# Chapter 2 The Common Cathode Gain Stage with grounded Cathode (CCS+Cc)

## 2.1 Circuit diagram

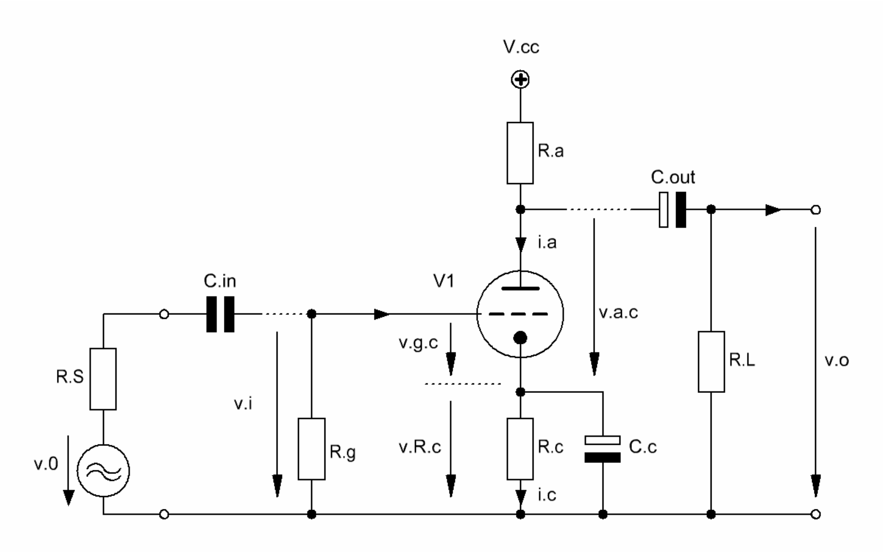


Figure 2.1 Basic design of the Common Cathode Gain Stage with grounded Cathode via $C_c$ (CCS+$C_c$)

## 2.2 Basic formulae (excl. stage load ($R_L$)-effect)

2.2.1 gain $G_t$ ($C_c$ acts as a complete short-circuit):

$$G_t = -\frac{v_o}{v_i} \tag{2.1}$$

gain $G_t$ in terms of $g_{m.t}$ and $\mu_t$ becomes:

$$\begin{aligned} G_t &= -g_{m.t}\frac{r_{a.t}\,R_a}{r_{a.t}+R_a} \\ &= -\mu_t\frac{R_a}{r_{a.t}+R_a} \end{aligned} \tag{2.2}$$

2.2.2 grid input resistance $R_g$, input capacitance $C_{i.tot}$ and input impedance $Z_{i.g}(f)$:

$$Z_{i.g}(f) = R_g \parallel C_{i.tot} \tag{2.3}$$

formulae for $C_{i.tot}$: see Chapter 1

2.2.3 plate output resistance $R_{o.a}$ and output impedance $Z_{o.a}(f)$:

$$R_{o.a} = r_{a.t} \parallel R_a \tag{2.4}$$

$$Z_{o.a}(f) = R_{o.a} \parallel C_{o.tot} \tag{2.5}$$

formulae for $C_o$: see Chapter 1

2.2.4 cathode output resistance $R_{o.c}$:

$$\begin{aligned} R_{o.c} &= r_{c.t} \parallel R_c \\ r_{c.t} &= \frac{r_{a.t}+R_a}{1+\mu_t} \end{aligned} \tag{2.6}$$

