0 | -0.1 | 0.6 | 0.5 | 0.0 | -0.5 | 0.6 |
| INAA2 | 0.0 | -0.2 | 0.0 | 0.4 | 0.5 | -0.1 | -0.2 | 0.3 |
| INA2 | 0.0 | -0.3 | 0.1 | 0.2 | 0.4 | 0.1 | -0.1 | 0.4 |
| INBBB2 | 0.2 | 0.0 | 0.2 | 1.0 | 1.1 | 0.5 | 0.5 | 0.9 |
| INBB2 | -0.2 | -0.5 | -0.4 | -0.3 | 0.3 | 0.5 | 0.4 | -0.3 |

Table 2.6. Skewness for absolute 1-day spread changes (in $\sigma^3$).

| Curve | 3M | 6M | 1Y | 2Y | 3Y | 4Y | 5Y | 10Y |
|---|---|---|---|---|---|---|---|---|
| INAAA | 12.7 | 6.0 | 8.1 | 10.1 | 16.8 | 9.1 | 11.2 | 12.8 |
| INAA2 | 10.5 | 6.4 | 7.8 | 10.1 | 15.8 | 7.8 | 9.5 | 10.0 |
| INA2 | 13.5 | 8.5 | 9.2 | 12.3 | 18.2 | 8.2 | 9.4 | 9.8 |
| INBBB2 | 13.7 | 7.0 | 9.9 | 14.5 | 21.8 | 10.5 | 13.9 | 14.7 |
| INBB2 | 11.2 | 13.0 | 11.0 | 15.8 | 12.3 | 13.2 | 11.0 | 11.3 |

Table 2.7. Kurtosis for absolute spread changes (in $\sigma^4$).

increase for all credit qualities in the last quarter of the observation period which coincides with the emerging market crises in the late 90s. The term

```
=========================================================
 Variable 10Y
=========================================================

 Mean          0.000354147
 Std.Error       0.0253712    Variance      0.000643697

 Minimum             -0.18    Maximum               0.2
 Range                0.38

 Lowest cases                 Highest cases
       1284:         -0.18           1246:         0.14
       1572:         -0.14           1283:         0.14
       1241:         -0.13           2110:         0.19
       1857:         -0.11           1062:         0.19
        598:          -0.1           2056:          0.2

 Median                  0
 25% Quartile        -0.01    75% Quartile         0.01

 Skewness         0.609321    Kurtosis          9.83974

 Observations                   2146
 Distinct observations            75

 Total number of {-Inf,Inf,NaN}    0
=========================================================
```

Table 2.8. Output of descriptive for the 10 years AAA spread.

| **Curve** | **3M** | **6M** | **1Y** | **2Y** | **3Y** | **4Y** | **5Y** | **7Y** | **10Y** |
|---|---|---|---|---|---|---|---|---|---|
| INAAA | 36.0 | 19.2 | 15.5 | 8.9 | 8.4 | 8.0 | 6.4 | 7.8 | 10.4 |
| INAA2 | 23.5 | 13.1 | 11.2 | 7.2 | 7.4 | 6.4 | 5.8 | 6.2 | 7.6 |
| INAA3 | 13.4 | - | 9.0 | 5.8 | 6.2 | 5.3 | 5.0 | 5.8 | 6.4 |
| INA1 | 13.9 | 9.2 | 7.7 | 5.7 | 5.6 | 4.7 | 4.5 | 4.6 | 5.7 |
| INA2 | 11.5 | 8.1 | 7.1 | 5.1 | 4.9 | 4.3 | 4.0 | 4.0 | 4.5 |
| INA3 | - | - | 6.4 | 4.6 | 4.3 | 3.8 | 3.5 | 3.5 | 4.1 |
| INBBB1 | 8.1 | 6.0 | 5.4 | 3.9 | 3.7 | 3.3 | 3.0 | 3.2 | 3.8 |
| INBBB2 | 7.0 | 5.3 | 5.0 | 3.3 | 3.3 | 2.9 | 2.8 | 2.9 | 3.3 |
| INBBB3 | 5.7 | 4.7 | 4.4 | 3.2 | 3.0 | 2.7 | 2.5 | 2.6 | 2.9 |
| INBB1 | 4.3 | 3.8 | 3.4 | 2.5 | 2.4 | 2.2 | 2.1 | 2.5 | 2.2 |
| INBB2 | 3.7 | 3.3 | 3.0 | 2.2 | 2.1 | 2.0 | 1.8 | 2.0 | 1.7 |
| INBB3 | 3.2 | 2.8 | 2.5 | 2.0 | 1.9 | 1.8 | 1.6 | 1.8 | 1.5 |
| INB1 | 3.0 | 2.4 | 2.1 | 1.7 | 1.7 | 1.6 | 1.5 | 1.6 | 1.5 |
| INB2 | 2.3 | 2.1 | 1.9 | 1.6 | 1.6 | 1.5 | 1.4 | 1.5 | 1.3 |
| INB3 | 1.8 | 2.2 | 2.3 | 1.3 | 1.3 | 1.2 | 1.2 | 1.3 | 1.1 |
| BNAAA | 37.0 | 36.6 | 16.9 | 9.8 | 9.0 | 8.2 | 6.1 | 5.9 | 6.5 |
| BNAA12 | 22.8 | 9.7 | 8.3 | 7.0 | 6.3 | 5.8 | 4.6 | 4.8 | 5.5 |
| BNA1 | 36.6 | 10.1 | 7.9 | 5.6 | 4.8 | 4.4 | 3.8 | 3.9 | 4.4 |
| BNA2 | 17.8 | 8.0 | 6.6 | 4.5 | 4.1 | 3.6 | 3.4 | 3.3 | 3.7 |
| BNA3 | 9.9 | 6.9 | 5.6 | 3.7 | 3.6 | 3.3 | 3.1 | 3.1 | 3.4 |

Table 2.9. Volatility for relative spread changes in %

structure of the long term averages of the rating-specific yield curves is also normal. The spreads over the benchmark curve increase with decreasing credit quality.

| Curve | 3M | 6M | 1Y | 2Y | 3Y | 4Y | 5Y | 10Y |
|---|---|---|---|---|---|---|---|---|
| INAAA | 2.3 | 4.6 | 4.3 | 2.2 | 2.3 | 2.1 | 0.6 | 4.6 |
| INAA2 | 5.4 | 2.6 | 3.7 | 1.6 | 2.0 | 0.6 | 0.8 | 1.8 |
| INA2 | 7.6 | 1.5 | 1.2 | 0.9 | 1.6 | 0.8 | 0.9 | 0.8 |
| INBBB2 | 5.5 | 0.7 | 0.8 | 0.8 | 1.4 | 0.8 | 0.7 | 0.8 |
| INBB2 | 0.8 | 0.4 | 0.6 | 0.3 | 0.4 | 0.5 | 0.3 | -0.2 |

Table 2.10. Skewness for relative spread changes (in $\sigma^3$).

| Curve | 3M | 6M | 1Y | 2Y | 3Y | 4Y | 5Y | 10Y |
|---|---|---|---|---|---|---|---|---|
| INAAA | 200.7 | 54.1 | 60.1 | 27.8 | 28.3 | 33.9 | 16.8 | 69.3 |
| INAA2 | 185.3 | 29.5 | 60.5 | 22.1 | 27.4 | 11.0 | 17.5 | 23.0 |
| INA2 | 131.1 | 22.1 | 18.0 | 13.9 | 26.5 | 16.4 | 18.5 | 13.9 |
| INBBB2 | 107.1 | 13.9 | 16.9 | 12.0 | 20.0 | 14.0 | 16.6 | 16.7 |
| INBB2 | 16.3 | 11.9 | 12.9 | 12.4 | 11.0 | 10.1 | 10.2 | 12.0 |

Table 2.11. Kurtosis for relative spread changes (in $\sigma^4$).

