2

The AS–AD Framework: Origins, Problems and Progress

2.1 Introduction

In this chapter we reformulate and extend the traditional AS–AD growth dynamics of the Neoclassical Synthesis, stage I with its traditional microfoundations, as it is for example treated in detail in Sargent (1987, Chap. 5).[1] Our extension in the first instance does not replace the LM curve with a now standard Taylor rule, as is done in the New Keynesian approaches (however this is treated in a later section of the chapter). The model exhibits sticky wages as well as sticky prices, underutilized labor as well as capital, myopic perfect foresight of current wage and price inflation rates and adaptively formed medium-run expectations concerning the investment and inflation climate in which the economy is operating. The resulting nonlinear 5D dynamics of labor and goods market disequilibrium (at first—in comparison with the old neoclassical synthesis—with a conventional LM treatment of the financial part of the economy) avoids striking anomalies of the conventional model of the Neoclassical synthesis, stage I. Instead it exhibits Keynesian feedback dynamics proper with in particular asymptotic stability of its unique interior steady state solution for low adjustment speeds of wages, prices, and expectations. The loss of stability occurs cyclically, by way of Hopf bifurcations, when these adjustment speeds are made sufficiently large, leading eventually to purely explosive dynamics.

[1] This chapter is based on Asada et al. (2006): "Keynesian Dynamics and the Wage-Price Spiral: A Baseline Disequilibrium Model". Journal of Macroeconomics, **28**, 90–130.

Locally we thus obtain and prove in detail (in the case of an interest rate policy rule in the place of the LM curve)—for a certain range of parameter values—the existence of damped or persistent fluctuations in the rates of capacity utilization of both labor and capital, and of wage and price inflation rates accompanied by interest rate fluctuations that (due to the conventional working of the Keynes-effect or later also in the case of an interest rate policy rule) move in line with the goods price level (or the inflation gap). Our modification and extension of traditional AS–AD growth dynamics, as investigated from the orthodox point of view in detail in Sargent (1987), see also Chiarella et al. (2005, Chap. 2), thus provides us with a Keynesian theory of the business cycle.[2] This is so even in the case of myopic perfect foresight, where the structure of the traditional approach dichotomizes into independent supply-side real dynamics—that cannot be influenced by monetary policy at all—and subsequently determined inflation dynamics that are purely explosive if the price level is taken as a predetermined variable. These dynamics are turned into a convergent process by an application of the jump variable technique of the rational expectations school (with unmotivated jumps in the money wage level however). In our new type of Keynesian labor and goods market dynamics we can, by contrast, treat myopic perfect foresight of both firms and wage earners without any need for the methodology of the rational expectations approach to unstable saddlepoint dynamics.

If the model loses asymptotic stability for higher adjustment speeds, it does so in a cyclical fashion, by way of so-called Hopf-bifurcations, which may give rise to persistent fluctuations around the steady state. However, this loss of stability (generated if some of the speed of adjustment parameters become sufficiently large) is only of a local nature (with respect to parameter changes), since eventually purely explosive behavior is the generally observed outcome, as is verified by means of numerical simulations. The model developed thus far cannot therefore be considered as being complete in such circumstances, since some additional mechanism is required to bound the fluctuations to economically viable regions. Downward money wage rigidity is the mechanism we use for this purpose. Extended in this way, we therefore obtain and study a

[2] Yet one, as must be stressed with respect to the results obtained in this chapter, with generally a long phase length for the implied cycles, due to the central role that is played by income distribution in the generation of the cycle and due to the lack of any fluctuations in the marginal propensity to consume, in investment efficiency and in the parameters characterizing the state of liquidity preference.

baseline model of the DAS–AD variety with a rich set of stability implications and a variety of patterns of the fluctuations that it can generate.

The dynamic outcomes of this baseline DAS–AD model can be usefully contrasted with those of the currently fashionable baseline or extended New Keynesian alternative (the Neoclassical synthesis, stage II) that in our view is more limited in scope, at least as far as interacting Keynesian feedback mechanisms and thereby implied dynamic possibilities are concerned. This comparison reveals in particular that one does not always end up with the typical (and in our view strange) dynamics of rational expectation models, due to certain types of forward looking behavior, if myopic perfect foresight is of cross-over type in the considered wage-price spiral, is based on Neoclassical dating of expectations, and is coupled with plausible backward looking behavior for the medium-run evolution of the economy. Furthermore, our dual Phillips-Curves approach to the wage-price spiral indeed also performs quite well from the empirical point of view,[3] and in particular does not give rise to the situation observed for the New (Keynesian) Phillips curve(s), found in the literature to be completely at odds with the facts.[4] In our approach, standard Keynesian feedback mechanisms are coupled with a wage-price spiral having a considerable degree of inertia, with the result that these feedback mechanisms work by and large (as is known from partial analysis) in their interaction with the added wage and price level dynamics.

In the next Sect. 2.2 we briefly reconsider the fully integrated Keynesian AS–AD model of the Neoclassical Synthesis, stage I, and show that it gives rise to an inconsistent real/nominal dichotomy under myopic perfect foresight—with appended explosive nominal dynamics, subsequently tamed by means of the jump variable technique of the "rational expectations approach". Money wage levels must then however be allowed to jump just as the price level, despite the presence of a conventional money wage Phillips curve, in order to overcome the observed nominal instability by means of the assumption of rational expectations (which indeed makes this solution procedure an inconsistent one in the chosen framework). We conclude that this model type—though still heavily used at the intermediate textbook level—is not suitable for a Keynesian approach to economic dynamics which (at least as a limit case of fast

[3] See Flaschel and Krolzig (2006), Flaschel et al. (2007) and Chen and Flaschel (2006).

[4] In this connection, see for example Mankiw (2001) and with much more emphasis Eller and Gordon (2003), whereas Galí et al. (2005) argue in favor of a hybrid form of the Phillips Curve in order to defend the New Phillips curve.

adaptive expectations) should allow for myopic perfect foresight on inflation rates without much change in its implications under normal circumstances.[5]

In Sect. 2.3 we briefly discuss the New Keynesian approach to economic dynamics on an extended level, with staggered wage and price adjustment. We find there too that it raises more questions than it helps to answer from the theoretical as well as from the empirical point of view, though it can be considered as a radical departure from the structural model of the old Neoclassical synthesis. Section 2.4 then proposes our new and nevertheless traditional (matured) approach to Keynesian dynamics, by taking note of the empirical facts that both labor and capital can be under- or overutilized, that both wages and prices adjust only gradually to such disequilibria and that there are certain climate expressions surrounding the current state of the economy which add sufficient inertia to the dynamics. This organic structural reformulation of the model of the old Neoclassical synthesis completely avoids its anomalies without representing a break with respect to the Keynesian part of the model, though the AS-curve in the narrow sense (of the old Neoclassical synthesis) is still present in the steady state of the model, but only of secondary importance in the adjustment processes surrounding this steady state.

The resulting 5D dynamical model is briefly analyzed with respect to its stability features in Sect. 2.5 and shown to give rise to local asymptotic stability when certain Keynesian feedback chains—to some extent well-known to be destabilizing from a partial perspective—are made sufficiently weak, including a real wage adjustment mechanism that is not so well established in the literature. The informal stability analysis presented there is made rigorous (for the case of an interest rate policy rule) in an appendix, where the calculation of the Routh–Hurwitz conditions for the relevant Jacobians is considered in great detail and where the occurrence of Hopf bifurcations (i.e. cyclical loss of stability) is also shown. Preparing the grounds for this appendix, Sect. 2.6 of the chapter replaces the LM curve view of financial markets in conventional AS–AD by a classic Taylor interest rate policy rule and also extends the wage and price Phillips curves of our baseline model such that they can be compared in a nearly one to one fashion with the New Keynesian approach towards staggered price as well as wage setting.

Section 2.7 then provides some numerical explorations of the model, which in particular illustrate the role of wage and price flexibility with respect to

[5] See Chiarella et al. (2005, Chap. 2) for the case of adaptive expectations formation.

their corresponding measures of demand pressure. This analysis does not always support the economic arguments based on the partial feedback structures considered in Sects. 2.4 and 2.5. In particular, although aggregate demand always depends negatively on the real wage, under certain conditions increasing wage flexibility may not lead to more stability. In such situations, downward money wage rigidity can indeed assist in stabilizing the economy and this in a way that creates economically still simple, but mathematically complex dynamics due to the “squeezed” working of the economy during the low inflation regime. Section 2.8 concludes.

2.2 Traditional AS–AD with Myopic Perfect Foresight. Classical Solutions in a Keynesian Setup?

In this section we briefly discuss the traditional AS–AD growth dynamics with prices set equal to marginal wage costs, and nominal wage inflation driven by an expectations augmented Phillips curve. Introducing myopic perfect foresight (i.e., the assumption of no errors with respect to the short-run rate of price inflation) into such a Phillips curve alters the dynamics implied by the model in a radical way, in fact towards a globally stable (neo-)classical real growth dynamics with real wage rigidity and thus fluctuating rates of under- or over-employment. Furthermore, price level dynamics no longer feed back into these real dynamics and are now unstable in the large. The mainstream approach in the literature is then to go on from myopic perfect foresight to “rational expectations” and to construct a purely forward looking solution (which incorporates the whole future of the economy) by way of the so-called jump-variable technique of Sargent and Wallace (1973). However in our view this does not represent a consistent solution to the dynamic results obtained in this model type under myopic perfect foresight, as we shall argue in this chapter.

The case of myopic perfect foresight in a dynamic AD–AS model of business fluctuations and growth has been considered in very detailed form in Sargent (1987, Chap. 5). The model of Sargent’s (1987, Chap. 5) so-called Keynesian dynamics is given by a standard combination of AD based on IS-LM, and AS based on the condition that prices always equal marginal wage costs, plus finally an expectations augmented money wage Phillips Curve or WPC. The specific features that characterize this textbook treatment of AS–AD–WPC are that investment includes profitability considerations besides the real rate of interest, that a reduced form PC is not immediately employed in this dynamic analysis, and most importantly that expectations are rational (i.e., of the myopic perfect foresight variety in the deterministic context). Consumption is based on current disposable income in the traditional way, the LM curve is of standard type and there is neoclassical smooth factor substitution along with the assumption that prices are set according to the marginal productivity principle—and thus optimal from the viewpoint of the firm. These more or less standard ingredients give rise to the following set of equations that determine the statically endogenous variables: consumption (C), investment (I), government expenditure (G), output (Y), interest (i), prices (p), taxes (T), the profit rate (ρ), employment (L^d) and the rate of employment (e). These statically endogenous variables feed into the dynamically endogenous variables: the capital stock (K), labor supply (L) and the nominal wage level (w), for which laws of motion are also provided in the equations shown below. The equations are

$$C = c(Y + iB/p - \delta K - T), \tag{2.1}$$

$$I/K = i_1(\rho - (r - \pi)) + n, \quad \rho = \frac{Y - \delta K - \omega L^d}{K}, \ \omega = \frac{w}{p}, \tag{2.2}$$

$$G = gK, \quad g = \text{ const.}, \tag{2.3}$$

$$Y \overset{IS}{=} C + I + \delta K + G, \tag{2.4}$$

$$M \overset{LM}{=} p\,(h_1 Y + h_2(i_o - i)W), \tag{2.5}$$

$$Y = F(K, L^d), \tag{2.6}$$

$$p \overset{AS}{=} w/F_L(K, L^d), \tag{2.7}$$

$$\hat{w} \overset{PC}{=} \beta_w(e - e_o) + \pi, \quad e = L^d/L, \tag{2.8}$$

$$\pi \overset{MPF}{=} \hat{p}, \tag{2.9}$$

$$\hat{K} = I/K, \tag{2.10}$$

$$\hat{L} = n \quad (= \hat{M} \quad \text{for analytical simplicity}). \tag{2.11}$$

We make the simplifying assumptions that all behavior is based on linear relationships in order to concentrate on the intrinsic nonlinearities of this type of AS–AD–WPC growth model. Furthermore, following Sargent (1987, Chap. 5), we assume that $t = (T - iB/p)/K$ is a given magnitude and thus, like real government expenditure per unit of capital, g, a parameter of the model. This excludes feedbacks from government bond accumulation and thus from the government budget equation on real economic activity. We thus concentrate on the working of the private sector with minimal interference from the side of fiscal policy, which is not an issue in this chapter. The model is fully backed-up by budget equations as in Sargent (1987): pure equity financing of firms, money and bond financing of the government budget deficit and money, bond and equity accumulation in the sector of private households. There is flow consistency, since the new inflow of money and bonds is always accepted by private households. Finally, Walras' Law of Stocks and the perfect substitute assumption for government bonds and equities ensure that equity price dynamics remain implicit. The LM-curve is thus the main representation of the financial part of the model, which is therefore still of a very simple type at this stage of its development.