2.2.5 capacitance $C_c$ for a specific hp corner frequency $f_{c.opt}$:

$$C_c = \frac{1}{2\pi f_{c.opt}\,R_{o.c}} \tag{2.7}$$

## 2.3 Derivations

2.3.1 Equivalent circuit for derivation purposes:

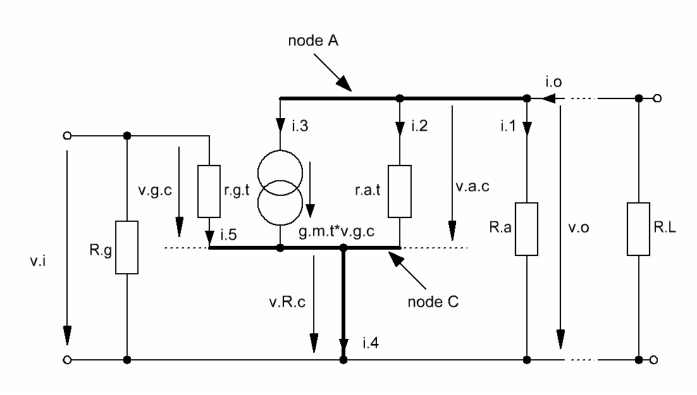


Figure 2.2 Equivalent circuit of Figure 2.1

2.3.2 Derivations:

With $C_c$ of a size that acts as a short-circuit for $v_{R.c}$ in $B_{20k}$ we can use all the respective formulae of Chapter 1 with $R_c = 0$, hence:

2.3.2.1 the gain $G_t$ and effective gain $G_{t.eff}$ become:

$$G_t = -g_{m.t}\left(r_{a.t} \| R_a\right) \tag{2.8}$$

$$R_{a.eff} = R_a \| R_L \tag{2.9}$$

$$G_{t.eff} = -g_{m.t}\left(r_{a.t} \| R_{a.eff}\right) \tag{2.10}$$

2.3.2.2 the grid input impedance $Z_{i.g}(f)$ becomes:

$$Z_{i.g}(f) = \left|\frac{R_g}{1 + 2j\pi f R_g C_{i.tot}}\right| \tag{2.11}$$

2.3.2.3 the effective plate output resistance $R_{o.a.eff}$ and impedance $Z_{o.a.eff}(f)$ become:

$$\begin{aligned} & R_{o.a} = r_{a.t} \| R_a \\ \Rightarrow\ & Z_{o.a}(f) = R_{o.a} \| C_{o.tot} \end{aligned} \tag{2.12}$$

$$\begin{aligned} & R_{o.a.eff} = r_{a.t} \| R_{a.eff} \\ \Rightarrow\ & Z_{o.a.eff}(f) = R_{o.a.eff} \| C_{o.tot} \end{aligned} \tag{2.13}$$

$$Z_{o.a.eff}(f) = \left| \frac{R_{o.a.eff}}{1 + 2j\pi f R_{o.a.eff} C_o} \right| \tag{2.14}$$

2.3.2.4 cathode resistance $r_{c.t.eff}$ [1] and effective cathode output resistance $R_{o.c.eff}$ become:

$$r_{c.t.eff} = \frac{r_{a.t} + R_{a.eff}}{1 + \mu_t} \tag{2.15}$$

$$\begin{aligned} R_{o.c.eff} &= r_{c.t.eff} \parallel R_c \\ &= \frac{\frac{r_{a.t} + R_{a.eff}}{1 + \mu_t} R_c}{\frac{r_{a.t} + R_{a.eff}}{1 + \mu_t} + R_c} \end{aligned} \tag{2.16}$$

MCD symbolic evaluation "simplify" leads to

$$R_{o.c.eff} = \frac{(r_{a.t} + R_{a.eff}) R_c}{r_{a.t} + R_{a.eff} + (1 + \mu_t) R_c} \tag{2.17}$$

2.3.2.5 cathode capacitance $C_c$ for a specific hp corner frequency $f_c$:

The audio band $B_{20k}$ spreads from 20Hz to 20kHz. To ensure a flat frequency response in $B_{20k}$ (±0.1dB) as well as a phase response deviation of less than 1° at $f_c$ = 20Hz the $C_c$ calculation should be based on a corner frequency $f_{c.opt}$ that is a $100^{th}$ of $f_c$.