*Mean Spread:* The term structure of the long term averages of the rating-specific yield curves, which is displayed in Figure 2.3, is normal (see also Table 2.4). The spreads over the benchmark curve increase with decreasing credit quality. For long maturities the mean spreads are larger than for intermediate maturities as expected. However, for short maturities the mean spreads are larger compared with intermediate maturities.

*Volatility:* The results for the volatility for absolute 1-day spread changes in basis points p.a. are listed in Table 2.5. From short to intermediate maturities the volatilities decrease. For long maturities a slight volatility increase can be observed compared to intermediate maturities. For equal maturities volatility is constant over the investment grade ratings, while for worse credit qualities a significant increase in absolute volatility can be observed. Volatility for relative spread changes is much larger for short maturities than for intermediate and long maturities. As in the case of absolute spread changes, a slight volatility increase exists for the transition from intermediate to long maturities. Since absolute spreads increase more strongly with decreasing credit quality than absolute spread volatility, relative spread volatility decreases with decreasing credit quality (see Table 2.9).

*Skewness:* The results for absolute 1-day changes (see Table 2.6) are all close to zero, which indicates that the distribution of changes is almost symmetric. The corresponding distribution of relative changes should have a positive skewness, which is indeed the conclusion from the results in Table 2.10.

*Kurtosis:* The absolute 1-day changes lead to a kurtosis, which is significantly larger than 3 (see Table 2.6). Thus, the distribution of absolute changes is leptokurtic. There is no significant dependence on credit quality or maturity. The distribution of relative 1-day changes is also leptokurtic (see Table 2.10). The deviation from normality increases with decreasing credit quality and decreasing maturity.

We visualize symmetry and leptokursis of the distribution of absolute spread changes for the `INAAA 10Y` data in Figure 2.4, where we plot the empirical distribution of absolute spreads around the mean spread in an averaged shifted histogram and the normal distribution with the variance estimated from historical data.

`XFGdist`

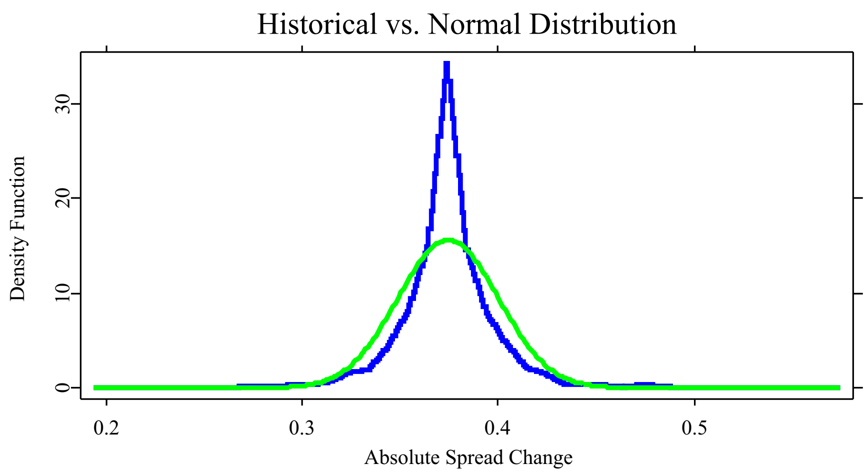


Figure 2.4. Historical distribution and estimated normal distribution. `XFGdist`

We note that by construction the area below both curves is normalized to one. We calculate the 1%, 10%, 90% and 99% quantiles of the spread distribution with the `quantile` command. Those quantiles are popular in market risk management. For the data used to generate Figure 2.4 the results are 0.30%, 0.35%, 0.40%, and 0.45%, respectively. The corresponding quantiles of the plotted normal distribution are 0.31%, 0.34%, 0.41%, 0.43%. The differences are less obvious than the difference in the shape of the distributions. However, in a portfolio with different financial instruments, which is exposed to different risk factors with different correlations, the difference in the shape of the distribution can play an important role. That is why a simple variance-covariance approach, J.P. Morgan (1996) and Kiesel et al. (1999), seems not adequate to capture spread risk.

## 2.4 Historical Simulation and Value at Risk

We investigate the behavior of a fictive zero-bond of a given credit quality with principal 1 USD, which matures after $T$ years. In all simulations $t = 0$ denotes the beginning and $t = T$ the end of the lifetime of the zero-bond. The starting point of the simulation is denoted by $t_0$, the end by $t_1$. The observation period, i.e., the time window investigated, consists of $N \geq 1$ trading days and the holding period of $h \geq 1$ trading days. The confidence level for the VaR is $\alpha \in [0, 1]$. At each point in time $0 \leq t \leq t_1$ the risky yields $R_i(t)$ (full yield curve) and the riskless treasury yields $B_i(t)$ (benchmark curve) for any time to maturity $0 < T_1 < \cdots < T_n$ are contained in our data set for $1 \leq i \leq n$, where $n$ is the number of different maturities. The corresponding spreads are defined by $S_i(t) = R_i(t) - B_i(t)$ for $1 \leq i \leq n$.

In the following subsections 2.4.1 to 2.4.5 we specify different variants of the historical simulation method which we use for estimating the distribution of losses from the zero-bond position. The estimate for the distribution of losses can then be used to calculate the quantile-based risk measure Value-at-Risk. The variants differ in the choice of risk factors, i.e., in our case the components of the historical yield time series. In Section 2.6 we describe how the VaR estimation is carried out with XploRe commands provided that the loss distribution has been estimated by means of one of the methods introduced and can be used as an input variable.

### 2.4.1 Risk Factor: Full Yield

**1. Basic Historical Simulation:**

We consider a historical simulation, where the risk factors are given by the full yield curve, $R_i(t)$ for $i = 1, \ldots, n$. The yield $R(t, T - t)$ at time $t_0 \leq t \leq t_1$ for the remaining time to maturity $T - t$ is determined by means of linear interpolation from the adjacent values $R_i(t) = R(t, T_i)$ and $R_{i+1}(t) = R(t, T_{i+1})$ with $T_i \leq T - t < T_{i+1}$ (for reasons of simplicity we do not consider remaining times to maturity $T - t < T_1$ and $T - t > T_n$):

$$R(t, T - t) = \frac{[T_{i+1} - (T - t)]R_i(t) + [(T - t) - T_i]R_{i+1}(t)}{T_{i+1} - T_i} . \tag{2.1}$$

The present value of the bond $PV(t)$ at time $t$ can be obtained by discounting,

$$PV(t) = \frac{1}{\big[1 + R(t, T-t)\big]^{T-t}}, \qquad t_0 \leq t \leq t_1. \tag{2.2}$$

In the historical simulation the relative risk factor changes

$$\Delta_i^{(k)}(t) = \frac{R_i\big(t - k/N\big) - R_i\big(t - (k+h)/N\big)}{R_i\big(t - (k+h)/N\big)}, \quad 0 \leq k \leq N-1, \tag{2.3}$$

are calculated for $t_0 \leq t \leq t_1$ and each $1 \leq i \leq n$. Thus, for each scenario $k$ we obtain a new fictive yield curve at time $t+h$, which can be determined from the observed yields and the risk factor changes,

$$R_i^{(k)}(t+h) = R_i(t)\big[1 + \Delta_i^{(k)}(t)\big], \qquad 1 \leq i \leq n, \tag{2.4}$$

by means of linear interpolation. This procedure implies that the distribution of risk factor changes is stationary between $t - (N-1+h)/N$ and $t$. Each scenario corresponds to a drawing from an identical and independent distribution, which can be related to an i.i.d. random variable $\varepsilon_i(t)$ with variance one via

$$\Delta_i(t) = \sigma_i \varepsilon_i(t). \tag{2.5}$$

This assumption implies homoscedasticity of the volatility of the risk factors, i.e., a constant volatility level within the observation period. If this were not the case, different drawings would originate from different underlying distributions. Consequently, a sequence of historically observed risk factor changes could not be used for estimating the future loss distribution.