The treatment of the resulting dynamics turns out to be not very difficult. In fact, (2.8) and (2.9) imply a real-wage dynamics of the type:

$$\hat{\omega} = \beta_w(l^d/l - e_o), \quad l^d = L^d/K, l = L/K.$$

From $\dot{K} = I = S = Y - \delta K - C - G$ and $\dot{L} = nL$ we furthermore get

$$\hat{l} = n - (y - \delta - c(y - \delta - t) - g) = n - (1-c)y - (1-c)\delta + ct - g,$$

with $y = Y/K = F(1, l^d) = f(l^d)$.

Finally, by (2.7) we obtain

$$\omega = f'(l^d), \text{ i.e.,} \quad l^d = (f')^{-1}(\omega) = h(\omega), \quad h' < 0.$$

Hence, the real dynamics of the model can be represented by the following autonomous 2D dynamical system:

$$\begin{aligned} \hat{\omega} &= \beta_w(h(\omega)/l - e_o), \\ \hat{l} &= n - (1-c)\delta - g + ct - (1-c)f(h(\omega)). \end{aligned}$$

It is easy to show, see e.g. Flaschel (1993), that this system is well-defined in the positive orthant of the phase space, has a unique interior steady–state, which moreover is globally asymptotically stable in the considered domain. In fact, this is just a Solow (1956) growth dynamics with a real-wage Phillips curve (real wage rigidity) and thus classical under- or over-employment dynamics if $e_o < 1$!. There may be a full-employment ceiling in this model type, but this is an issue of secondary importance here.

The unique interior steady state is given by

$$\begin{aligned} y_o &= \frac{1}{1-c}[(1-c)\delta + n + g - ct] = \frac{1}{1-c}[n + g - t] + \delta + t, \\ l_o^d &= f^{-1}(y_o), \quad \omega_o = f'(l_o^d), \quad l_o = l_o^d/e_o, \\ m_o &= h_1 y_o, \quad \hat{p}_o = 0, \quad r_o = \rho_o = f(l_o^d) - \delta - \omega_o l_o^d. \end{aligned}$$

Keynes' (1936) approach is almost entirely absent in this type of analysis, which seems to be Keynesian in nature (AS–AD), but which—due to the neglect of short-run errors in inflation forecasting—has become in fact of very (neo-)classical type. The marginal propensity of consume, the stabilizing element in Keynesian theory, is still present, but neither investment nor money demand plays a role in the real dynamics we have obtained from (2.1)–(2.11). Volatile investment decisions and financial markets are thus simply irrelevant for the real dynamics of this AS–AD growth model when *myopic* perfect foresight on the current rate of price inflation is assumed. What, then, remains for the role of traditional Keynesian "troublemakers", the marginal efficiency of investment and liquidity preference schedule? The answer again is, in technical terms, a very simple one:

We have for given $\omega = \omega(t) = (w/p)(t)$ as implied by the real dynamics (due to the $I = S$ assumption):

$$(1-c)f(h(\omega)) - (1-c)\delta + ct - g = i_1(f(l) - \delta - \omega h(\omega) - i + \hat{p}) + n, \quad \text{i.e.}$$

$$\begin{aligned}\hat{p} &= \frac{1}{i_1}[(1-c)f(h(\omega)) - (1-c)\delta + ct - g - n] - (f(l) - \delta - \omega h(\omega)) + i \\ &= g(\omega, l) + i,\end{aligned}$$

with an added reduced-form LM-equation of the type

$$i = (h_1 f(h(\omega)) - m)/h_2 + i_o, \quad m = \frac{M}{pK}.$$

The foregoing equations imply

$$\hat{m} = \hat{l}(\omega) - g(\omega, l) - i_o + \frac{m - h_1 f(h(\omega))}{h_2},$$

as the non-autonomous[6] differential equation for the evolution of real money balances, as the reduced form representation of the nominal dynamics.[7] Due to this feedback chain, $\hat{m}$ depends positively on the level of m and it seems as if the jump-variable technique needs to be implemented in order to tame such explosive nominal processes; see Flaschel (1993), Turnovsky (1997) and Flaschel et al. (1997) for details on this technique. Advocates of the jump-variable technique, therefore are led to conclude that investment efficiency and liquidity preference only play a role in appended purely nominal processes and

[6] Since the independent (ω, l) block will feed into the RHS as a time function.

[7] Note that we have $g(\omega, l) = -\rho_o$ in the steady state.

this solely in a stabilizing way, though with initially accelerating phases in the case of anticipated monetary and other shocks. A truly neoclassical synthesis.

By contrast, we believe that Keynesian IS-LM growth dynamics proper (demand driven growth and business fluctuations) must remain intact if (generally minor) errors in inflationary expectations are excluded from consideration in order to reduce the dimension and to simplify the analysis of the dynamical system to be considered. A correctly formulated Keynesian approach to economic dynamics and fluctuating growth should not give rise to such a strange dichotomized system with classical real and purely nominal IS-LM inflation dynamics, here in fact of the most basic jump variable type, namely

$$\hat{m} = \frac{m - h_1 y_o}{h_2} \quad \left[\hat{p} = -\frac{(M/K)_o \frac{1}{p} - h_1 y_o}{h_2}\right],$$

if it is assumed for simplicity that the real part is already at its steady state. This dynamic equation is of the same kind as the one for the Cagan monetary model and can be treated with respect to its forward-looking solution in the same way, as it is discussed in detail for example in Turnovsky (1997, Sect. 3.3/4), i.e., the nominal dynamics assumed to hold under the jump-variable hypothesis in AS–AD–WPC is then of a very well-known type.

However, the basic fact that the AS–AD–WPC model under myopic perfect foresight is not a consistently formulated one and also not consistently solved arises from its ad hoc assumption that nominal wages must here jump with the price level p $(w = \omega p)$, since the real wage ω is now moving continuously in time according to the derived real wage dynamics. The level of money wages is thus now capable of adjusting instantaneously, which is in contradiction to the assumption of only sluggishly adjusting nominal wages according to the assumed money wage PC.[8] Furthermore, a properly formulated Keynesian growth dynamics should—besides allowing for un- or over-employed labor—also allow for un- or over-employment of the capital stock, at least in certain episodes. Thus the price level, like the wage level, should better and alternatively be assumed to adjust somewhat sluggishly; see also Barro (1994) in this regard. We will come back to this observation after the next section which is devoted to new developments in the area of Keynesian dynamics, the so-called New Keynesian approach of the macrodynamic literature.

[8] See Flaschel (1993) and Flaschel et al. (1997) for further investigations along these lines.

The conclusion of this section is that the Neoclassical synthesis, stage I, must be considered a failure on logical grounds and not a valid attempt "to formalize for students the relationships among the various hypotheses advanced in Milton Friedman's AEA presidential address (1968)", see Sargent (1987, p. 117).

2.3 New Keynesian AS–AD Dynamics with Staggered Wage and Price Setting

In this section we consider briefly the modern analog to the old neoclassical synthesis (with gradually adjusting money wages), the New Keynesian approach to macrodynamics, and this already in its advanced form, where both staggered price setting and staggered wage setting are assumed.[9] We here follow Woodford (2003, p. 225) in his formulation of staggered wages and prices, where their joint evolution is coupled with the usual forward-looking output dynamics and now in addition augmented by a derived law of motion for real wages. Here we shall only briefly look at this extended approach and leave to a later sections of the chapter a consideration of the similarities and differences between these New Keynesian dynamics and our own approach.

Woodford (2003, p. 225) makes use of the following two loglinear equations for describing the joint evolution of wages and prices (d the backward-oriented difference operator).[10]

$$\begin{aligned} \mathrm{d}\ln w_t &\overset{NWPC}{=} \beta E_t(\mathrm{d}\ln w_{t+1}) + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln \omega_t, \\ \mathrm{d}\ln p_t &\overset{NPPC}{=} \beta E_t(\mathrm{d}\ln p_{t+1}) + \beta_{py}\ln Y_t + \beta_{p\omega}\ln \omega_t, \end{aligned}$$

where all parameters are assumed to be positive. Based on theories of staggered wage and price setting, output gaps act positively on current rates of wage and price inflation, while the wage gap is influencing negatively the current wage inflation rate and positively the current price inflation rate. Our first aim here is to derive the continuous time analog to these two equations (and the other equations of the full model) and to show on this basis how this extended model is solved by the methods of the rational expectations school.

[9] The baseline case of only staggered price setting is considered in Asada et al. (2006), while the extended case considered here is further discussed in Chiarella et al. (2005, Chap. 1).

[10] We make use of this convention throughout this chapter and thus have to write $r_{t-1} - dp_t$ to denote for example the real rate of interest.

In a deterministic setting we obtain from the above

$$\begin{aligned} d\ln w_{t+1} &\overset{WPC}{=} \frac{1}{\beta}[d\ln w_t - \beta_{wy}\ln Y_t + \beta_{w\omega}\ln \omega_t], \\ d\ln p_{t+1} &\overset{PPC}{=} \frac{1}{\beta}[d\ln p_t - \beta_{py}\ln Y_t - \beta_{p\omega}\ln \omega_t]. \end{aligned}$$

If we assume (as we do in all of the following and without much loss in generality) that the parameter β is not only close to one, but in fact set equal to one, then the last two equations can be expressed as

$$\begin{aligned} d\ln w_{t+1} - d\ln w_t &\overset{WPC}{=} -\beta_{wy}\ln Y_t + \beta_{w\omega}\ln \omega_t, \\ d\ln p_{t+1} - d\ln p_t &\overset{PPC}{=} -\beta_{py}\ln Y_t - \beta_{p\omega}\ln \omega_t. \end{aligned}$$

Denoting by π^w the rate of wage inflation and by π^p the rate of price inflation (both indexed by the end of the corresponding period) we therefrom obtain the continuous time dynamics, (with $\ln Y = y$ and $\theta = \ln \omega$):

$$\begin{aligned} \dot{\pi}^w &\overset{WPC}{=} -\beta_{wy}y + \beta_{w\omega}\theta, \\ \dot{\pi}^p &\overset{PPC}{=} -\beta_{py}y - \beta_{p\omega}\theta. \end{aligned}$$

From the output dynamics of the New Keynesian approach, namely

$$y_t = y_{t+1} - \alpha_{yr}(i_t - \pi^p_{t+1} - i_0), \quad \text{i.e.,} \quad y_{t+1} - y_t = \alpha_{yr}(i_t - \pi^p_{t+1} - i_0),$$

we moreover obtain the continuous time reduced form law of motion

$$\dot{y} \overset{IS}{=} \alpha_{yr}[(\phi_{ip} - 1)\pi^p + \phi_{iy}y]$$

where we have already inserted the interest rate policy rule shown below in order to (hopefully) obtain dynamic determinacy as in the New Keynesian baseline model, which is known to be indeterminate for the case of an interest rate peg, but determinate in the case of a suitably chosen active interest rate policy rule. For the following we choose the simple textbook Taylor interest rate policy rule:

$$i = i_{\mathrm{T}} = i_o + \phi_{ip}\,\pi + \phi_{iy}\,y,$$

see Walsh (2003, p. 247), which is of a classical Taylor rule type (though without interest rate smoothing yet).

There remains finally the law of motion for real wages to be determined, which setting $\theta = \ln \omega$ simply reads

$$\dot{\theta} = \pi^w - \pi^p.$$

We thus get from this extended New Keynesian model an autonomous linear dynamical system, in the variables π^w, π^p, y and θ. The uniquely determined steady state of the dynamics is given by $(0, 0, 0, 0)$. From the definition of θ we see that the model exhibits four forward-looking variables, in direct generalization of the baseline New Keynesian model with only staggered price setting. Searching for a zone of determinacy of the dynamics (appropriate parameter values that make the steady state the only bounded solution to which the economy then immediately returns after isolated shocks of any type) thus requires establishing conditions under which all roots of the Jacobian have positive real parts.