$$\begin{aligned} f_c &= 20Hz \\ f_{c.opt} &= \frac{20Hz}{100} = 0.2Hz \end{aligned} \tag{2.18}$$

$$C_c = \frac{1}{2\pi f_{c.opt} R_{o.c.eff}} \tag{2.19}$$

2.3.2.6 total input capacitance $C_{i.tot}$ and total output capacitance $C_{o.tot}$:

see respective equations and footnote[2] in Chapter 1 - 1.3.2.5

1 detailed derivation see Chapter 4 "The common grid stage (CGS)"
2 see footnote 2 in Chapter 1

## 2.4 Gain stage frequency and phase response calculations

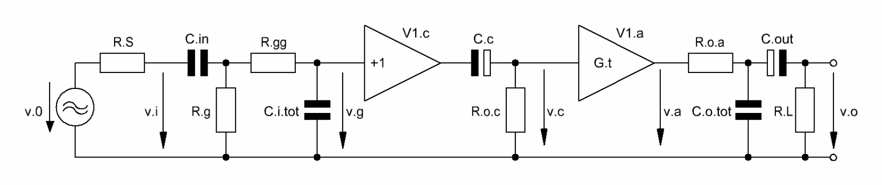


Figure 2.3 Simplified[3] equivalent circuit of Figure 2.1 - including all frequency and phase response relevant components

2.4.1 gain stage input transfer function $T_i(f)$ and phase $\varphi_i(f)$ - including source resistance $R_S$ and an oscillation preventing resistor $R_{gg}<<R_g$:

see Chapter 1 - 1.4.1

2.4.2 gain stage output transfer function $T_o(f)$ and phase $\varphi_o(f)$:

see Chapter 1 - 1.4.2

2.4.3 gain stage cathode transfer function $T_c(f)$ and phase $\varphi_c(f)$:

$$T_c(f)=\left(\frac{R_{o.c.eff}}{R_{o.c.eff}+\frac{1}{2j\pi f C_c}}\right) \tag{2.20}$$

$$\varphi_c(f)=\arctan\left\{\frac{Im\left[T_c(f)\right]}{Re\left[T_c(f)\right]}\right\} \tag{2.21}$$

2.4.4 fundamental gain stage phase shift $\varphi_{G.t}(f)$:

$$\varphi_{G.t}(f)=-180° \tag{2.22}$$

2.4.5 gain stage transfer function $T_{tot}(f)$ and phase $\varphi_{tot}(f)$:

$$T_{tot}(f)=T_i(f)\ T_o(f)\ T_c(f)\ G_t \tag{2.23}$$

$$\varphi_{tot}(f)=\varphi_i(f)+\varphi_o(f)+\varphi_c(f)+\varphi_{G.t}(f) \tag{2.24}$$

---

[3] "Simplified" because of footnote 2 of Chapter 1. In addition: The simple $C_c$ and $R_{o.c}$ network after the +1 amp $V1_c$ works correct the shown way only in the frequency range of $f_{c.opt}$ minus one octave. In lower frequency ranges than that the gain of the whole gain stage will reach the gain of a CCS gain stage with the same components but without $C_c$. But this frequency range lies far outside $B_{20k}$.

➢ MCD Worksheet II CCS+Cc calculations Page 1

**2.5 Example with ECC83 / 12AX7 (83):**

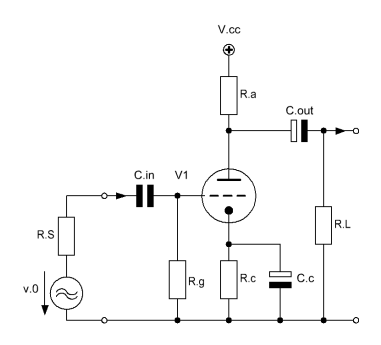


Figure 2.4 CCS+Cc example circuitry

**2.5.1 Triode bias data:**

$I_{a.83} = 1.2\cdot 10^{-3}A$  $V_{cc.83} = 370V$  $V_{a.83} = 250V$  $V_{g.83} = -2V$