In analogy to (2.1) for time $t+h$ and remaining time to maturity $T-t$ one obtains

$$R^{(k)}(t+h, T-t) = \frac{[T_{i+1} - (T-t)]R_i^{(k)}(t) + [(T-t) - T_i]R_{i+1}^{(k)}(t)}{T_{i+1} - T_i}$$

for the yield. With (2.2) we obtain a new fictive present value at time $t+h$:

$$PV^{(k)}(t+h) = \frac{1}{\big[1 + R^{(k)}(t+h, T-t)\big]^{T-t}}. \tag{2.6}$$

In this equation we neglected the effect of the shortening of the time to maturity in the transition from $t$ to $t+h$ on the present value. Such an approximation should be refined for financial instruments whose time to maturity/time to expiration is of the order of $h$, which is not relevant for the constellations investigated in the following.

Now the fictive present value $PV^{(k)}(t+h)$ is compared with the present value for unchanged yield $R(t+h, T-t) = R(t, T-t)$ for each scenario $k$ (here the remaining time to maturity is not changed, either).

$$PV(t+h) = \frac{1}{\left\{1 + R(t+h, T-t)\right\}^{T-t}} . \tag{2.7}$$

The loss occurring is

$$L^{(k)}(t+h) = PV(t+h) - PV^{(k)}(t+h) \qquad 0 \le k \le N-1, \tag{2.8}$$

i.e., losses in the economic sense are positive while profits are negative. The VaR is the loss which is not exceeded with a probability $\alpha$ and is estimated as the $[(1-\alpha)N+1]$-th-largest value in the set

$$\{L^{(k)}(t+h) \mid 0 \le k \le N-1\}.$$

This is the $(1-\alpha)$-quantile of the corresponding empirical distribution.

**2. Mean Adjustment:**

A refined historical simulation includes an adjustment for the average of those relative changes in the observation period which are used for generating the scenarios according to (2.3). If for fixed $1 \le i \le n$ the average of relative changes $\Delta_i^{(k)}(t)$ is different from 0, a trend is projected from the past to the future in the generation of fictive yields in (2.4). Thus the relative changes are corrected for the mean by replacing the relative change $\Delta_i^{(k)}(t)$ with $\Delta_i^{(k)}(t) - \overline{\Delta}_i(t)$ for $1 \le i \le n$ in (2.4):

$$\overline{\Delta}_i(t) = \frac{1}{N} \sum_{k=0}^{N-1} \Delta_i^{(k)}(t), \tag{2.9}$$

This mean correction is presented in Hull (1998).

**3. Volatility Updating:**

An important variant of historical simulation uses volatility updating Hull (1998). At each point in time $t$ the exponentially weighted volatility of relative historical changes is estimated for $t_0 \le t \le t_1$ by

$$\sigma_i^2(t) = (1-\gamma) \sum_{k=0}^{N-1} \gamma^k \left\{\Delta_i^{(k)}(t)\right\}^2, \qquad 1 \le i \le n. \tag{2.10}$$

The parameter $\gamma \in [0,1]$ is a decay factor, which must be calibrated to generate a best fit to empirical data. The recursion formula

$$\sigma_i^2(t) = (1-\gamma)\sigma_i^2(t-1/N) + \gamma\left\{\Delta_i^{(0)}(t)\right\}^2, \qquad 1 \le i \le n, \tag{2.11}$$

is valid for $t_0 \leq t \leq t_1$. The idea of volatility updating consists in adjusting the historical risk factor changes to the present volatility level. This is achieved by a renormalization of the relative risk factor changes from (2.3) with the corresponding estimation of volatility for the observation day and a multiplication with the estimate for the volatility valid at time $t$. Thus, we calculate the quantity

$$\delta_i^{(k)}(t) = \sigma_i(t) \cdot \frac{\Delta_i^{(k)}(t)}{\sigma_i(t-(k+h)/N)}, \qquad 0 \leq k \leq N-1. \tag{2.12}$$

In a situation, where risk factor volatility is heteroscedastic and, thus, the process of risk factor changes is not stationary, volatility updating cures this violation of the assumptions made in basic historical simulation, because the process of re-scaled risk factor changes $\Delta_i(t)/\sigma_i(t))$ is stationary. For each $k$ these renormalized relative changes are used in analogy to (2.4) for the determination of fictive scenarios:

$$R_i^{(k)}(t+h) = R_i(t)\big\{1 + \delta_i^{(k)}(t)\big\}, \qquad 1 \leq i \leq n, \tag{2.13}$$

The other considerations concerning the VaR calculation in historical simulation remain unchanged.

**4. Volatility Updating and Mean Adjustment:**

Within the volatility updating framework, we can also apply a correction for the average change according to 2.4.1(2). For this purpose, we calculate the average

$$\overline{\delta}_i(t) = \frac{1}{N}\sum_{k=0}^{N-1} \delta_i^{(k)}(t), \tag{2.14}$$

and use the adjusted relative risk factor change $\delta_i^{(k)}(t) - \overline{\delta}_i(t)$ instead of $\delta_i^{(k)}(t)$ in (2.13).

### 2.4.2 Risk Factor: Benchmark

In this subsection the risk factors are relative changes of the benchmark curve instead of the full yield curve. This restriction is adequate for quantifying general market risk, when there is no need to include spread risk. The risk factors are the yields $B_i(t)$ for $i = 1, \ldots, n$. The yield $B(t, T-t)$ at time $t$ for remaining time to maturity $T-t$ is calculated similarly to (2.1) from adjacent values by linear interpolation,

$$B(t, T-t) = \frac{\{T_{i+1} - (T-t)\}B_i(t) + \{(T-t) - T_i\}B_{i+1}(t)}{T_{i+1} - T_i}. \tag{2.15}$$

The generation of scenarios and the interpolation of the fictive benchmark curve is carried out in analogy to the procedure for the full yield curve. We use

$$\Delta_i^{(k)}(t) = \frac{B_i\big(t - k/N\big) - B_i\big(t - (k+h)/N\big)}{B_i\big(t - (k+h)/N\big)}, \quad 0 \le k \le N-1, \tag{2.16}$$

and

$$B_i^{(k)}(t+h) = B_i(t)\big[1 + \Delta_i^{(k)}(t)\big], \qquad 1 \le i \le n. \tag{2.17}$$

Linear interpolation yields

$$B^{(k)}(t+h, T-t) = \frac{\{T_{i+1} - (T-t)\}B_i^{(k)}(t) + \{(T-t) - T_i\}B_{i+1}^{(k)}(t)}{T_{i+1} - T_i}.$$

In the determination of the fictive full yield we now assume that the spread remains unchanged within the holding period. Thus, for the $k$-th scenario we obtain the representation

$$R^{(k)}(t+h, T-t) = B^{(k)}(t+h, T-t) + S(t, T-t), \tag{2.18}$$

which is used for the calculation of a new fictive present value and the corresponding loss. With this choice of risk factors we can introduce an adjustment for the average relative changes or/and volatility updating in complete analogy to the four variants described in the preceding subsection.