The Jacobian of the 4D dynamical system under consideration reads:

$$J = \begin{pmatrix} 0 & 0 & -\beta_{wy} & \beta_{w\omega} \\ 0 & 0 & -\beta_{py} & -\beta_{p\omega} \\ 0 & \alpha_{yr}(\phi_{ip} - 1) & \alpha_{yr}\phi_{iy} & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

For the determinant of this Jacobian we calculate

$$-|J| = (\beta_{wy}\beta_{p\omega} + \beta_{py}\beta_{w\omega})\alpha_{yr}(\phi_{ip} - 1) \gtreqless 0 \quad \text{iff} \quad \beta_{i\pi} \gtreqless 1.$$

We thus get that an active monetary policy of the conventional type (with $\phi_{ip} > 1$) is—compared to the baseline New Keynesian model—no longer appropriate to ensure determinacy (for which a positive determinant of J is a necessary condition). One can show in addition, see Chen et al. (2005), via the minors of order 3 of the Jacobian J, that the same holds true for a passive monetary policy rule, i.e., the model in this form must be blocked out from consideration, at least from a continuous time perspective. There consequently arises the necessity to specify an extended or modified active Taylor interest rate policy rule from which one can then obtain determinacy for the resulting dynamics, where the steady state as again the only bounded solution and therefore, according to the logic of the rational expectations approach, the only realized situation in this deterministic set-up. This would then generalize the New Keynesian baseline model with only staggered prices, which is known to be indeterminate in the case of an interest rate peg or a passive monetary policy rule, but which exhibits determinacy for the above conventional Taylor rule with $\phi_{ip} > 1$.

The situations of unanticipated (and anticipated) shocks and their implications under the assumptions made by the jump variable method now have a long tradition in macrodynamics, so long in fact that economic, and not only mathematical justifications for this type of approach are no longer given, see Turnovsky (1997, part II) for an exception. Yet, authors working in the tradition of the present chapter have expressed doubts on the economic meaningfulness of the jump variable procedure on various occasions. These authors point to a variety of weaknesses in this contrived procedure to overcome the explosive forces of saddlepoint dynamics, or even purely explosive dynamics, and this in particular if such explosiveness is used to traverse smoothly to convergent solutions in the case of anticipated events, see Flaschel et al. (1997), Chiarella and Flaschel (2000) and Asada et al. (2003) in particular. We believe that in a fully specified, then necessarily nonlinear model of economic dynamics, an analysis along the lines of the jump variable technique represents not only an exceptional case with hyper-perfect foresight on the whole set of future possibilities of the economy (that in particular in the case of anticipated events cannot be learned), but that it is generally intractable from the mathematical point of view, and that in a nonlinear world is not unambiguously motivated through certain boundedness conditions.

We acknowledge that the jump variable technique of the *rational expectations* approach is a rigorous approach to forward looking behavior, too often however restricted to loglinear approximations, with potentially very demanding calculational capabilities even in nonlinear baseline situations. Our aim in this chapter is to demonstrate that acceptable situations of myopic perfect foresight can be handled in general without employing non-predetermined variables in order to place the economy on some stable manifold in a unique fashion such that (temporary) processes of accelerating instability can only occur until anticipated shocks are assumed to occur. Instead, local instability will be an integral part in the adjustment processes we consider, here however tamed not by imposing jumps on some non-predetermined variables that bypass instability, but by making use of certain nonlinearities in the behavior of economic agents when the economy departs too much from its steady state position. In sum, we therefore find with respect to the dynamic situation we have sketched above, that it may provide a rigorous way out of certain instability scenarios, one that does not fail on logical grounds as the one considered in the preceding section, but nevertheless one with a variety of questionable features of theoretical (as well as empirical) content that demand other solution procedures with respect to the local instability features of models with forward looking components.

We are fairly skeptical as to whether the New PC's really represent an improvement over conventional structural approaches with separate equations for wage and price inflation (as often used in macroeconometric model building). Further skepticism is expressed in Mankiw (2001) where the New (price) Phillips curve is characterized as being completely at odds with the facts. Eller and Gordon (2003) go even further and state that "the NKPC approach is an empirical failure by every measure". Galí et al. (2005) by contrast defend this NKPC by now basing it on real marginal costs in the place of an output gap and what they call a simple hybrid variant of the NKPC as derived from Calvo's staggered price setting framework. They find in such a framework "that forward-looking behavior is highly important; the coefficient on expected future inflation is large and highly significant." The criticism just quoted also applies to the extended wage and price dynamics considered above.

In order to overcome the questionable features of the New Keynesian approach to price and wage formation we now propose some modifications to the above presentation of the wage-price dynamics which will completely remove from it the problematic feature of a sign reversal (in their reduced-form representation) in front of output as well as wage gaps. This sign reversal is caused by the fact that future values of the considered state values are used on the right hand side of their determining equations, which implies that the time rates of change of these variables depend on output and wage gaps with a reversed sign in front on them. These sign reversals are at the root of the problem when the empirical relevance of such NPC's is investigated. We instead will make use of the following expectations augmented wage and price Phillips curves in the remainder of this chapter:

$$
\begin{aligned}
\mathrm{d}\ln w_{t+1} &\overset{WPC}{=} \kappa_w \mathrm{d}\ln p_{t+1} + (1-\kappa_w)\pi_t^c + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln\omega_t,\\
\mathrm{d}\ln p_{t+1} &\overset{PPC}{=} \kappa_p \mathrm{d}\ln w_{t+1} + (1-\kappa_p)\pi_t^c + \beta_{py}\ln Y_t + \beta_{p\omega}\ln\omega_t.
\end{aligned}
$$

We have modified the New Keynesian approach to wage and price dynamics here only with respect to the terms that concern expectations, in order to generate the potential for a wage-price spiral mechanism. We first assume that expectations formation is of a crossover type, with perfectly foreseen price inflation in the wage PC of workers and perfectly foreseen wage inflation in the price PC of firms. Furthermore, we make use in this regard of a neoclassical dating of expectations in the considered PC's, which means that we have the same dating for expectations and actual wage and price formation on both sides of the PC's. Finally, following Chiarella and Flaschel (1996) and later work, we assume expectations formation to be of a hybrid type, where a certain weight is given to current (perfectly foreseen) inflation rates and the counterweight attached to a concept that we have dubbed the inflationary climate π^c that is surrounding the currently evolving wage-price spiral. We thus assume that workers as well as firms to a certain degree pay attention to whether the current situation is embedded in a high inflation regime or in a low inflation one.

These relatively straightforward modifications of the New Keynesian approach to expectations formation will imply for the dynamics of what we call a matured Keynesian approach—to be started in the next section and completed in Sect. 2.7—radically different orbits and stability features, with in particular no need to single out the steady state as the only relevant situation for economic analysis in the deterministic set-up. Concerning microfoundations for the assumed wage-price spiral we here only note that the postulated wage PC can be microfounded as in Blanchard and Katz (1999), using standard labor market theories, giving rise to nearly exactly the form shown above (with the unemployment gap in the place of the logarithm of the output gap) if hybrid expectations formation is in addition embedded into their approach. Concerning the price PC a similar procedure may be applied based on desired markups of firms, see Flaschel and Krolzig (2006). Along these lines one in particular gets an economic motivation for the inclusion of—indeed the logarithm of—the real wage (or wage share) with negative sign into the wage PC and with positive sign into the price PC, without any need for loglinear approximations. We furthermore will use the (un-)employment gap and the capacity utilization gap in these two PC's, respectively, in the place of a single measure (the log of the output gap). We conclude that the above wage-price spiral is an interesting alternative to the—theoretically rarely discussed and empirically questionable—New Keynesian form of wage-price dynamics. This wage-price spiral will at first be embedded in a somewhat simplified form into a complete Keynesian approach in the next section, exhibiting a dynamic IS-equation as in Rudebusch and Svensson (1999), but now also including real wage effects and thus a role for income distribution in aggregate demand, exhibiting furthermore Okun's law as the link between goods and labor markets, and exhibiting of course (later on, see Sect. 2.6) the classical type of Taylor interest rate policy rule in the place of LM-curve employed for the time being.

2.4 Matured Keynesian AD–AS Analysis: A Baseline Model

We have already remarked that a Keynesian model of aggregate demand fluctuations should (independently of whether justification can be found in Keynes' General Theory) allow for under- (or over-)utilized labor as well as capital in order to be general enough from the descriptive point of view. As Barro (1994) for example observes IS-LM is (or should be) based on imperfectly flexible wages *and* prices and thus on the consideration of wage as well as price Phillips Curves. This is precisely what we will do in the following, augmented by the observation that medium-run aspects count both in wage and price adjustment as well as in investment behavior, here still expressed in simple terms by the introduction of the concept of an inflation as well as an investment climate. These economic climate terms are based on past observation, while we have model-consistent expectations with respect to short-run wage and price inflation. The modification of the traditional AS–AD model of Sect. 2.2 that we shall introduce now thus treats expectations in a hybrid way, myopic perfect foresight on the current rates of wage and price inflation on the one hand and an adaptive updating of economic climate expressions, with an exponential weighting scheme, on the other hand.

In light of the foregoing discussion, we assume here two Phillips Curves or PC's in the place of only one. In this way we provide wage and price dynamics separately, both based on measures of demand pressure $e - e_o, u - u_o$, in the market for labor and for goods, respectively. We denote by e the rate of employment on the labor market and by e_o the NAIRU-level of this rate, and similarly by u the rate of capacity utilization of the capital stock and u_o the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation, $\hat{w}, \hat{p}$, are both augmented by a weighted average of cost-pressure terms based on forward-looking perfectly foreseen price and wage inflation rates, respectively, and a backward looking measure of the prevailing inflationary climate, symbolized by π^c. Cost pressure perceived by workers is thus a weighted average of the currently evolving price inflation rate $\hat{p}$ and some longer-run concept of price inflation, π^c, based on past observations. Similarly, cost pressure perceived by firms is given by a weighted average of the currently evolving (perfectly foreseen) wage inflation rate $\hat{w}$ and again the measure of the inflationary climate in which the economy is operating. We thus arrive at the following two Phillips Curves for wage and price inflation, here formulated in a fairly symmetric way.

Structural form of the wage-price dynamics:

$$\hat{w} = \beta_w(e - e_o) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c,$$
$$\hat{p} = \beta_p(u - u_o) + \kappa_p \hat{w} + (1 - \kappa_p)\pi^c.$$

Inflationary expectations over the medium run, π^c, i.e., the *inflationary climate* in which current wage and price inflation is operating, may be adaptively following the actual rate of inflation (by use of some exponential weighting scheme), may be based on a rolling sample (with hump-shaped weighting schemes), or on other possibilities for updating expectations. For simplicity of exposition we shall here make use of the conventional adaptive expectations mechanism. Besides demand pressure we thus use (as cost pressure expressions) in the two PC's weighted averages of this economic climate and the (foreseen) relevant cost pressure term for wage setting and price setting. In this way we get two PC's with very analogous building blocks, which despite their traditional outlook turn out to have interesting and novel implications. These two Phillips curves have been estimated for the US-economy in various ways in Flaschel and Krolzig (2006), Flaschel et al. (2007) and Chen and Flaschel (2006) and found to represent a significant improvement over single reduced-form price Phillips curves, with wage flexibility being greater than price flexibility with respect to demand pressure in the market for goods and for labor, respectively. Such a finding is not possible in the conventional framework of a single reduced-form Phillips curve.

Note that for our current version, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$, the law of motion of which is given by:

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_w(e - e_o) - (1 - \kappa_w)\beta_p(u - u_o)], \quad \kappa = 1/(1 - \kappa_w\kappa_p).$$

This follows easily from the obviously equivalent representation of the above two PC's:

$$\hat{w} - \pi^c = \beta_w(e - e_o) + \kappa_w(\hat{p} - \pi^c),$$
$$\hat{p} - \pi^c = \beta_p(u - u_o) + \kappa_p(\hat{w} - \pi^c),$$

by solving for the variables $\hat{w} - \pi^c$ and $\hat{p} - \pi^c$. It also implies the two cross-markets or *reduced form PC's* are given by:

$$\hat{p} = \kappa[\beta_p(u - u_o) + \kappa_p \beta_w(e - e_o)] + \pi^c, \tag{2.12}$$

$$\hat{w} = \kappa[\beta_w(e - e_o) + \kappa_w \beta_p(u - u_o)] + \pi^c, \tag{2.13}$$

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market. This traditional expectations-augmented PC formally resembles the above reduced form $\hat{p}$-equation if Okun's Law holds in the sense of a strict positive correlation between $u - u_o,\ u = Y/Y^p$ and $e - e_o,\ e = L^d/L$, our measures of demand pressures on the market for goods and for labor. Yet, the coefficient in front of the traditional PC would even in this situation be a mixture of all of the β's and κ's of the two originally given PC's and thus represent a synthesis of goods and labor market characteristics.