**2.5.2 Triode valve constants:**

$g_{m.83} := 1.6\cdot 10^{-3}\cdot S$  $\mu_{83} := 100$  $r_{a.83} := 62.5\cdot 10^{3}\Omega$

$C_{g.c.83} := 1.65\cdot 10^{-12}F$  $C_{g.a.83} := 1.6\cdot 10^{-12}F$  $C_{a.c.83} := 0.33\cdot 10^{-12}F$

$C_{o.tot.83} := C_{a.c.83} + C_{g.a.83}$  $C_{o.tot.83} = 1.93\times 10^{-12}F$

**2.5.3 Circuit variables:**

$R_{a.83} := 100\cdot 10^{3}\Omega$  $R_{c.83} := 1.6\cdot 10^{3}\Omega$  $R_{g.83} := 1\cdot 10^{6}\Omega$

$R_S := 1\cdot 10^{3}\Omega$  $R_L := 1\cdot 10^{6}\Omega$  $R_{gg.83} := 2.2\cdot 10^{3}\Omega$

$C_{stray.83} := 10\cdot 10^{-12}F$  $C_{in.83} := 100\cdot 10^{-9}F$  $C_{out.83} := 100\cdot 10^{-9}F$

**2.5.4 Calculation relevant data:**

frequency range f for the below shown graphs: $f := 10Hz, 20Hz.. 20000Hz$

$h := 1000Hz$

**2.5.5 Gain $G_t$ and effective gain $G_{t.eff}$ :**

$$G_{83} := -g_{m.83}\cdot\frac{R_{a.83}\cdot r_{a.83}}{R_{a.83} + r_{a.83}}$$  $$G_{83} = -61.538\times 10^{0}$$

➢ MCD Worksheet II CCS+Cc calculations Page 2

$G_{83.e} := 20\cdot\log\left(\left|G_{83}\right|\right)$ $G_{83.e} = 35.783\times10^{0}$ [dB]

$R_{a.83.eff} := \left(\frac{1}{R_{a.83}}+\frac{1}{R_L}\right)^{-1}$ $R_{a.83.eff} = 90.909\times10^{3}\,\Omega$

$G_{83.eff} := -\mu_{83}\cdot\frac{R_{a.83.eff}}{R_{a.83.eff}+r_{a.83}}$ $G_{83.eff} = -59.259\times10^{0}$

$G_{83.eff.e} := 20\cdot\log\left(\left|G_{83.eff}\right|\right)$ $G_{83.eff.e} = 35.455\times10^{0}$ [dB]

**2.5.6 Specific resistances:**

$R_{o.a.83} := \left[\left(\frac{1}{r_{a.83}}+\frac{1}{R_{a.83}}\right)^{-1}\right]$ $R_{o.a.83} = 38.462\times10^{3}\,\Omega$

$R_{o.a.83.eff} := \left(\frac{1}{r_{a.83}}+\frac{1}{R_{a.83.eff}}\right)^{-1}$ $R_{o.a.83.eff} = 37.037\times10^{3}\,\Omega$

$r_{c.83} := \frac{r_{a.83}+R_{a.83}}{1+\mu_{83}}$ $r_{c.83} = 1.609\times10^{3}\,\Omega$

$R_{o.c.83} := \left(\frac{1}{r_{c.83}}+\frac{1}{R_{c.83}}\right)^{-1}$ $R_{o.c.83} = 802.222\times10^{0}\,\Omega$

$r_{c.83.eff} := \frac{r_{a.83}+R_{a.83.eff}}{1+\mu_{83}}$ $r_{c.83.eff} = 1.519\times10^{3}\,\Omega$

$R_{o.c.83.eff} := \left(\frac{1}{r_{c.83.eff}}+\frac{1}{R_{c.83}}\right)^{-1}$ $R_{o.c.83.eff} = 779.198\times10^{0}\,\Omega$