### 2.4.3 Risk Factor: Spread over Benchmark Yield

When we take the view that risk is only caused by spread changes but not by changes of the benchmark curve, we investigate the behavior of the spread risk factors $S_i(t)$ for $i = 1, \ldots, n$. The spread $S(t, T-t)$ at time $t$ for time to maturity $T-t$ is again obtained by linear interpolation. We now use

$$\Delta_i^{(k)}(t) = \frac{S_i\big(t - k/N\big) - S_i\big(t - (k+h)/N\big)}{S_i\big(t - (k+h)/N\big)}, \quad 0 \le k \le N-1, \tag{2.19}$$

and

$$S_i^{(k)}(t+h) = S_i(t)\big\{1 + \Delta_i^{(k)}(t)\big\}, \qquad 1 \le i \le n. \tag{2.20}$$

Here, linear interpolation yields

$$S^{(k)}(t+h, T-t) = \frac{\{T_{i+1} - (T-t)\}S_i^{(k)}(t) + \{(T-t) - T_i\}S_{i+1}^{(k)}(t)}{T_{i+1} - T_i}.$$

Thus, in the determination of the fictive full yield the benchmark curve is considered deterministic and the spread stochastic. This constellation is the opposite of the constellation in the preceding subsection. For the $k$-th scenario one obtains

$$R^{(k)}(t+h, T-t) = B(t, T-t) + S^{(k)}(t+h, T-t). \tag{2.21}$$

In this context we can also work with adjustment for average relative spread changes and volatility updating.

### 2.4.4 Conservative Approach

In the conservative approach we assume full correlation between risk from the benchmark curve and risk from the spread changes. In this worst case scenario we add (ordered) losses, which are calculated as in the two preceding sections from each scenario. From this loss distribution the VaR is determined.

### 2.4.5 Simultaneous Simulation

Finally, we consider simultaneous relative changes of the benchmark curve and the spreads. For this purpose (2.18) and (2.21) are replaced with

$$R^{(k)}(t+h, T-t) = B^{(k)}(t+h, T-t) + S^{(k)}(t, T-t), \tag{2.22}$$

where, again, corrections for average risk factor changes or/and volatility updating can be added. We note that the use of relative risk factor changes is the reason for different results of the variants in subsection 2.4.1 and this subsection.

## 2.5 Mark-to-Model Backtesting

A backtesting procedure compares the VaR prediction with the observed loss. In a mark-to-model backtesting the observed loss is determined by calculation of the present value before and after consideration of the actually observed risk factor changes. For $t_0 \leq t \leq t_1$ the present value at time $t+h$ is calculated with the yield $R(t+h, T-t)$, which is obtained from observed data for $R_i(t+h)$ by linear interpolation, according to

$$PV(t) = \frac{1}{\left\{1 + R(t+h, T-t)\right\}^{T-t}}. \tag{2.23}$$

This corresponds to a loss $L(t) = PV(t) - PV(t+h)$, where, again, the shortening of the time to maturity is not taken into account.

The different frameworks for the VaR estimation can easily be integrated into the backtesting procedure. When we, e.g., only consider changes of the benchmark curve, $R(t+h, T-t)$ in (2.23) is replaced with $B(t+h, T-t) + S(t, T-t)$. On an average $(1-\alpha) \cdot 100$ per cent of the observed losses in a given time interval should exceed the corresponding VaR (outliers). Thus, the percentage of observed losses is a measure for the predictive power of historical simulation.

## 2.6 VaR Estimation and Backtesting

In this section we explain, how a VaR can be calculated and a backtesting can be implemented with the help of XploRe routines. We present numerical results for the different yield curves. The VaR estimation is carried out with the help of the `VaRest` command. The `VaRest` command calculates a VaR for historical simulation, if one specifies the method parameter as "EDF" (empirical distribution function). However, one has to be careful when specifying the sequence of asset returns which are used as input for the estimation procedure. If one calculates zero-bond returns from relative risk factor changes (interest rates or spreads) the complete empirical distribution of the profits and losses must be estimated anew for each day from the $N$ relative risk factor changes, because the profit/loss observations are not identical with the risk factor changes.

For each day the $N$ profit/loss observations generated with one of the methods described in subsections 2.4.1 to 2.4.5 are stored to a new row in an array `PL`. The actual profit and loss data from a mark-to-model calculation for holding period $h$ are stored to a one-column-vector `MMPL`. It is not possible to use a continuous sequence of profit/loss data with overlapping time windows for the VaR estimation. Instead the `VaRest` command must be called separately for each day. The consequence is that the data the `VaRest` command operates on consists of a row of $N+1$ numbers: $N$ profit/loss values contained in the vector `(PL[t,])'`, which has one column and $N$ rows followed by the actual mark-to-model profit or loss `MMPL[t,1]` within holding period $h$ in the last row. The procedure is implemented in the quantlet `XFGpl` which can be downloaded from quantlet download page of this book.

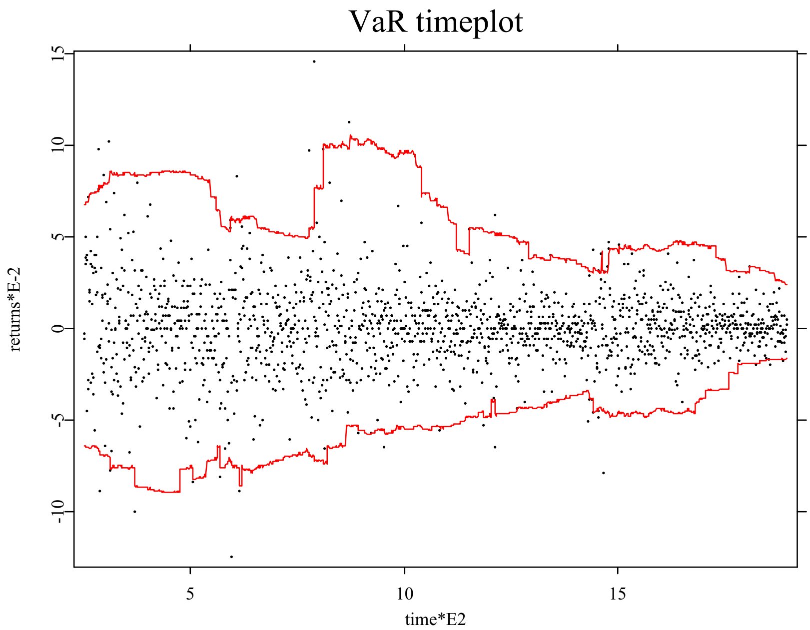


Figure 2.5. VaR time plot basic historical simulation. `XFGtimeseries`

The result is displayed for the `INAAA` curve in Figures. 2.5 (basic historical simulation) and 2.6 (historical simulation with volatility updating). The time plots allow for a quick detection of violations of the VaR prediction. A striking feature in the basic historical simulation with the full yield curve as risk factor is the platform-shaped VaR prediction, while with volatility updating the VaR prediction decays exponentially after the occurrence of peak events in the market data. This is a consequence of the exponentially weighted historical volatility in the scenarios. The peak VaR values are much larger for volatility updating than for the basic historical simulation.