With respect to the investment climate we proceed similarly and assume that this climate is adaptively following the current risk premium $\epsilon (= \rho - (i - \hat{p}))$, the excess of the actual profit rate over the actual real rate of interest (which is perfectly foreseen). This gives[11]

$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\epsilon - \epsilon^m), \quad \epsilon = \rho + \hat{p} - i,$$

which is directly comparable to

$$\dot{\pi}^c = \beta_{\pi^c}(\pi - \pi^c), \quad \pi = \hat{p}.$$

We believe that it is very natural to assume that economic climate expressions evolve sluggishly towards their observed short-run counter-parts. It is however easily possible to introduce also forward looking components into the updating of the climate expressions, for example based on the p^* concept of central banks and related potential output calculations. The investment function of the model of this section is now given simply by $i_1(\epsilon^m)$ in the place of $i_1(\epsilon)$.

We have now covered all modifications needed to overcome the extreme conclusions of the traditional AS–AD approach under myopic perfect foresight as they were sketched in Sect. 2.2. The model simply incorporates sluggish price adjustment besides sluggish wage adjustment and makes use of certain

[11] Chiarella et al. (2003) in response to Velupillai (2003), have used a slightly different expression for the updating of the investment climate, in this regard see the introductory observation in Sect. 2.6.

delays in the cost pressure terms of its wage and price PC and in its investment function. In the Sargent (1987) approach to Keynesian dynamics the $\beta_{\epsilon^m}, \beta_{\pi^c}, \beta_p$ are all set equal to infinity and u_o set equal to one, which implies that only current inflation rate and excess profitabilities matter for the evolution of the economy and that prices are perfectly flexible, so that full capacity utilization, not only normal capacity utilization, is always achieved. This limit case has however little in common with the properties of the model of this section.

This brings us to one point that still needs definition and explanation, namely the concept of the rate of capacity utilization that we will be using in the presence of neoclassical smooth factor substitution, but Keynesian over- or under-employment of the capital stock. Actual use of productive capacity is of course defined in reference to actual output Y. As measure of potential output Y^p we associate with actual output Y the profit-maximizing output with respect to currently given wages and prices. Capacity utilization u is therefore measured relative to the profit maximizing output level and thus given by[12]

$$u = Y/Y^p \quad \text{with} \quad Y^p = F(K, L^p),\ \omega = F_L(K, L^p),$$

where Y is determined from the IS-LM equilibrium block in the usual way. We have assumed in the price PC as normal rate of capacity utilization a rate that is less than one and thus assume in general that demand pressure leads to price inflation, before potential output has been reached, in line with what is assumed in the wage PC and demand pressure on the labor market. The idea behind this assumption is that there is imperfect competition on the market for goods so that firms raise prices before profits become zero at the margin.

Sargent (1987, Chap. 5) not only assumes myopic perfect foresight ($\beta_{\pi^c} = \infty$), but also always the perfect—but empirically questionable—establishment of the condition that the price level is given by marginal wage costs ($\beta_p = \infty, u_o = 1$). This "limit case" of the dynamic AS–AD model of this section does not represent a meaningful model, in particular since its dynamic properties are not at all closely related to situations of very fast adjustment of prices and climate expressions to currently correctly observed inflation rates and excess profitability.

[12] In intensive form expressions the following gives rise to $u = y/y^p$ with $y^p = f((f')^{-1}(\omega))$ in terms of the notation we introduced in Sect. 2.2.

There is still another motivation available for the imperfect price level adjustment we are assuming. For reasons of simplicity, we here consider the case of a Cobb-Douglas production function, given by $Y = K^\alpha L^{1-\alpha}$. According to the above we have

$$p = w/F_L(K, L^p) = w/[(1-\alpha)K^\alpha(L^p)^{-\alpha}]$$

which for given wages and prices defines potential employment. Similarly, we define competitive prices as the level of prices p_c such that

$$p_c = w/F_L(K, L^d) = w/[(1-\alpha)K^\alpha(L^d)^{-\alpha}].$$

From these definitions we get the relationship

$$\frac{p}{p_c} = \frac{(1-\alpha)K^\alpha(L^d)^{-\alpha}}{(1-\alpha)K^\alpha(L^p)^{-\alpha}} = (L^p/L^d)^\alpha.$$

Due to this we obtain from the definitions of L^d, L^p and their implication $Y/Y^p = (L^d/L^p)^{1-\alpha}$ an expression that relates the above price ratio to the rate of capacity utilization as defined in this section:

$$\frac{p}{p_c} = \left(\frac{Y}{Y^p}\right)^{\frac{-\alpha}{1-\alpha}} \quad \text{or} \quad \frac{p_c}{p} = \left(\frac{Y}{Y^p}\right)^{\frac{\alpha}{1-\alpha}} = u^{\frac{\alpha}{1-\alpha}}.$$

We thus get that (for $u_o = 1$) upward adjustment of the rate of capacity utilization to full capacity utilization is positively correlated with downward adjustment of actual prices to their competitive value and vice versa. In particular in the special case $\alpha = 0.5$ we would get as reformulated price dynamics (see (2.12) with $\bar{u}$ being replaced by $(p_c/p)_o$):

$$\hat{p} = \beta_p(p_c/p - (p_c/p)_o) + \kappa_p\hat{w} + (1-\kappa_p)\pi^c,$$

which resembles the New Phillips curve of the New Keynesian approach as far as the reflection of demand pressure forces by means of real marginal wage costs are concerned. Price inflation is thus increasing when competitive prices (and thus nominal marginal wage costs) are above the actual ones and decreasing otherwise (neglecting the cost-push terms for the moment). This shows that our understanding of the rate of capacity utilization in the framework of neoclassical smooth factor substitution is related to demand pressure terms as used in New Keynesian approaches[13] and thus further motivating

[13] See also Powell and Murphy (1997) for a closely related approach, there applied to an empirical study of the Australian economy. We would like to stress here

its adoption. Actual prices will fall if they are above marginal wage costs to a sufficient degree. However, our approach suggests that actual prices start rising before marginal wage costs are in fact established, i.e. in particular, we have that actual prices are always higher than the competitive ones in the steady state.

We note that the steady state of the now considered Keynesian dynamics is the same as the one of the dynamics of Sect. 2.2 (with $\epsilon_o^m = 0$, $y_o^p = y_o/u_o$, $l_o^p = f^{-1}(y_o^p)$ in addition). Furthermore, the dynamical equations considered above have of course to be augmented by the ones that have remained unchanged by the modifications just considered. The intensive form of all resulting static and dynamic equations is presented below, from which we then start the stability analysis of the baseline model of the next section. The modifications of the AS–AD model of Sect. 2.2 proposed in the present section imply that it no longer dichotomizes and there is no need here to apply the poorly motivated jump-variable technique. Instead, the steady state of the dynamics is locally asymptotically stable under conditions that are reasonable from a Keynesian perspective, loses its asymptotic stability by way of cycles (by way of so-called Hopf-bifurcations) and becomes sooner or later globally unstable if (generally speaking) adjustment speeds become too high.

We no longer have state variables in the model that can be considered as being not predetermined, but in fact can reduce the dynamics to an autonomous system in the five predetermined state variables: the real wage, real balances per unit of capital, full employment labor intensity, and the expressions for the inflation and the investment climate. When the model is subject to explosive forces, it requires extrinsic nonlinearities in economic behavior, assumed to come into affect far off the steady state, that bound the dynamics to an economically meaningful domain in the 5D state space. Chen et al. (2005) provide details of such an approach and its numerical investigation.

Summing up we can state that we have arrived at a model type that is much more complex, but also much more convincing, that the labor market dynamics of the traditional AS–AD dynamics of the Neoclassical synthesis, stage I. We now have five in the place of only three laws of motion, which incorporate myopic perfect foresight without any significant impact on the resulting Keynesian dynamics. We can handle factor utilization problems both for labor

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that this property of our model represents an important further similarity with the New Keynesian approach, yet here in a form that gives substitution (with moderate elasticity of substitution) no major role to play in the overall dynamics.

and capital without necessarily assuming a fixed proportions technology, i.e., we can treat AS–AD growth with neoclassical smooth factor substitution. We have sluggish wage as well as price adjustment processes with cost pressure terms that are both forward and backward looking, and that allow for the distinction between temporary and permanent inflationary shocks. We have a unique interior steady state solution of (one must stress) supply side type, generally surrounded by business fluctuations of Keynesian short-run as well as medium-run type. Our DAS–AD growth dynamics therefore exhibits a variety of features that are much more in line with a Keynesian understanding of the features of the trade cycle than is the case for the conventional modelling of AS–AD growth dynamics.

Taken together the model of this section consists of the following five laws of motion for real wages, real balances, the investment climate, labor intensity and the inflationary climate:

$$\hat{\omega} = \kappa[(1-\kappa_p)\beta_w(l^d/l - e_o) - (1-\kappa_w)\beta_p(y/y^p - u_o)], \tag{2.14}$$

$$\hat{m} = -\hat{p} - i_1\epsilon^m, \tag{2.15}$$

$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\rho + \hat{p} - i - \epsilon^m), \tag{2.16}$$

$$\hat{l} = -i_1\epsilon^m, \tag{2.17}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c), \tag{2.18}$$

with $\hat{p} = \kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)] + \pi^c$.

We here already employ reduced-form expressions throughout and consider the dynamics of the real wage, ω, real balances per unit of capital, m, the investment climate ϵ^m, labor intensity, l, and the inflationary climate, π^c on the basis of the simplifying assumptions that natural growth n determines also the trend growth term in the investment function as well as money supply growth. The above dynamical system is to be supplemented by the following static relationships for output, potential output and employment (all per unit of capital) and the rate of interest and the rate of profit:

$$y = \frac{1}{1-c}[i_1\epsilon^m + n + g - t] + \delta + t, \tag{2.19}$$

$$y^p = f((f')^{-1}(\omega)), \tag{2.20}$$

$$F(1, L^p/K) = f(l^p) = y^p, F_L(1, L^p/K) = f'(l^p) = \omega,$$

$$l^d = f^{-1}(y), \tag{2.21}$$

$$i = i_o + (h_1 y - m)/h_2, \tag{2.22}$$

$$\rho = y - \delta - \omega l^d, \tag{2.23}$$

which have to be inserted into the right-hand sides in order to obtain an autonomous system of 5 differential equations that is nonlinear in a natural or intrinsic way. We note however that there are many items that reappear in various equations, or are similar to each other, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. This dynamical system will be investigated in the next section in somewhat informal terms and, with slight modifications, in a rigorous way in the Appendix A to this chapter.