**2.5.7 Capacitance $C_c$ :**

$f_c := 20\text{Hz}$ $f_{c.opt} := \frac{f_c}{100}$ $f_{c.opt} = 200\times10^{-3}\,\text{Hz}$

$C_c := \frac{1}{2\cdot\pi\cdot f_{c.opt}\cdot R_{o.c.83.eff}}$ $C_c = 1.021\times10^{-3}\,\text{F}$

**2.5.8 Specific other capacitances:**

$C_{i.tot.83} := \left(1+\left|G_{83}\right|\right)\cdot C_{g.a.83}+C_{g.c.83}+C_{stray.83}$ $C_{i.tot.83} = 111.712\times10^{-12}\,\text{F}$

➢ MCD Worksheet II CCS+Cc calculations Page 3

### 2.5.9 Gain stage frequency and phase response calculations:

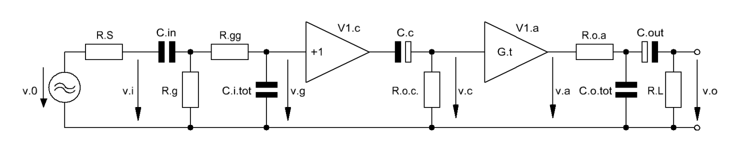


Figure 2.5 = Figure 2.3

$$Z1(f) := \frac{1}{2j \cdot \pi \cdot f \cdot C_{in.83}} \qquad Z2(f) := \frac{1}{2j \cdot \pi \cdot f \cdot C_{i.tot.83}}$$

$$T_i(f) := \frac{Z2(f) \cdot \left(\frac{1}{R_{g.83}} + \frac{1}{R_{gg.83} + Z2(f)}\right)^{-1}}{\left(Z2(f) + R_{gg.83}\right) \cdot \left[R_S + Z1(f) + \left(\frac{1}{R_{g.83}} + \frac{1}{R_{gg.83} + Z2(f)}\right)^{-1}\right]} \qquad \phi_i(f) := \operatorname{atan}\left(\frac{\operatorname{Im}(T_i(f))}{\operatorname{Re}(T_i(f))}\right)$$

$$T_{i.e}(f) := 20 \cdot \log\left(\left|T_i(f)\right|\right)$$

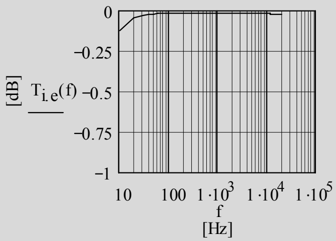


Figure 2.6 Transfer of i/p network

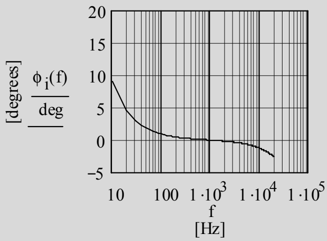


Figure 2.7 Phase of i/p network

$$Z3(f) := \frac{1}{2j \cdot \pi \cdot f \cdot C_{o.tot.83}} \qquad Z4(f) := \frac{1}{2j \cdot \pi \cdot f \cdot C_{out.83}}$$

$$T_o(f) := \frac{\left(\frac{1}{Z3(f)} + \frac{1}{Z4(f) + R_L}\right)^{-1}}{R_{o.a.83} + \left(\frac{1}{Z3(f)} + \frac{1}{Z4(f) + R_L}\right)^{-1}} \cdot \frac{R_L}{R_L + Z4(f)} \qquad \phi_o(f) := \operatorname{atan}\left(\frac{\operatorname{Im}(T_o(f))}{\operatorname{Re}(T_o(f))}\right)$$