In order to find out, which framework for VaR estimation has the best predictive power, we count the number of violations of the VaR prediction and divide it by the number of actually observed losses. We use the 99% quantile, for which we would expect an violation rate of 1% for an optimal VaR estimator. The history used for the drawings of the scenarios consists of $N = 250$ days, and the holding period is $h = 1$ day. For the volatility updating we use

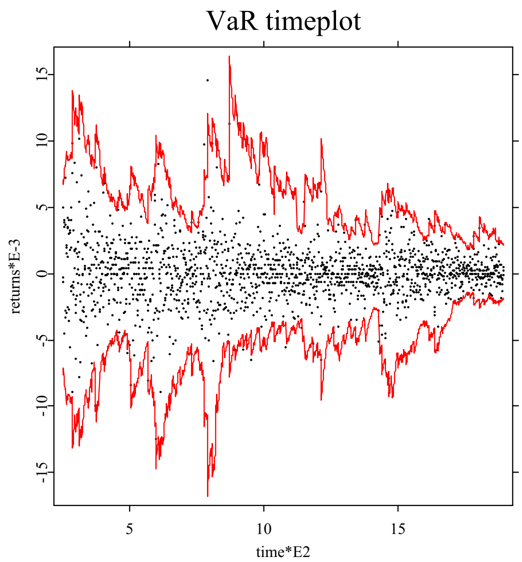


Figure 2.6. VaR time plot historical simulation with volatility updating. XFGtimeseries2

a decay factor of $\gamma = 0.94$, J.P. Morgan (1996). For the simulation we assume that the synthetic zero-bond has a remaining time to maturity of 10 years at the beginning of the simulations. For the calculation of the first scenario of a basic historical simulation $N + h - 1$ observations are required. A historical simulation with volatility updating requires $2(N + h - 1)$ observations preceding the trading day the first scenario refers to. In order to allow for a comparison between different methods for the VaR calculation, the beginning of the simulations is $t_0 = [2(N+h-1)/N]$. With these simulation parameters we obtain 1646 observations for a zero-bond in the industry sector and 1454 observations for a zero-bond in the banking sector.

In Tables 2.12 to 2.14 we list the percentage of violations for all yield curves and the four variants of historical simulation V1 to V4 (V1 = Basic Historical Simulation; V2 = Basic Historical Simulation with Mean Adjustment; V3 = Historical Simulation with Mean Adjustment; V4 = Historical Simulation with Volatility Updating and Mean Adjustment). In the last row we display the average of the violations of all curves. Table 2.12 contains the results for the simulation with relative changes of the full yield curves and of the yield spreads over the benchmark curve as risk factors. In Table 2.13 the risk factors are changes of the benchmark curves. The violations in the conservative approach and in the simultaneous simulation of relative spread and benchmark changes are listed in Table 2.14.

XFGexc

| | Full yield | | | | Spread curve | | | |
|---|---|---|---|---|---|---|---|---|
| **Curve** | **V1** | **V2** | **V3** | **V4** | **V1** | **V2** | **V3** | **V4** |
| INAAA | 1,34 | 1,34 | 1,09 | 1,28 | 1,34 | 1,34 | 1,34 | 1,34 |
| INAA2 | 1,34 | 1,22 | 1,22 | 1,22 | 1,46 | 1,52 | 1,22 | 1,22 |
| INAA3 | 1,15 | 1,22 | 1,15 | 1,15 | 1,09 | 1,09 | 0,85 | 0,91 |
| INA1 | 1,09 | 1,09 | 1,46 | 1,52 | 1,40 | 1,46 | 1,03 | 1,09 |
| INA2 | 1,28 | 1,28 | 1,28 | 1,28 | 1,15 | 1,15 | 0,91 | 0,91 |
| INA3 | 1,22 | 1,22 | 1,15 | 1,22 | 1,15 | 1,22 | 1,09 | 1,15 |
| INBBB1 | 1,28 | 1,22 | 1,09 | 1,15 | 1,46 | 1,46 | 1,40 | 1,40 |
| INBBB2 | 1,09 | 1,15 | 0,91 | 0,91 | 1,28 | 1,28 | 0,91 | 0,91 |
| INBBB3 | 1,15 | 1,15 | 1,09 | 1,09 | 1,34 | 1,34 | 1,46 | 1,52 |
| INBB1 | 1,34 | 1,28 | 1,03 | 1,03 | 1,28 | 1,28 | 0,97 | 0,97 |
| INBB2 | 1,22 | 1,22 | 1,22 | 1,34 | 1,22 | 1,22 | 1,09 | 1,09 |
| INBB3 | 1,34 | 1,28 | 1,28 | 1,22 | 1,09 | 1,28 | 1,09 | 1,09 |
| INB1 | 1,40 | 1,40 | 1,34 | 1,34 | 1,52 | 1,46 | 1,09 | 1,03 |
| INB2 | 1,52 | 1,46 | 1,28 | 1,28 | 1,34 | 1,40 | 1,15 | 1,15 |
| INB3 | 1,40 | 1,40 | 1,15 | 1,15 | 1,46 | 1,34 | 1,09 | 1,15 |
| BNAAA | 1,24 | 1,38 | 1,10 | 1,10 | 0,89 | 0,89 | 1,03 | 1,31 |
| BNAA1/2 | 1,38 | 1,24 | 1,31 | 1,31 | 1,03 | 1,10 | 1,38 | 1,38 |
| BNA1 | 1,03 | 1,03 | 1,10 | 1,17 | 1,03 | 1,10 | 1,24 | 1,24 |
| BNA2 | 1,24 | 1,31 | 1,24 | 1,17 | 0,76 | 0,83 | 1,03 | 1,03 |
| BNA3 | 1,31 | 1,24 | 1,17 | 1,10 | 1,03 | 1,10 | 1,24 | 1,17 |
| **Average** | **1,27** | **1,25** | **1,18** | **1,20** | **1,22** | **1,24** | **1,13** | **1,15** |

Table 2.12. Violations full yield and spread curve (in %)

| **Curve** | **V1** | **V2** | **V3** | **V4** |
|---|---|---|---|---|
| INAAA, INAA2, INAA3, INA1, INA2, INA3, INBBB1, INBBB2, INBBB3, INBB1, INBB2, INBB3, INB1, INB2, INB3 | 1,52 | 1,28 | 1,22 | 1,15 |
| BNAAA, BNAA1/2, BNA1, BNA2, BNA3 | 1,72 | 1,44 | 1,17 | 1,10 |
| **Average** | **1,57** | **1,32** | **1,20** | **1,14** |

Table 2.13. Violations benchmark curve (in %)

| | conservative approach | | | | simultaneous simulation | | | |
|---|---|---|---|---|---|---|---|---|
| **Curve** | **V1** | **V2** | **V3** | **V4** | **V1** | **V2** | **V3** | **V4** |
| INAAA | 0,24 | 0,24 | 0,30 | 0,30 | 1,22 | 1,28 | 0,97 | 1,03 |
| INAA2 | 0,24 | 0,30 | 0,36 | 0,30 | 1,22 | 1,28 | 1,03 | 1,15 |
| INAA3 | 0,43 | 0,36 | 0,30 | 0,30 | 1,22 | 1,15 | 1,09 | 1,09 |
| INA1 | 0,36 | 0,43 | 0,55 | 0,55 | 1,03 | 1,03 | 1,03 | 1,09 |
| INA2 | 0,49 | 0,43 | 0,49 | 0,49 | 1,34 | 1,28 | 0,97 | 0,97 |
| INA3 | 0,30 | 0,36 | 0,30 | 0,30 | 1,22 | 1,15 | 1,09 | 1,09 |
| INBBB1 | 0,43 | 0,49 | 0,36 | 0,36 | 1,09 | 1,09 | 1,03 | 1,03 |
| INBBB2 | 0,49 | 0,49 | 0,30 | 0,30 | 1,03 | 1,03 | 0,85 | 0,79 |
| INBBB3 | 0,30 | 0,30 | 0,36 | 0,36 | 1,15 | 1,22 | 1,03 | 1,03 |
| INBB1 | 0,36 | 0,30 | 0,43 | 0,43 | 1,34 | 1,34 | 1,03 | 0,97 |
| INBB2 | 0,43 | 0,36 | 0,43 | 0,43 | 1,40 | 1,34 | 1,15 | 1,09 |
| INBB3 | 0,30 | 0,30 | 0,36 | 0,36 | 1,15 | 1,15 | 0,91 | 0,91 |
| INB1 | 0,43 | 0,43 | 0,43 | 0,43 | 1,34 | 1,34 | 0,91 | 0,97 |
| INB2 | 0,30 | 0,30 | 0,30 | 0,30 | 1,34 | 1,34 | 0,97 | 1,03 |
| INB3 | 0,30 | 0,30 | 0,36 | 0,30 | 1,46 | 1,40 | 1,22 | 1,22 |
| BNAAA | 0,62 | 0,62 | 0,48 | 0,48 | 1,31 | 1,31 | 1,10 | 1,03 |
| BNAA1/2 | 0,55 | 0,55 | 0,55 | 0,48 | 1,24 | 1,31 | 1,10 | 1,17 |
| BNA1 | 0,62 | 0,62 | 0,55 | 0,55 | 0,96 | 1,03 | 1,10 | 1,17 |
| BNA2 | 0,55 | 0,62 | 0,69 | 0,69 | 0,89 | 1,96 | 1,03 | 1,03 |
| BNA3 | 0,55 | 0,55 | 0,28 | 0,28 | 1,38 | 1,31 | 1,03 | 1,10 |
| **Average** | **0,41** | **0,42** | **0,41** | **0,40** | **1,22** | **1,22** | **1,03** | **1,05** |