As the model is now formulated it exhibits—besides the well-known real rate of interest channel (giving rise to destabilizing Mundell-effects that are traditionally tamed by the application of the jump variable technique—another real feedback channel, see Fig. 2.1, which we have called the Rose real wage effect (based on the work of Rose (1967)) in Chiarella and Flaschel

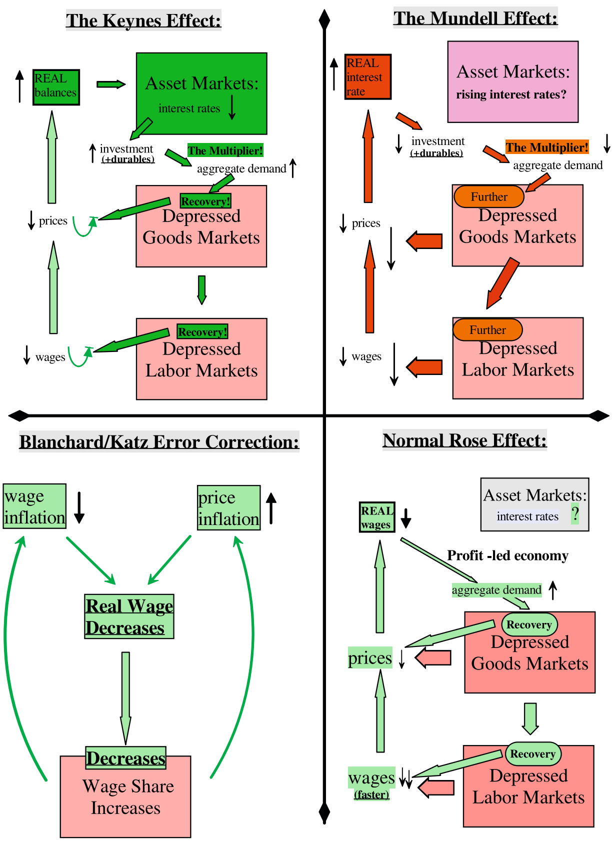

Fig. 2.1. The feedback channels of matured Keynesian macrodynamics

(2000). This channel is completely absent from the considered New Keynesian approach and it is in a weak form present in the model of the Neoclassical synthesis, stage I, due to the inclusion of the rate of profit into the considered investment function. The Rose effect only gives rise to a clearly distinguishable and significant feedback channel, however, if wage and price flexibilities are both finite and if aggregate demand depends on the income distribution between wages and profits. In the traditional AS–AD model of Sect. 2.2 it only gives rise to a directly stabilizing dependence of the growth rate of real wages on their level, while in our mature form of this AS–AD analysis it works through the interaction of the law of motion (2.14) for real wages, the investment climate and the IS-curve we have derived on this basis. The real marginal costs effect of the New Keynesian approach is here present in addition, in the denominator of the expression we are using for rate of capacity utilization, ($u = y/y^p$) and contributes to some extent to stability should the Rose effect by itself be destabilizing.

We thus have now two feedback channels interacting in our extended DAS–AD dynamics which in specific ways exhibit stabilizing as well as destabilizing features (Keynes vs. Mundell effects and normal vs. adverse Rose effects). A variety of further feedback channels of Keynesian macrodynamics are investigated in Chiarella et al. (2000). The careful analysis of these channels and the partial insights that can be related with them form the basis of the 5D stability analysis of the next section and the appendix to this chapter. Such an analysis differs radically from the always convergent jump-variable analysis of the rational expectations school in models of the Neoclassical synthesis, stage I and stage II and many other approaches to macrodynamics.

In Fig. 2.1 we summarize the basic feedback channels of our approach to DAS–AD dynamics. We have the textbook Keynes-effect or stabilizing nominal rate of interest rate channel top-left and the therewith interacting destabilizing Mundell- or inflationary expectations effect which together with the Keynes-effect works through the (expected) real rate of interest channel. In addition we have Rose (1967)-effects working though the real wage channel. Figure 2.1 indicates that the real wage channel will be stabilizing when investment reacts more strongly than consumption to real wage changes (which is the case in our model type, since here consumption does not depend at all on the real wage) if this is coupled with wages being more flexible than prices, in the sense that (2.14) then establishes a positive link between economic activity and induced real wage changes. However if this latter relationship becomes a negative one, due to a sufficient degree of price level flexibility, this will destabilize the economy, since shrinking economic activity due to real wage increases will then indeed induce further real wage increases, due to a price level that is falling faster than the wage level in this state of depressed markets for goods and for labor (representing an adverse type of Rose-effect). We stress here that the degree of forward looking behavior in both the wage and the price level dynamics is also important, since these weights also enter the crucial equation (2.14) describing the dynamics of real wages for any changing states of economic activity. Figure 2.1 finally also shows the Blanchard and Katz wage share correction mechanism (bottom left) which will be added to the considered dynamics in Sect. 2.6.

2.5 Feedback-Guided Local Stability Investigation

In this section, we illustrate an important method used to prove local asymptotic stability of the interior steady state of the considered dynamical system, through partial motivations from the feedback chains that characterize our baseline model of Keynesian macrodynamics. Since the model is an extension of the traditional AS–AD growth model we know that there is a real rate of interest effect involved, first analyzed by formal methods in Tobin (1975), see also Groth (1992). There is therefore the stabilizing Keynes-effect based on activity-reducing nominal interest rate increases following price level increases, which provides a check to further price increases. Secondly, if the expected real rate of interest is driving investment and consumption decisions (increases leading to decreased aggregate demand), there is the stimulating (partial) effect of increases in the expected rate of inflation that may lead to further inflation and further increases in expected inflation under appropriate conditions. This is the Mundell-effect that works opposite to the Keynes-effect, but also through the real rate of interest channel as just seen; we refer the reader again to Fig. 2.1.

The Keynes-effect is the stronger the smaller the parameter h_2 characterizing the interest rate sensitivity of money demand becomes, since the reduced-form LM equation reads:

$$i = i_o + (h_1 y - m)/h_2, \quad y = Y/K, \; m = M/(pK).$$

The Mundell-effect is the stronger the faster the inflationary climate adjusts to the present level of price inflation, since we have

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c) = \beta_{\pi^c}\kappa[\beta_p(u - u_o) + \kappa_p\beta_w(e - e_o)],$$

and since both rates of capacity utilization depend positively on the investment climate ϵ^m which in turn (see (2.16)) is driven by excess profitability $\epsilon = \rho + \hat{p} - i$. Excess profitability in turn depends positively on the inflation rate and thus on the inflationary climate as the reduced-form price Phillips curve shows.

There is—as we already know—a further potentially (at least partially) destabilizing feedback mechanism as the model is formulated. Excess profitability depends positively on the rate of return on capital ρ and thus negatively on the real wage ω. We thus get—since consumption does not yet depend on the real wage—that real wage increases depress economic activity (though with the delay that is caused by our concept of an investment climate transmitting excess profitability to investment behavior). From our reduced-form real wage dynamics

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_w(e - e_o) - (1 - \kappa_w)\beta_p(u - u_o)],$$

we thus obtain that price flexibility should be bad for economic stability due to the minus sign in front of the parameter β_p while the opposite should hold true for the parameter that characterizes wage flexibility. This is a situation already investigated in Rose (1967). It gives the reason for our statement that wage flexibility gives rise to normal, and price flexibility to adverse, Rose effects as far as real wage adjustments are concerned. Besides real rate of interest effect, establishing opposing Keynes- and Mundell-effects, we thus have also another real adjustment process in the considered model where now wage and price flexibility are in opposition to each other, see Chiarella and Flaschel (2000) and Chiarella et al. (2000) for further discussion of these as well as other feedback mechanisms in Keynesian growth dynamics.

There is still another adjustment speed parameter in the model, the one (β_{ϵ^m}) that determines how fast the investment climate is updated in the light of current excess profitability. This parameter will play no decisive role in the stability investigations that follow, but will become important in the more detailed and rigorous stability analysis to be considered in the appendix to the chapter. In the present stability analysis we will however focus on the role played by h_2, β_w, β_p, β_{π^c} in order to provide one example of asymptotic stability of the interior steady state position by appropriate choices of these parameter values, basically in line with the above feedback channels of partial Keynesian macrodynamics.

The above adds to the understanding of the dynamical system (2.14)–(2.18) whose stability properties are now briefly investigated by means of varying adjustment speed parameters. With the feedback scenarios considered above in mind, we first observe that the inflationary climate can be frozen at its steady state value, here $\pi_o^c = \hat{M} - n = 0$, if $\beta_{\pi^c} = 0$ is assumed. The system thereby becomes 4D and it can indeed be further reduced to 3D if in addition $\beta_w = 0$ is assumed, since this decouples the l-dynamics from the remaining dynamical system with state variables ω, m, ϵ^m.

We intentionally will consider the stability of these 3D subdynamics—and its subsequent extensions—in very informal terms here, leaving rigorous calculations of stability criteria to the appendix (there however for the case of an interest rate policy rule in the place of our standard LM-curve). In this way we hope to demonstrate to the reader how one can proceed in a systematic way from low to high dimensional analysis in such stability investigations. This method has been already applied to various other often much more complicated dynamical systems, see Asada et al. (2003) for a variety of typical examples.

Proposition 2.1. *Assume that $\beta_{\pi^c} = 0$, $\beta_w = 0$ holds. Assume furthermore that the parameters h_2, β_p are chosen sufficiently small and that the κ_w, κ_p parameters do not equal* 1. *Then: The interior steady state of the reduced* 3D *dynamical system*

$$\begin{aligned}
\hat{\omega} &= -\kappa(1-\kappa_w)\beta_p(y/y^p(\omega) - u_o),\\
\hat{m} &= -i_1\epsilon^m - \kappa\beta_p(y/y^p(\omega) - u_o),\\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\rho + \kappa\beta_p(y/y^p(\omega) - u_o) - i - \epsilon^m),
\end{aligned}$$

is locally asymptotically stable.

Sketch of proof. The assumptions made imply that the Mundell-effect is absent from the reduced dynamics, since inflationary expectations are kept constant, and that the destabilizing component of the Rose-effect is weak. Due to the further assumption of a strong Keynes-effect, the steady state of the system is thus surrounded by centripetal forces,

Proposition 2.2. *Assume in addition that the parameter β_w is now positive and chosen sufficiently small. Then: The interior steady state of the implied* 4D *dynamical system (where the law of motion for l has now been incorporated)*

$$\begin{aligned}
\hat{\omega} &= \kappa[(1-\kappa_p)\beta_w(l^d/l - e_o) - (1-\kappa_w)\beta_p(y/y^p - u_o)],\\
\hat{m} &= -i_1\epsilon^m - \kappa[\beta_p(y/y^p - u_o) + \kappa_p\beta_w(l^d/l - e_o)],\\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\rho + \kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)], -i - \epsilon^m),\\
\hat{l} &= -i_1\epsilon^m,
\end{aligned}$$

is locally asymptotically stable.

Sketch of proof. In the considered situation we do no apply the Routh–Hurwitz conditions to 4D dynamical systems, as in the appendix to this chapter, but instead proceed by simple continuity arguments. Eigenvalues are continuous functions of the parameters of the model. It therefore suffices to show that the determinant of the Jacobian matrix of the 4D dynamics that is generated when the parameter β_w is made positive is positive in sign. The zero eigenvalue of the case $\beta_w = 0$ must then become positive and the three other eigenvalues continue to exhibit negative real parts if the parameter β_w is changing by a small amount solely. We conjecture—in view what is shown in the appendix in the case of an interest rate policy rule—that this proposition holds for all changes of the parameter β_w.

Proposition 2.3. *Assume in addition that the parameters β_{π^c} is now positive and chosen sufficiently small. Then: The interior steady state of the full* 5D *dynamical system (where the differential equation for π^c is now included)*

$$\begin{aligned}
\hat{\omega} &= \kappa[(1-\kappa_p)\beta_w(l^d/l - e_o) - (1-\kappa_w)\beta_p(y/y^p - u_o)],\\
\hat{m} &= -\pi^c - i_1\epsilon^m - \kappa[\beta_p(y/y^p - u_o) + \kappa_p\beta_w(l^d/l - e_o)],\\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\rho + \kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)] + \pi^c - i - \epsilon^m),\\
\hat{l} &= -i_1\epsilon^m,\\
\dot{\pi}^c &= \beta_{\pi^c}(\kappa[\beta_p(y/y^p(\omega) - u_o) + \kappa_p\beta_w(l^d/l - e_o)]),
\end{aligned}$$

is locally asymptotically stable.

Sketch of proof. As for Proposition 2.2, now simply making use of the rows corresponding to the laws of motion for l and m in order to reduce the row corresponding to the law of motion for π^c to the form $(0,0,0,0,-)$, again without change in the sign of the determinants of the accompanying Jacobians, allows to show here that the determinant of the full 5D dynamics is always negative. The fifth eigenvalue must therefore change from zero to a negative value if the parameter β_π is made slightly positive (but not too large), while the remaining real parts of eigenvalues do not experience a change in sign.

A weak Mundell-effect does consequently not disturb the proven asymptotic stability.

We stress again that the parameters β_p and β_{π^c} have been chosen such that adverse Rose and destabilizing Mundell-effects are both weak and accompanied by a strongly stabilizing Keynes-effect.

We formulate as a corollary to Proposition 2.3 that, due to the always negative sign of the just considered 5D determinant, loss of stability can only occur by way of Hopf-bifurcations, i.e., through the generation of cycles in the real-nominal interactions of the model.

