Preface

Mathematics is undoubtedly the key to state-of-the-art high technology. It is an international technical language and proves to be an eternally young science to those who have learned its ways. Long an indispensable part of research thanks to modeling and simulation, mathematics is enjoying particular vitality now more than ever. Nevertheless, this stormy development is resulting in increasingly high requirements for students in technical disciplines, while general interest in mathematics continues to wane at the same time. This book and its appendices on the *Internet* seek to deal with this issue, helping students master the difficult transition from the receptive to the productive phase of their education.

The author has repeatedly held a three-semester introductory course entitled *Higher Mathematics* at the University of Stuttgart and used a series of *"handouts"* to show further aspects, make the course contents more motivating, and connect with the mechanics lectures taking place at the same time. *One* part of the book has more or less evolved from this on its own. True to the original objective, this part treats a variety of separate topics of varying degrees of difficulty; nevertheless, all these topics are oriented to mechanics.

Another part of this book seeks to offer a selection of understandable realistic models that can be implemented directly from the multitude of mathematical resources. The author does not attempt to hide his preference of *Numerical Mathematics* and thus places importance on careful theoretical preparation. Proofs are only shown when they are necessary for comprehension; additional proofs are available on the *Web pages* that are part of this book. Overall, the book is divided into four parts of varying lengths:

- a summary of the aids used in the book,
- general methods and mathematical methods in particular,
- the fascinating topic of mechanics from a mathematician's point of view,
- a survey of tensor calculus.

Physics programs at the university levels have always focused on the real experiment and continue to do so in the age of virtual worlds while experiments have been a rare exception in lectures on numerical and applied mathematics. This was at the great expense of comprehension, much like the way that reading music is a sorry substitute for the sound produced or even for one's own intonation. However, the situation has changed fundamentally in the last two decades. Problems that no one would have dared to imagine in the past can now be solved on a *laptop*, and these days anyone can test numerical methods themselves. Taking into account this development, the book provides experiments for *all* of the numerical methods that it treats. If more complex case studies are involved, bear in mind that it is a difficult road from the formula to the illustration and that detailed instructions are required here. Many well-established algorithms are known not to develop the performance required of them until after they have been subjected to careful *parameter tuning*, which is something that the computer cannot do. Sheer lack of space does not permit simultaneous treatment of theory and numerics in their entirety, and still maintain clarity. In order to deal with this dilemma, the author has relegated many proofs and all the algorithms to the internet. An extensive library of MATLAB programs are available there that is continually supplemented and expanded. The development environment of MATLAB allows algorithms to be displayed without all the unnecessary ballast in a clearly structured form that is often more impressive than a longwinded description and that — by the way – unconvers all errors without shred of mercy. These programs and the way the materials are presented are not only meant to unite reader's desire to play with their inherent curiosity but also to break a lance for the experiment in mathematics as the way to indings. In extreme cases, the user can even experience a certain esthetic charm from *the algorithm itself* and start viewing the problem from an entirely different angle.

All the illustrations and charts can be reproduced by readers themselves if they download the appropriate programs.

The *first* chapter introduces the mathematical toolbox. Terms are defined for later use, and some calculus topics are reviewed and worked through.

The *second* chapter provides concise treatment of the classic problems of numerical mathematics, in particular inital and boundary value problems for ordinary differential systems .

The *third and fourth* chapters focus on optimization and its sister in the continuum, control theory. Many questions in technical disciplines and business alike lead quite naturally to an extremal problem with constraints. Formulation as an optimization problem is often the last resort when a numerical approximation problem refuse to be solved in the usual way. Incidentally, optimization under constraints follows its own laws, which can be tracked throughout the whole of theoretical mechanics. In control theory, the calculus of variations and the LAGRANGE theory enter into a fruitful symbiosis. Concrete problems are solved numerically after discretization using the discrete optimization methods treated.

The *fifth* chapter attempts to make theory of branching and path continuation understandable for practically oriented users. This basically involves the task of calculating non-trivial solutions of a system when they exist simultaneously alongside the trivial solution, as in the case of the classical eigenvalue problem. The methods discussed provide accesss to new, somewhat exotic types of solutions that remain unattainable under the usual assumptions on existence and uniqueness. For numerical approximation, the trivial solution has an almost irresistible power of attraction that must be overcome. The usefulness of the numerical methods presented here is tested by means of some benchmark problems from the literature.

Chapters *six* and *seven* treat problems known from introductory mechanics such as planetary orbits, n-body problem, gyroscope theory, and framing. These chapters also place particular value on visualization that readers can comprehend.

Chapter *eight* presents the fundamentals of continuum theory for later use, allowing readers to understand how differential systems such as the POISSON equation and the NAVIER-STOKES equations evolved from conservation laws.

Chapter *nine* uses numerical methods to solve problems in continuum mechanics for solids and *incompressible* flow. At first, some general aspects of the finite element method are considered. Then its implementation by fundamental matrices and by shape functions is elaborated more thoroughly, and some special examples from the family of finite elements are presented in detail. For the numerical treatment of the NAVIER-STOKES equations, stream-function vorticity form with the most simple triangular elements is chosen for the most part; although it places high demands on the smoothness of the solution, it allows easier access to related problems such as convection and mass transport (and is also not as bad as its reputation would suggest, by the way). A selection of algorithm examples available on the Internet provides further explanations.

Everone lives with at least two sets of coordinate systems, the absolute coordinate system and one's own relative system; other systems are added constantly. Switching back and forth between coordinate systems has to be described mathematically, leading to tensor calculus and differential geometry. The *tenth* chapter introduces the disciplines, applying dual pairing of vector spaces.

The *eleventh* chapter includes, in addition to a model-like example from gas dynamics and three examples of multibody problems, a section on rolling discs and cogwheels with its numeric implementation on the corresponding Web page.

Many innovative impulses for Numerical Mathematics originate from the engineering fields. The author hopes that this book will raise interest in the numerical components of technical and physical problems and encourage readers to do some experimenting of their own.

The book is strictly divided into topics, and the individual parts are presented as separately from each other as possible to allow the book to serve as a reference volume and maintain clarity despite the multitude of material. It should more or less build a bridge from introductory studies to the requirements of advanced studies that paves the way to the "upper class" of the community.

The finite elements presented for the static problems are for the major part MATLAB versions of FORTRAN programs from the books by H.R. SCHWARZ. Their reliability has made them a great help over the course of developing and testing further elements, and they should also be made accessible to the MATLAB community in this way.

Warning: As the interest in experimenting grows, readers will observe that every numerical method can be undermined by a suitable example. For many problems such as non-convex optimization problems, convergence is not a matter of course and extreme caution is advised; even for bifurcation problems, iteration can quickly lead to non-realistic regions or go back to zero; spectacular accidents have been known to happen when finite elements are used in a wrong way. This is why results always need to be checked carefully.

Hint for using the MATLAB programs: make first the directory `AAMESH` permanent.

Reutlingen, July 2008 *Eckart W. Gekeler*

Springer

http://www.springer.com/978-3-540-69278-2

Mathematical Methods for Mechanics
A Handbook with MATLAB Experiments
Gekeler, E.W.
2008, XVI, 624 p. 218 illus., Hardcover
ISBN: 978-3-540-69278-2