Table 2.14. Violations in the conservative approach and simultaneous simulation(in %)

## 2.7 P-P Plots

The evaluation of the predictive power across all possible confidence levels $\alpha \in [0,1]$ can be carried out with the help of a transformation of the empirical distribution $\{L^{(k)} \mid 0 \leq k \leq N-1\}$. If $F$ is the true distribution function of the loss $L$ within the holding period $h$, then the random quantity $F(L)$ is (approximately) uniformly distributed on $[0,1]$. Therefore we check the values $F_e\big[L(t)\big]$ for $t_0 \leq t \leq t_1$, where $F_e$ is the empirical distribution. If the prediction quality of the model is adequate, these values should not differ significantly from a sample with size $250\,(t_1 - t_0 + 1)$ from a uniform distribution on $[0,1]$.

The P-P plot of the transformed distribution against the uniform distribution (which represents the distribution function of the transformed empirical distribution) should therefore be located as closely to the main diagonal as possible. The mean squared deviation from the uniform distribution (MSD) summed over all quantile levels can serve as an indicator of the predictive power of a quantile-based risk measure like VaR. The XFGpp quantlet creates a P-P plot and calculates the MSD indicator.

## 2.8 Q-Q Plots

With a quantile plot (Q-Q plot) it is possible to visualize whether an ordered sample is distributed according to a given distribution function. If, e.g., a sample is normally distributed, the plot of the empirical quantiles vs. the quantiles of a normal distribution should result in an approximately linear plot. Q-Q plots vs. a normal distribution can be generated with the following command:

```
VaRqqplot (matrix(N,1)|MMPL,VaR,opt)
```

## 2.9 Discussion of Simulation Results

In Figure 2.7 the P-P plots for the historical simulation with the full yield curve (`INAAA`) as risk factor are displayed for the different variants of the simulation. From the P-P plots it is apparent that mean adjustment significantly improves the predictive power in particular for intermediate confidence levels (i.e., for small risk factor changes).

Figure 2.8 displays the P-P plots for the same data set and the basic historical simulation with different choices of risk factors. A striking feature is the poor predictive power for a model with the spread as risk factor. Moreover, the over-estimation of the risk in the conservative approach is clearly reflected by a sine-shaped function, which is superposed on the ideal diagonal function.

In Figs. 2.9 and 2.10 we show the Q-Q plots for basic historic simulation and volatility updating using the `INAAA` data set and the full yield curve as risk factors. A striking feature of all Q-Q plots is the deviation from linearity (and, thus, normality) for extreme quantiles. This observation corresponds to the leptokurtic distributions of time series of market data changes (e.g. spread changes as discussed in section 2.3.2).

### 2.9.1 Risk Factor: Full Yield

The results in Table 2.12 indicate a small under-estimation of the actually observed losses. While volatility updating leads to a reduction of violations,

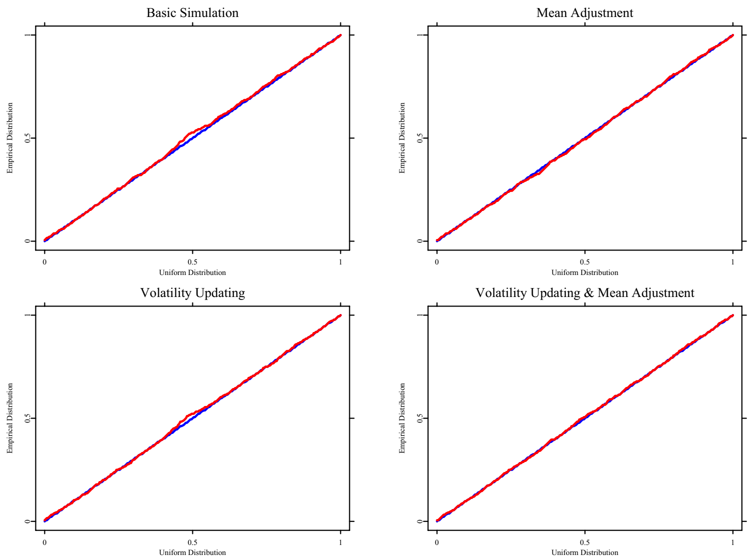


Figure 2.7. P-P Plots variants of the simulation. XFGpp

this effect is not clearly recognizable for the mean adjustment. The positive results for volatility updating are also reflected in the corresponding mean squared deviations in Table 2.15. Compared with the basic simulation, the model quality can be improved. There is also a positive effect of the mean adjustment.

### 2.9.2 Risk Factor: Benchmark

The results for the number of violations in Table 2.13 and the mean squared deviations in Table 2.16 are comparable to the analysis, where risk factors are changes of the full yield. Since the same relative changes are applied for all yield curves, the results are the same for all yield curves. Again, the application of volatility updating improves the predictive power and mean adjustment also has a positive effect.

### 2.9.3 Risk Factor: Spread over Benchmark Yield

The number of violations (see Table 2.12) is comparable to the latter two variants. Volatility updating leads to better results, while the effect of mean

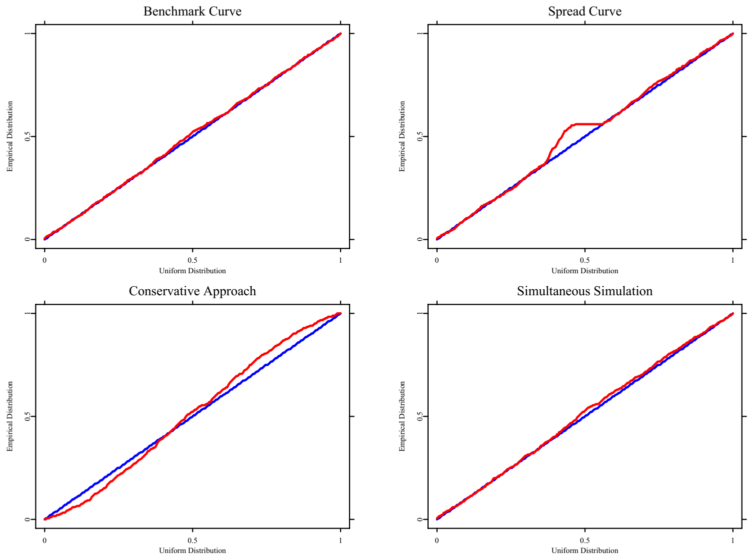


Figure 2.8. P-P Plots choice of risk factors. XFGpp

adjustment is only marginal. However, the mean squared deviations (see Table 2.15) in the P-P plots are significantly larger than in the case, where the risk factors are contained in the benchmark curve. This can be traced back to a partly poor predictive power for intermediate confidence levels (see Figure 2.8). Mean adjustment leads to larger errors in the P-P plots.