Corollary. *Assume an asymptotically stable steady state on the basis of Proposition 2.3. Then: The interior steady state of the full* 5D *dynamical system will lose its stability (generally) by way of a sub- or supercritical Hopf-bifurcation if the parameters β_p or β_{π^c} are chosen sufficiently large.*

Since the model is in a natural way a non-linear one, we know from the Hopf-bifurcation theorem[14] that usually loss of stability will occur through the death of an unstable limit cycle (the subcritical case) or the birth of a stable one (the supercritical case), when destabilizing parameters pass through their bifurcation values. Such loss of stability is here possible if prices become sufficiently flexible compared to wage flexibility, leading to an adverse type of real wage adjustment, or if the inflationary climate expression is updated sufficiently fast, i.e., if the system loses the inflationary inertia—we have built into it—to a sufficient degree. These are typical feedback structures of a properly formulated Keynesian dynamics that may give rise to global instability, directly in the case of subcritical Hopf-bifurcations and sooner or later in the case of supercritical bifurcations, and thus give rise to the need to add further extrinsic behavioral nonlinearities to the model in order to bound the generated business fluctuations. Such issues will be briefly explored in the following section. They are further investigated in the next chapter of the book.

We conclude from this section that a properly specified Keynesian disequilibrium dynamics—with labor and capital both over- or underutilized in the course of the generated business fluctuations—integrates important feedback channels based on partial perspectives into a consistent whole, where all behavioral and budget restrictions fully specified. We can have damped oscillations, persistent fluctuations or even explosive oscillations in such a framework. The latter necessitate the introduction of certain behavioral non-

[14] See the mathematical appendix in Asada et al. (2003) for details.

linearities in order to allow for viable business fluctuations. However, a variety of well-known stabilizing or destabilizing feedback channels of Keynesian macrodynamics are still excluded from the present stage of the modelling of Keynesian macrodynamics, such as wealth effects in consumption or Fisher debt effects in investment behavior, all of which define the agenda for future extensions of this model type.[15]

2.6 Wage Share Error Corrections and Interest Rate Policy Rules

We have considered in Sect. 2.3 the New Keynesian approach to wage and price dynamics and have compared this approach already briefly with the two Phillips curve wage-price spiral of this chapter there (without use of real wage gaps in this baseline DAS–AD model). We recapitulate this extended wage-price spiral here briefly and include thereby Blanchard and Katz (1999) type error correction terms into our baseline DAS–AD dynamics, together with a Taylor interest rate policy rule now in the place of the LM-curve representation of the financial markets of Sect. 2.4, in order to fully show how our matured Keynesian AS–AD dynamics is differentiated from the New Keynesian approach when both approaches make use of two Phillips curves and an interest rate policy rule. In the New Keynesian model of wage and price dynamics we had:

$$\begin{aligned}
\mathrm{d}\ln w_t &\overset{NWPC}{=} E_t(\mathrm{d}\ln w_{t+1}) + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln\omega_t,\\
\mathrm{d}\ln p_t &\overset{NPPC}{=} E_t(\mathrm{d}\ln p_{t+1}) + \beta_{py}\ln Y_t + \beta_{p\omega}\ln\omega_t.
\end{aligned}$$

Current wage and price inflation depend on expected future wage and price inflation, respectively, and in the usual way on output gaps, augmented by a negative (positive) dependence on the real wage gap in the case of the wage (price) Phillips curve. Assuming again a deterministic framework and myopic perfect foresight allows to suppress the expectations operator.

In order to get from these two laws of motion the corresponding Phillips curves of our matured, but conventional DAS–AD dynamics, we use Neoclassical dating of expectations in a crossover fashion, i.e., perfectly foreseen wage inflation in the price Phillips curve and perfectly foreseen price inflation in the wage Phillips curve, now coupled with hybrid expectations formation as

[15] See Chiarella et al. (2000) for a survey on such feedback channels.

in the DAS–AD model of the preceding sections. We furthermore replace the output gap in the NWPC by the employment rate gap and by the capacity utilization gap in the NPPC as in the matured Keynesian macrodynamics introduced in Sect. 2.4. Finally, we now also use real wage gaps in the MWPC and the MPPC, here based on microfoundations of Blanchard and Katz type, as in the paper by Flaschel and Krolzig (2006). In this way we arrive at the following general form of our M(atured)WPC and M(atured)PPC, formally discriminated from the New Keynesian case of both staggered wage and price setting solely by a different treatment of wage and price inflation expectations.

$$\begin{aligned}
\mathrm{d}\ln w_{t+1} &\overset{\mathrm{MWPC}}{=} \kappa_w \mathrm{d}\ln p_{t+1} + (1-\kappa_w)\pi_t^c + \beta_{we}(e_t - e_o) - \beta_{w\omega}\ln(\omega_t/\omega_o), \\
\mathrm{d}\ln p_{t+1} &\overset{\mathrm{MPPC}}{=} \kappa_p \mathrm{d}\ln w_{t+1} + (1-\kappa_p)\pi_t^c + \beta_{pu}(u_t - u_o) + \beta_{p\omega}\ln(\omega_t/\omega_o).
\end{aligned}$$

In continuous time these wage and price dynamics read

$$\begin{aligned}
\hat{w} &= \kappa_w \hat{p} + (1-\kappa_w)\pi^c + \beta_{we}(e - e_o) - \beta_{w\omega}\ln(\omega/\omega_o), \\
\hat{p} &= \kappa_p \hat{w} + (1-\kappa_p)\pi^c + \beta_{pu}(u - u_o) + \beta_{p\omega}\ln(\omega/\omega_o).
\end{aligned}$$

Reformulated as reduced-form expressions, these equations give rise to the following linear system of differential equations ($\theta = \ln\omega$):

$$\begin{aligned}
\hat{w} &= \kappa[\beta_{we}(e - e_o) - \beta_{w\omega}(\theta - \theta_o) + \kappa_w(\beta_{pu}(u - u_o) + \beta_{p\omega}(\theta - \theta_o))] + \pi^c, \\
\hat{p} &= \kappa[\beta_{pu}(u - u_o) + \beta_{p\omega}(\theta - \theta_o) + \kappa_p(\beta_{we}(e - e_o) - \beta_{w\omega}(\theta - \theta_o))] + \pi^c, \\
\dot{\theta} &= \kappa[(1-\kappa_p)(\beta_{we}(e - e_o) - \beta_{w\omega}(\theta - \theta_o)), \\
&\quad - (1-\kappa_w)(\beta_{pu}(u - u_o) + \beta_{w\omega}(\theta - \theta_o))].
\end{aligned}$$

As monetary policy we now in addition employ a Taylor interest rate rule, in the place of an LM-curve, given by:

$$i_{\mathrm{T}} = (i_o - \bar{\pi}) + \hat{p} + \phi_{ip}(\hat{p} - \bar{\pi}) + \phi_{iu}(u - u_o), \tag{2.24}$$

$$\dot{i} = \alpha_{ii}(i_{\mathrm{T}} - i). \tag{2.25}$$

These equation describe the interest rate target i_{T} and the interest rate smoothing dynamics chosen by the central bank. The target rate of the central bank i_{T} is here made dependent on the steady state real rate of interest, augmented by actual inflation towards to a specific nominal rate of interest, and is as usually dependent on the inflation gap with respect to the target inflation rate $\bar{\pi}$ and the capacity utilization gap (our measure of the output gap). With respect to this interest rate target, there is then interest rate smoothing with strength α_{ii}. Inserting i_{T} and rearranging terms we get from this latter expression the following form of a Taylor rule

$$\dot{i} = -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o),$$

where we have $\gamma_{ii} = \alpha_{ii}$, $\gamma_{ip} = \alpha_{ii}(1 + \phi_{ip})$, i.e., $\phi_{ip} = \gamma_{ip}/\alpha_{ii} - 1$ and $\gamma_{iu} = \alpha_{ii}\phi_{iu}$.

Since the interest rate is temporarily fixed by the central bank, we must have an endogenous money supply now and get that the law of motion of the original model

$$\hat{m} = -\hat{p} - i_1\epsilon^m,$$

does now no longer feed back into the rest of the dynamics.

Taken together the revised AS–AD model of this section consists of the following five laws of motion for the log of real wages, the nominal rate of interest, the investment climate, labor intensity and the inflationary climate:

$$\begin{aligned}
\dot{\theta} &= \frac{1}{1-\kappa_w\kappa_p}[(1-\kappa_p)(\beta_{we}(e-e_o) - \beta_{w\omega}(\theta-\theta_o)),\\
&\quad -(1-\kappa_w)(\beta_{pu}(u-u_o) + \beta_{w\omega}(\theta-\theta_o))],\\
\dot{i} &= -\gamma_{ii}(i - i_o) + \gamma_{ip}(\hat{p} - \bar{\pi}) + \gamma_{iu}(u - u_o),\\
\dot{\epsilon}^m &= \beta_{\epsilon^m}(\epsilon - \epsilon^m),\\
\hat{l} &= -i_1\epsilon^m,\\
\dot{\pi}^c &= \beta_{\pi^c}(\hat{p} - \pi^c),
\end{aligned}$$

with $\hat{p} = \kappa[\beta_{pu}(u-u_o) + \beta_{p\omega}(\theta-\theta_o) + \kappa_p(\beta_{we}(e-e_o) - \beta_{w\omega}(\theta-\theta_o))]$.

This dynamical system is to be supplemented by the following static relationships for output, potential output and employment (all per unit of capital), the rate of interest and the rate of profit:

$$\begin{aligned}
y &= \frac{1}{1-c}[i_1\epsilon^m + n + g - t] + \delta + t,\\
y^p &= f((f')^{-1}(\exp\theta)),\\
&\qquad F(1, L^p/K) = f(l^p) = y^p, F_L(1, L^p/K)) = f'(l^p) = \omega,\\
l^d &= f^{-1}(y),\\
u &= y/y^p, \quad e = l^d/l,\\
\rho &= y - \delta - \omega l^d, \quad \epsilon = \rho - (i - \hat{p}),\\
i_o &= \rho_o + \bar{\pi},
\end{aligned}$$

which have to be inserted into the right-hand sides of the dynamics in order to obtain an autonomous system of five differential equations that is nonlinear in a natural or intrinsic way.

The interior steady state solution of the above dynamics is given by:

$$
\begin{aligned}
& y_o = \frac{1}{1-c}[n+g-t]+\delta+t,\ l_o^d = f^{-1}(y_o),\ l_o = l_o^d/e_o,\ y_o^p = y_o/u_o, \\
& l_o^p = f^{-1}(y_o^p),\ \omega_o = f'(l_o^p),\ \hat{p}_o = \pi_o^c = \bar{\pi}, \\
& \rho_o = f(l_o^d)-\delta-\omega_o l_o^d,\ i_o = \rho_o+\hat{p}_o,\ \epsilon_o = \epsilon_o^m = 0.
\end{aligned}
$$

Note that income distribution in the steady state is still determined by marginal productivity theory, since it does not yet play a role in aggregate demand in the steady state.

Despite formal similarities in the building blocks of the New Keynesian AS–AD dynamics and the above matured Keynesian DAS–AD dynamics, the resulting reduced form laws of motion, see Sect. 2.3, have not much in common in their structure and nothing in common in the applied solution strategies. The New Keynesian model has four forward-looking variables and thus demands for its determinacy four unstable roots, while our approach only exhibits myopic perfect foresight of a crossover type and thus allows again, with respect to its all variables, for predeterminacy and for stability results as in the preceding section and as shown in the mathematical appendix of this chapter.

We note in this regard that there are many items that reappear in various equations, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability. Using such linear dependencies and the knowledge we have about the feedback structure of the dynamics we can then show the following proposition:

Proposition 2.4. *Assume that the parameters β_{pu}, $\beta_{p\omega}$ in the price PC are not chosen too large and that the parameters κ_p, β_{p^m} and i_1, γ_{ii} are chosen sufficiently small. Then: The interior steady state of the above 5D dynamical system is locally asymptotically stable.*

Proof. See the mathematical Appendix A of this chapter.