$$T_{o.e}(f) := 20 \cdot \log\left(\left|T_o(f)\right|\right)$$

➢ MCD Worksheet II CCS+Cc calculations Page 4

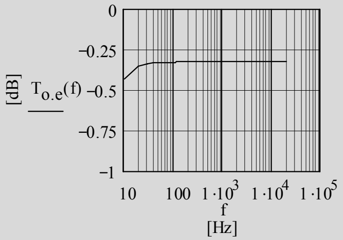


Figure 2.8 Transfer of cathode network

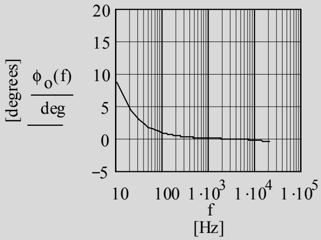


Figure 2.9 Phase of cathode network

$$T_c(f) := \left( \frac{R_{o.c.83.eff}}{R_{o.c.83.eff} + \frac{1}{2j \cdot \pi \cdot f \cdot C_c}} \right)$$

$$\phi_c(f) := \operatorname{atan}\left( \frac{\operatorname{Im}(T_c(f))}{\operatorname{Re}(T_c(f))} \right)$$

$$T_{c.e}(f) := 20 \cdot \log\left( \left| T_c(f) \right| \right)$$

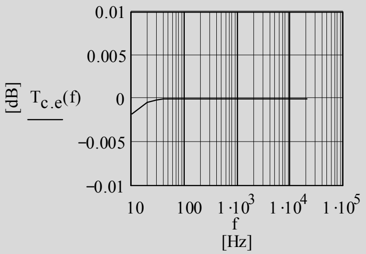


Figure 2.10 Transfer of o/p network

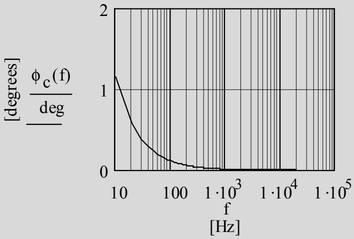


Figure 2.11 Phase of o/p network

$$T_{tot.83}(f) := T_i(f) \cdot T_o(f) \cdot T_c(f) \cdot G_{83}$$

$$\phi_{G.83}(f) := -180deg$$

$$T_{tot.83.e}(f) := 20 \cdot \log\left( \left| T_{tot.83}(f) \right| \right)$$

$$\phi_{tot.83}(f) := \phi_i(f) + \phi_o(f) + \phi_c(f) + \phi_{G.83}(f)$$

➢ MCD Worksheet II CCS+Cc calculations Page 5

**2.2.10 Frequency and phase response plots:**

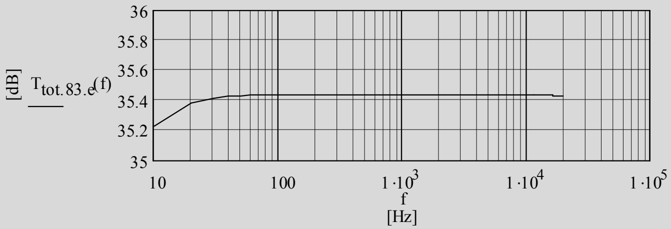


Figure 2.12 Frequency response of the whole CCS+Cc gain stage

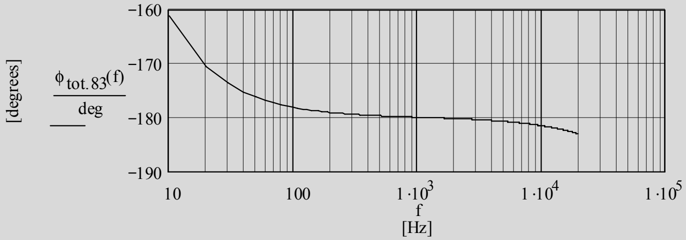


Figure 2.13 Phase response of the whole CCS+Cc gain stage

Springer

http://www.springer.com/978-3-540-69502-8

How to gain gain
A Reference Book on Triodes in Audio Pre-Amps
Vogel, B.
2008, XII, 312 p. 297 illus., Hardcover
ISBN: 978-3-540-69502-8