### 2.9.4 Conservative Approach

From Table 2.14 the conclusion can be drawn, that the conservative approach significantly over-estimates the risk for all credit qualities. Table 2.17 indicates the poor predictive power of the conservative approach over the full range of confidence levels. The mean squared deviations are the worst of all approaches. Volatility updating and/or mean adjustment does not lead to any significant improvements.

### 2.9.5 Simultaneous Simulation

From Tables 2.14 and 2.17 it is apparent that simultaneous simulation leads to much better results than the model with risk factors from the full yield curve,

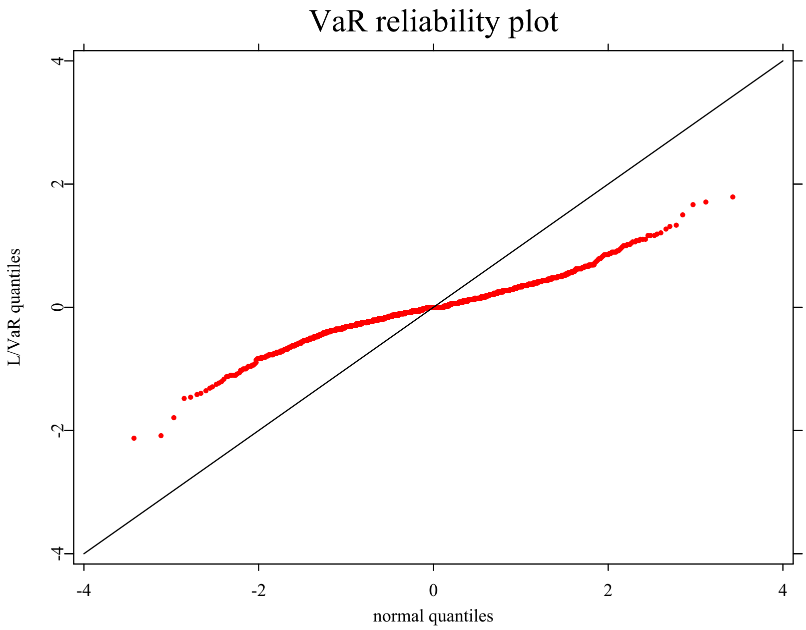


Figure 2.9. Q-Q Plot for basic historical simulation.

when volatility updating is included. Again, the effect of mean adjustment does not in general lead to a significant improvement. These results lead to the conclusion that general market risk and spread risk should be modeled independently, i.e., that the yield curve of an instrument exposed to credit risk should be modeled with two risk factors: benchmark changes and spread changes.

## 2.10 Internal Risk Models

In this contribution it is demonstrated that XploRe can be used as a tool in the analysis of time series of market data and empirical loss distributions. The focus of this contribution is on the analysis of spread risk. Yield spreads are an indicator of an obligor's credit risk. The distributions of spread changes are leptokurtic with typical fat tails, which makes the application of conventional variance-covariance risk models problematic. That is why in this contribution we prefer the analysis of spread risk by means of historical simulation. Since it is not a priori clear, how spread risk should be inte-

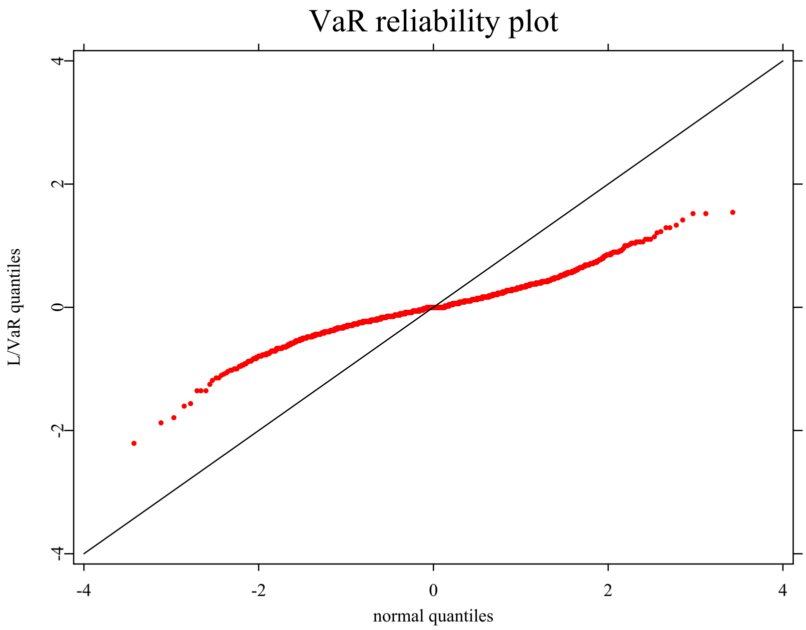


Figure 2.10. Q-Q plot for volatility updating.

grated in a risk model for interest rate products and how it can be separated from general market risk, we investigate several possibilities, which include modelling the full yield curve (i.e., consideration of only one risk factor category, which covers both benchmark and spread risk) as well as separately modelling spread risk and benchmark risk. The aggregation of both risk categories is carried out in a conservative way (addition of the risk measure for both risk categories) as well as coherently (simultaneous simulation of spread and benchmark risk). Moreover, in addition to the basic historical simulation method we add additional features like mean adjustment and volatility updating. Risk is quantified by means of a quantile-based risk measure in this contribution - the VaR. We demonstrate the differences between the different methods by calculating the VaR for a fictive zero-bond.

The numerical results indicate, that the conservative approach over-estimates the risk of our fictive position, while the simulation results for the full yield as single risk factor are quite convincing. The best result, however, is delivered by a combination of simultaneous simulation of spread and benchmark risk and volatility updating, which compensates for non-stationarity in the risk factor time series. The conclusion from this contribution for model-builders