We thus see that the assumption about the price PC, the Mundell effect, the degree of interest rate smoothing and the speed with respect to which investment is adjusted to profitability changes can be decisive for the stability of the dynamics. However, this is only one set of sufficient stability conditions for the considered dynamics, which and not all a necessary selection yet. Further combination of the working of destabilizing Mundell-effects, Rose real-wage effects and the strength the inflation targeting process may be found that ensure stability, yet relevant parameter choices can also be found where the dynamics are not viable without the addition of extra behavioral nonlinearities, a topic that is considered in the next section by means of numerical simulations of the dynamics (in the case of an LM-curve as well of a Taylor interest rate rule).

2.7 Downward Nominal Wage Rigidities

Let us now turn to some numerical simulations of our matured Keynesian analysis of the working of the wage-price spiral. In Fig. 2.2 we show the maximum real parts of eigenvalues as functions of the crucial adjustment speeds of

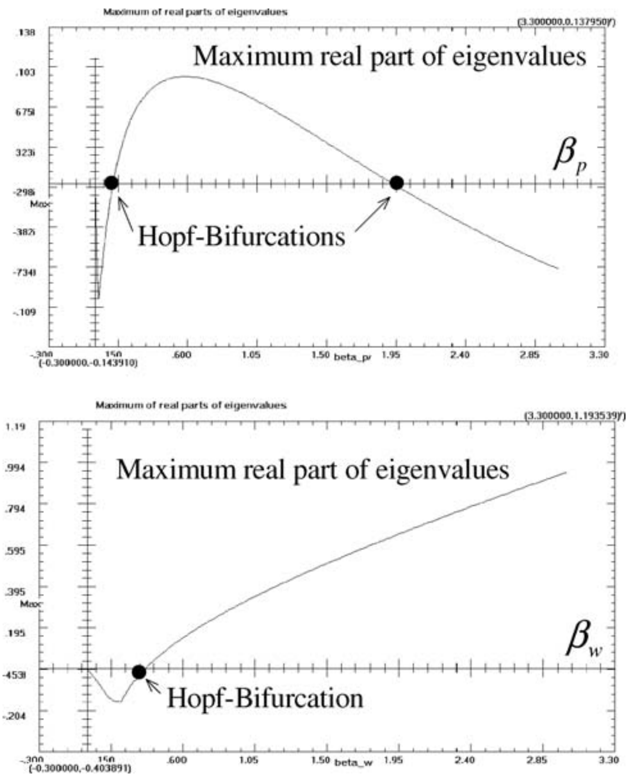

Fig. 2.2. Loss of stability and reestablishment of stability by way of Hopf-bifurcations

prices and wages with respect to the demand pressure on their corresponding markets. We see from these graphs that increasing wage flexibility is initially stabilizing and increasing price flexibility destabilizing (as expected from our partial consideration of the real wage channel). But fairly soon the role played by these parameters becomes reversed, approaching thereafter in fact a second Hopf-bifurcation point in each case. Thus, sooner or later, the partial insights gained from our consideration of Rose-effects are overturned and further wage flexibility and price flexibility then start to do just the opposite of what these partial arguments would suggest. This shows that a 5D dynamical system (and the numerous local asymptotic stability conditions it exhibits) can be much more complicated than is suggested by partial formal or even verbal economic reasoning.

Starting from this observation we now consider situations where the loss of stability has become a total one, giving rise to economic fluctuations, the amplitude of which increases without bound. From the perspective of previous work of ours[16]—and the reversal in the stability features just observed—we expect that complete or partial downward rigidity of money wages may be the cure in such a situation, in line with what has been suggested already by Keynes (1936), since wage adjustment is then destabilizing, while price adjustment is not. We thus now consider situations where money wages can fall at most with the rate $f \leq 0$, which alters our WPC in an obvious way, leading to a kink in it if the floor f is reached. Figure 2.3 provides a typical outcome of the dynamics if downwardly rigid money wages are added to an explosive situation where the economy is not at all a viable one and in fact subject to immediate breakdown without such rigidity.

If the money wage Phillips curve is augmented by the assumption that money wages can rise as described, but cannot fall at all, we get a situation of a continuum of steady states (for money supply growth equal to natural growth $\hat{M} = n$ and thus no steady state inflation). This is due to the zero root hysteresis that then occurs, and the thereby implied strong result that the economy will then converge rapidly to the situation of a stable depression where wages become stationary. This stable depression depends in its depth on the initial shock the economy was subject to and is indeed a persistent one, since money wages do not fall (whereby on the one hand economic breakdown is avoided, at the cost of more or less massive underemployment on the other hand). If, by contrast, money wages can fall, but will do so at most at the rate

[16] See e.g. Chiarella and Flaschel (2000).

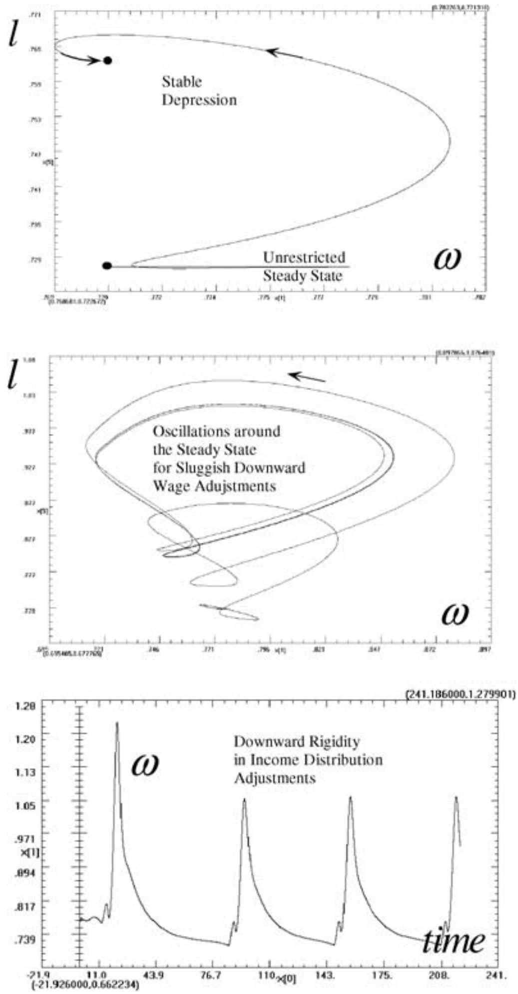

Fig. 2.3. Stable depressions or persistent fluctuations through downwardly rigid money wages (phase length approximately sixty years). *Note:* The parameter set used was: $\omega(0) = 0.770$, $m(0) = 9.088$, $\epsilon^m(0) = 0$, $\ell(0) = 0.727$, $\pi^c(0) = 0$, $\beta_\omega = 0.5$, $\beta_p = 0.5$, $\beta_{\pi^c} = 0.32$, $\beta_{\epsilon^m} = 0.3$, $\alpha = 0.3$, $\kappa_w = 0.5$, $\kappa_p = 0.5$, $s_c = 0.2$, $t^n = 0.25$, $\delta = 0.05$, $n = 0.05$, $g = 0.3$, $\bar{u} = 1.0$, $\bar{e} = 1.0$, $h_1 = 0.1$, $h_2 = 0.1$, $i = 0.25$, $wage\text{-}floor = 0.0$, $w_{shock} = 1.01$

of for example -0.01, the steady state instead remains uniquely determined (as shown in this chapter) and—though surrounded by strongly explosive forces—it is not totally unstable, due to the limit cycle situation that is then generated by the operation of the floor to the speed of money wage declines.

This type of floor makes depressions much longer than recoveries, but avoids the situation where the economy can be trapped in a stable depression as in the case of complete downward rigidity of money wages. The two situations just discussed are illustrated by the Fig. 2.3 in the real wage and labor intensity phase subspace of the full 5D dynamics. In this figure, the latter situation is also augmented by a time series plot for the real wage with its characteristic asymmetry between booms and depressions, with a total phase length of around sixty years of the generated income distribution dynamics. This is in broad agreement with observed empirical phase plots of this sort for example for the U.S. economy, see Chen et al. (2005). Money supply policy rules can dampen the fluctuations shown, but are in general too weak to allow for a disappearance of such endogenously generated and very persistent business cycles in the private sector.

Note that the employment rate of an economy is inversely related to the fluctuations in the full employment labor intensity ratio l that is shown in Fig. 2.3. A high value of l therefore signifies a low employment rate and thus the situation of a long-lasting depression from where the economy is slowly recovering. Normal employment, by contrast, is given when the state variable l exhibits a low value and is accompanied by the instability the economy is subject to if the kink in the money wage PC is not in operation. The economy is then in a very volatile state, which is however moving into a new depression the more the kink in the money wage PC comes into operation again. The working of the kink is clearly shown in the bottom Fig. 2.3 where we have only sluggishly falling real wages until a new recovery phase sets in.

It is one important implication of such a downward floor to the speed of money wage declines that it can easily generate complex dynamics from the mathematical point of view. This is due to the fact that the economy is hitting the kink often in slightly distinct situations after each unstable recovery and thus works each time through the depression phase in a different way, leading to a clearly distinguishable upswing thereafter. Such a situation is exemplified in Fig. 2.4 where the irregularity of the fluctuations in the real wage ω is shown over a time horizon of four thousand years in the top figure. In the bottom figure we in addition show the projection of the cycle into the $\omega - l$ phase plane. One can see there the small corridor through which the dynamics are squeezed on the left hand side for low real wages and the in principle explosive fluctuations that are generated thereafter, but kept under control again and in an increasing manner through the kink in the money wage PC.

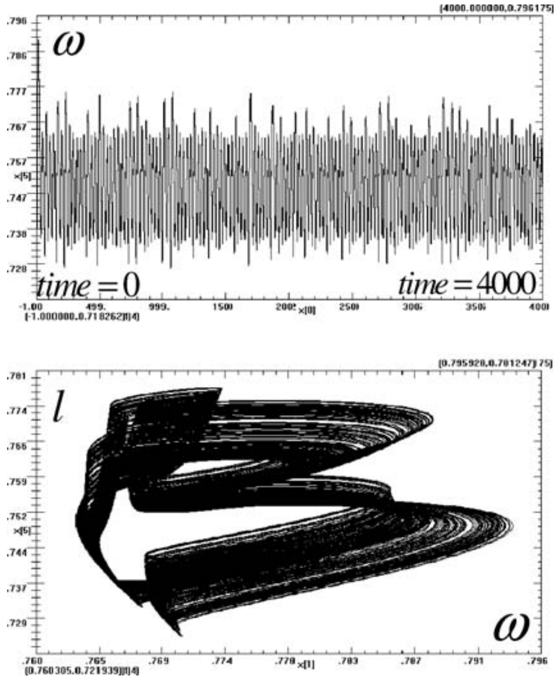

Fig. 2.4. Mathematically complex dynamics with basically economically similar long-term fluctuations in growth and income distribution. *Note:* The parameter set used was: $\omega(0) = 0.770$, $m(0) = 9.088$, $\epsilon^m(0) = 0$, $\ell(0) = 0.727$, $\pi^c(0) = 0$, $\beta_\omega = 0.2$, $\beta_p = 0.5$, $\beta_{\pi^c} = 1.1$, $\beta_{\epsilon^m} = 0.3$, $\alpha = 0.3$, $\kappa_w = 0.7$, $\kappa_p = 0.3$, $s_c = 0.2$, $t^n = 0.25$, $\delta = 0.05$, $n = 0.05$, $g = 0.3$, $\bar{u} = 1.0$, $\bar{e} = 1.0$, $h_1 = 0.1$, $h_2 = 0.1$, $i = 1$, $wage\text{-}floor = -0.0049$, $w_{shock} = 1.01$

An indication of the range of complex behavior can be obtained by considering bifurcation diagrams (showing the local maxima and minima of a state variable as one parameter of the model is increased along the horizontal axis). In Fig. 2.5 we show such a diagram for ω with respect to the savings rate $s(= 1 - c)$. As s increases the bifurcation diagram indicates that two-cycles for ω give way to periods of complex behavior interspersed with period of high order cycles. Of course, average savings ratios above 25-percent are not too likely from the economic point of view, so that the economic range for the savings parameter is significantly smaller than the one shown in Fig. 2.5. Visible is however that higher savings ratios increase the tension in our model economy. This also holds true for the case of an interest rate policy, as shown