| | full yield | | | | spread curve | | | |
|---|---|---|---|---|---|---|---|---|
| **Curve** | **V1** | **V2** | **V3** | **V4** | **V1** | **V2** | **V3** | **V4** |
| INAAA | 0,87 | 0,28 | 0,50 | 0,14 | 8,13 | 22,19 | 8,14 | 16,15 |
| INAA2 | 0,45 | 0,36 | 0,32 | 0,16 | 6,96 | 21,41 | 7,25 | 15,62 |
| INAA3 | 0,54 | 0,41 | 0,43 | 0,23 | 7,91 | 21,98 | 7,97 | 15,89 |
| INA1 | 0,71 | 0,27 | 0,41 | 0,13 | 7,90 | 15,32 | 8,10 | 8,39 |
| INA2 | 0,50 | 0,39 | 0,42 | 0,17 | 9,16 | 15,15 | 9,51 | 6,19 |
| INA3 | 0,81 | 0,24 | 0,58 | 0,24 | 9,53 | 12,96 | 9,61 | 7,09 |
| INBBB1 | 0,71 | 0,29 | 0,54 | 0,13 | 9,59 | 15,71 | 9,65 | 11,13 |
| INBBB2 | 0,33 | 0,34 | 0,26 | 0,12 | 11,82 | 14,58 | 11,59 | 10,72 |
| INBBB3 | 0,35 | 0,59 | 0,40 | 0,34 | 7,52 | 11,49 | 7,78 | 6,32 |
| INBB1 | 0,31 | 0,95 | 0,26 | 0,28 | 4,14 | 4,57 | 3,90 | 1,61 |
| INBB2 | 0,52 | 0,49 | 0,36 | 0,19 | 6,03 | 3,63 | 5,89 | 2,12 |
| INBB3 | 0,53 | 0,41 | 0,36 | 0,17 | 3,11 | 3,65 | 3,09 | 1,67 |
| INB1 | 0,51 | 0,29 | 0,38 | 0,15 | 3,59 | 1,92 | 2,85 | 1,16 |
| INB2 | 0,51 | 0,48 | 0,31 | 0,22 | 4,29 | 2,31 | 3,41 | 1,42 |
| INB3 | 0,72 | 0,38 | 0,32 | 0,16 | 3,70 | 2,10 | 2,99 | 3,02 |
| BNAAA | 0,59 | 0,19 | 0,48 | 0,56 | 10,13 | 17,64 | 9,74 | 11,10 |
| BNAA1/2 | 0,54 | 0,21 | 0,45 | 0,46 | 5,43 | 13,40 | 5,73 | 7,50 |
| BNA1 | 0,31 | 0,12 | 0,29 | 0,25 | 8,65 | 17,19 | 8,09 | 8,21 |
| BNA2 | 0,65 | 0,19 | 0,57 | 0,59 | 6,52 | 12,52 | 6,95 | 6,45 |
| BNA3 | 0,31 | 0,19 | 0,32 | 0,29 | 6,62 | 9,62 | 6,59 | 3,80 |
| **Average** | **0,54** | **0,35** | **0,40** | **0,25** | **7,04** | **11,97** | **6,94** | **7,28** |

Table 2.15. MSD P-P Plot for the full yield and the spread curve($\times 10\,000$)

| **Curve** | **V1** | **V2** | **V3** | **V4** |
|---|---|---|---|---|
| INAAA, INAA2, INAA3 | 0,49 | 0,23 | 0,26 | 0,12 |
| INA1 | 0,48 | 0,23 | 0,26 | 0,12 |
| INA2, INA3, INBBB1, INBBB2, INBBB3, INBB1, INBB2 | 0,49 | 0,23 | 0,26 | 0,12 |
| INBB3 | 0,47 | 0,23 | 0,25 | 0,12 |
| INB1 | 0,49 | 0,23 | 0,26 | 0,12 |
| INB2 | 0,47 | 0,23 | 0,25 | 0,12 |
| INB3 | 0,48 | 0,23 | 0,26 | 0,12 |
| BNAAA, BNAA1/2 | 0,42 | 0,18 | 0,25 | 0,33 |
| BNA1 | 0,41 | 0,18 | 0,23 | 0,33 |
| BNA2 | 0,42 | 0,18 | 0,25 | 0,33 |
| BNA3 | 0,41 | 0,18 | 0,24 | 0,33 |
| **Average** | **0,47** | **0,22** | **0,25** | **0,17** |

Table 2.16. MSD P-P-Plot benchmark curve ($\times 10\,000$)

in the banking community is, that it should be checked, whether the full yield curve or the simultaneous simulation with volatility updating yield satisfactory results for the portfolio considered.

| | conservative approach | | | | simultaneous simulation | | | |
|---|---|---|---|---|---|---|---|---|
| **Curve** | **V1** | **V2** | **V3** | **V4** | **V1** | **V2** | **V3** | **V4** |
| INAAA | 14,94 | 14,56 | 14,00 | 13,88 | 1,52 | 0,64 | 0,75 | 0,40 |
| INAA2 | 13,65 | 13,51 | 14,29 | 14,31 | 0,79 | 0,38 | 0,40 | 0,23 |
| INAA3 | 14,34 | 13,99 | 13,66 | 13,44 | 0,79 | 0,32 | 0,49 | 0,27 |
| INA1 | 15,39 | 15,60 | 15,60 | 15,60 | 0,95 | 0,40 | 0,52 | 0,29 |
| INA2 | 13,95 | 14,20 | 14,32 | 14,10 | 0,71 | 0,55 | 0,50 | 0,39 |
| INA3 | 14,73 | 14,95 | 14,45 | 14,53 | 0,94 | 0,30 | 0,59 | 0,35 |
| INBBB1 | 13,94 | 14,59 | 14,05 | 14,10 | 1,00 | 0,33 | 0,43 | 0,17 |
| INBBB2 | 13,74 | 13,91 | 13,67 | 13,73 | 0,64 | 0,52 | 0,45 | 0,29 |
| INBBB3 | 13,68 | 14,24 | 14,10 | 14,09 | 0,36 | 0,78 | 0,31 | 0,31 |
| INBB1 | 19,19 | 20,68 | 18,93 | 19,40 | 0,73 | 1,37 | 0,52 | 0,70 |
| INBB2 | 13,21 | 14,17 | 14,79 | 15,15 | 0,30 | 0,82 | 0,35 | 0,51 |
| INBB3 | 15,19 | 16,47 | 15,40 | 15,67 | 0,55 | 0,65 | 0,15 | 0,21 |
| INB1 | 15,47 | 15,64 | 15,29 | 15,51 | 0,53 | 0,44 | 0,19 | 0,26 |
| INB2 | 14,47 | 14,93 | 15,46 | 15,77 | 0,24 | 0,55 | 0,24 | 0,24 |
| INB3 | 14,78 | 14,67 | 16,77 | 17,03 | 0,38 | 0,44 | 0,27 | 0,22 |
| BNAAA | 14,80 | 15,30 | 16,30 | 16,64 | 1,13 | 0,33 | 0,99 | 0,96 |
| BNAA1/2 | 13,06 | 13,45 | 14,97 | 15,43 | 0,73 | 0,16 | 0,57 | 0,50 |
| BNA1 | 11,95 | 11,83 | 12,84 | 13,08 | 0,52 | 0,26 | 0,44 | 0,41 |
| BNA2 | 13,04 | 12,58 | 14,31 | 14,56 | 0,78 | 0,13 | 0,51 | 0,58 |
| BNA3 | 12,99 | 12,70 | 15,19 | 15,42 | 0,34 | 0,18 | 0,58 | 0,70 |
| **Average** | **14,33** | **14,60** | **14,92** | **15,07** | **0,70** | **0,48** | **0,46** | **0,40** |

Table 2.17. MSD P-P Plot for the conservative approach and the simultaneous simulation($\times 10\,000$)

# Bibliography

Bank for International Settlements (1998a). Amendment to the Capital Accord to incorporate market risks, www.bis.org. (January 1996, updated to April 1998).

Bank for International Settlements (1998b). Overview of the Amendment to the Capital Accord to incorporate market risk, www.bis.org. (January 1996, updated to April 1998).

Bundesaufsichtsamt für das Kreditwesen (2001). Grundsatz I/Modellierung des besonderen Kursrisikos, Rundschreiben 1/2001, www.bakred.de.

Gaumert, U. (1999). *Zur Diskussion um die Modellierung besonderer Kursrisiken in VaR-Modellen, Handbuch Bankenaufsicht und Interne Risikosteuerungsmodelle*, Schäffer-Poeschel.

Hull, J. C. (1998). Integrating Volatility Updating into the Historical Simulation Method for Value at Risk, *Journal of Risk* .

J.P. Morgan (1996). RiskMetrics, *Technical report*, J.P. Morgan, New York.

Kiesel, R., Perraudin, W. and Taylor, A. (1999). The Structure of Credit Risk. Working Paper, London School of Economics.

http://www.springer.com/978-3-540-69177-8

Applied Quantitative Finance
Härdle, W.; Hautsch, N.; Overbeck, L. (Eds.)
2008, XXVI, 447 p., Hardcover
ISBN: 978-3-540-69177-8