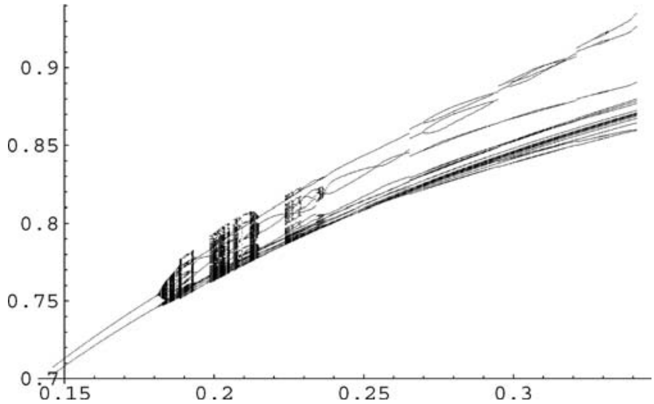

Fig. 2.5. Mathematically complex dynamics: Bifurcation diagram 2.4

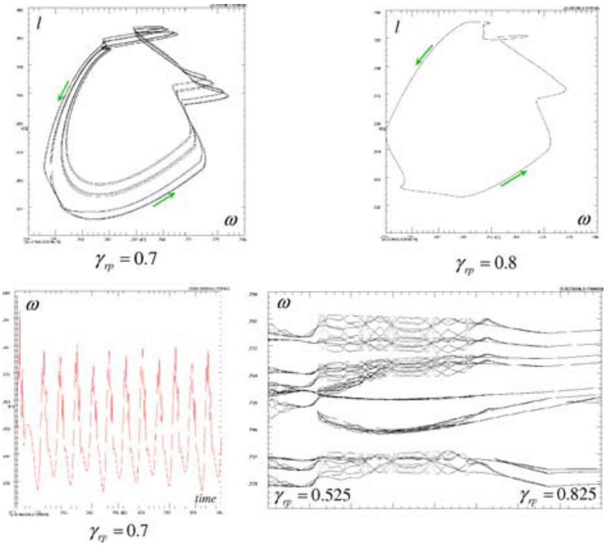

Fig. 2.6. Complexity reducing interest rate policy?

in Fig. 2.6, where we in fact would get convergence for saving rates below $s = 0.03$-percent solely.

In Fig. 2.6,[17] we instead show for a higher savings rate period doubling situations (top-left and bottom-left) that can be reduced to simple limit cycles (top-right) by increasing the strength of the reaction of the Central Bank with

[17] The parameter set here is: $\beta_w = 0.2, \beta_p = 0.1, \beta_{pi^m} = 0.72, \beta_{\epsilon^m} = 0.8, \gamma_{ii} = 0.1, \gamma_{ip} = 0.7, \gamma_{iu} = 0.1, \alpha = 0.3, \kappa_w = 0.5, \kappa_p = 0.5, s = 0.1, t^n = 0.3, \delta = 0.05, n = 0.05, g = 0.3, u_o = 1.0, e_o = 1.0, \bar{\pi} = 0, i = 0.2.$

respect to the inflation gap. Yet, due to the fast adjustment of the inflationary climate with respect to current inflation rates that is here assumed, not much more can be achieved by monetary policy in the considered case. Figure 2.6, bottom-right shows in this respect again a bifurcation diagram which indicates complex types of limit cycle behavior for parameter values γ_{ip} below 0.7, but thereafter the establishment of simpler limit cycles which cannot made simpler however or even turned into convergent dynamics as the parameter γ_{ip} is further increased, even when increased much beyond 0.825 (not shown in this figure). Monetary policy may reduce dynamic complexity to a certain degree, but may be incapable to turn persistent business fluctuations generated in the private sector of the economy into damped oscillations.

Underlying Fig. 2.6 is a floor parameter $f = 0.02$, i.e., wage inflation cannot even be reduced below 2 percent. In addition we, however, here assume that wages regain their assumed flexibility if the rate of employment falls below 80 percent. Without this latter assumption cycles would be much larger than shown in Fig. 2.6, i.e., we here have a case where a return to wage flexibility in deep depressions improves the stability of the dynamics, though the floor to money wage inflation in between is indeed of help, since its removal would lead to explosive business fluctuations. A wage Phillips curve with three regimes (two regime changes) as investigated empirically in Filardo (1998) may therefore be better than one with only two in a situation where partial Rose effects indicate that wage flexibility is stabilizing while price flexibility is not.

2.8 Conclusion

Summing up the main results of this chapter, we have been able to generate damped business fluctuations, persistent oscillations or even complex dynamics from a matured, but conventional synthesis of Keynesian AS–AD dynamics with an advanced description of its wage-price module as a wage-price spiral, when in addition simple (plausible) regime changes in the money wage Phillips curve are taken into account. There are thus no fancy nonlinearities necessary in a by and large conventional type of AS–AD disequilibrium dynamics in order to obtain interesting dynamic outcomes. Some further stability may be achieved through monetary policy to a certain degree. However the cycle generating mechanisms in the private sector are often too strong to be overcome completely by the mechanisms analyzed in this chapter. This is so since in a situation of possibly fairly explosive dynamics the downward money wage rigidity provides a stabilizing influence on the dangers for economic breakdown arising from inflationary or deflationary spirals and their implications, but not on other broader destabilizing tendencies.

We stress that we have achieved viable or bounded dynamics through behavioral assumptions that concern the private sector and not—as in the New Keynesian approach of Sect. 2.3—only by way of an interest rate policy of the Central Bank that is sufficiently advanced and active such that all roots of the Jacobian of the dynamics become unstable. In the latter case, boundedness comes about by assumption in a totally unstable linear(ized) environment and not by changes in agents' behavior when the economy departs too much from the steady state.

Appendix A

Rigorous Stability Analysis (Interest Rate Policy Case)

In this appendix we provide the proof for Proposition 2.4 of the chapter. We refer the reader to Sect. 2.6 for the original formulation of the laws of motion to be considered here and their interior steady state solution. The static relationships supplementing the laws of motion introduced in Sect. 2.6 and their partial derivatives are reformulated for the subsequent proof as follows:

$$y = \frac{1}{1-c}[i_1\varepsilon^m + n + g - t] + \delta + t = y(\varepsilon^m), \tag{A.1}$$

$$y_\varepsilon = dy/\varepsilon^m = \frac{1}{1-c} i_1 > 0,$$

$$y^p = f((f')^{-1}(\exp\theta)) = y^p(\theta); \tag{A.2}$$

$$y^p_\theta = dy^p/d\theta = (f'(l^p)/f''(l^p))(\exp\theta) < 0,$$

$$l^d = f^{-1}(y(\varepsilon^m)) = l^d(\varepsilon^m); \quad l^d_\varepsilon = dl^d/d\varepsilon^m = y_\varepsilon/f' > 0, \tag{A.3}$$

$$u = y(\varepsilon^m)/y^p(\theta) = u(\varepsilon^m, \theta); \quad u_\varepsilon = \partial u/\partial\varepsilon^m = y_\varepsilon/y^p > 0,$$

$$u_\theta = \partial u/\partial\theta = -y y^p_\theta/(y^p)^2 > 0, \tag{A.4}$$

$$e = l^d(\varepsilon^m)/l = e(\varepsilon^m, l); \quad e_\varepsilon = \partial e/\partial\varepsilon^m = l^d_\varepsilon/l > 0,$$

$$e_l = \partial e/\partial l = -l^d/l^2 < 0, \tag{A.5}$$

$$\rho = y - \delta - \omega l^d = y(\varepsilon^m) - \delta - (\exp\theta) l^d(\varepsilon^m) = \rho(\varepsilon^m, \theta);$$

$$\rho_\varepsilon = \partial\rho/\partial\varepsilon^m = \{1 - (\exp\theta)/f'(l^d)\} y_\varepsilon$$

$$= \{1 - f'(l^p)/f'(l^d)\} y_\varepsilon > 0 \ \text{ if } \ l^d < l^p,$$

$$\rho_\theta = \partial\rho/\partial\theta = -(\exp\theta) l^d < 0, \tag{A.6}$$

$$i_o = \rho_o + \bar{\pi}, \tag{A.7}$$

$$\hat{p} = \frac{1}{1-\kappa_w\kappa_p}[\beta_{pu}\{u(\varepsilon^m, \theta) - u_o\} + \beta_{p\omega}(\theta - \theta_o)$$

$$+\kappa_p\{\beta_{we}(e(\varepsilon^m, l) - e_o) - \beta_{w\omega}(\theta - \theta_o)\}] + \pi^m = f(\theta, \varepsilon^m, l) + \pi^m,$$

$$\begin{aligned}
f_\theta &= \partial f/\partial\theta = \frac{1}{1-\kappa_w\kappa_p}(\beta_{pu}u_\theta + \beta_{p\omega} - \kappa_p\beta_{w\omega}),\\
f_\varepsilon &= \partial f/\partial\varepsilon^m = \frac{1}{1-\kappa_w\kappa_p}(\beta_{pu}u_\varepsilon + \kappa_p\beta_{we}e_\varepsilon) > 0,\\
f_l &= \partial f/\partial l = \frac{1}{1-\kappa_w\kappa_p}\kappa_p\beta_{we}e_l < 0,
\end{aligned} \tag{A.8}$$

because of the inequalities $0 < \kappa_w < 1$ and $0 < \kappa_p < 1$. In this case we have

$$\begin{aligned}
&\varepsilon = \rho - (i - \hat{p}) = \rho(\varepsilon^m, \theta) - i + f(\theta, \varepsilon^m, l) + \pi^m = \varepsilon(\theta, r, \varepsilon^m, l, \pi^m);\\
&\quad \varepsilon_\theta = \partial\varepsilon/\theta = \rho_\theta \lim_{(-)} + f_\theta \lim_{(?)},\ \varepsilon_i = \partial\varepsilon/\partial i = -1 < 0,\\
&\quad \varepsilon_\varepsilon = \partial\varepsilon/\partial\varepsilon^m = \rho_\varepsilon + f_\varepsilon > 0,\\
&\quad \varepsilon_l = \partial\varepsilon/\partial l = f_l < 0,\ \varepsilon_\pi = \partial\varepsilon/\partial\pi^m = 1 > 0.
\end{aligned} \tag{A.9}$$

We will make the following two assumptions in the derivation of the propositions of this appendix:

Assumption A.1. The parameters β_{pu} and $\beta_{p\omega}$ are not extremely large so that we can have $\varepsilon_\theta = \rho_\theta + f_\theta < 0$. Substituting the above static relationships into the dynamic equations of Sect. 2.6, we have the following five dimensional system of nonlinear differential equations

$$\left.\begin{aligned}
&\text{(i) } \dot{\theta} &&= \tfrac{1}{1-\kappa_w\kappa_p}\,[(1-\kappa_p)\{\beta_{we}(e(\varepsilon^m, l) - e_o) - \beta_{w\omega}(\theta - \theta_o)\}\\
& &&\quad -(1-\kappa_w)\{\beta_{pu}(u(\varepsilon^m, \theta) - u_o) + \beta_{w\omega}(\theta - \theta_o)\}]\\
& &&\quad = F_1(\theta, \varepsilon^m, l),\\
&\text{(ii) } \dot{i} &&= -\gamma_{ii}(i - i_0) + \gamma_{ip}\{f(\theta, \varepsilon^m, l) + \pi^m - \bar{\pi}\}\\
& &&\quad +\gamma_{iu}\{u(\varepsilon^m, \theta) - u_o\} = F_2(\theta, i, \varepsilon^m, l, \pi^m),\\
&\text{(iii) } \dot{\varepsilon}^m &&= \beta_{\varepsilon^m}\{\varepsilon(\theta, i, \varepsilon^m, l, \pi^m) - \varepsilon^m\} = F_3(\theta, i, \varepsilon^m, l, \pi^m),\\
&\text{(iv) } \dot{l} &&= -i_1\varepsilon^m l = F_4(\varepsilon^m, l),\\
&\text{(v) } \dot{\pi}^m &&= \beta_{\pi^m} f(\theta, \varepsilon^m, l) = F_5(\theta, \varepsilon^m, l).
\end{aligned}\right\} \tag{A.10}$$

The equilibrium solution of this system is given in Sect. 2.6. We assume that

Assumption A.2. At the equilibrium point we have $l^d < l^p$ so that $\rho_\varepsilon > 0$ holds true.

Now, let us investigate the local stability/instability of the equilibrium point of the system given by (A.10). We can write the Jacobian matrix of this system *at the equilibrium point* as follows

$$J = \begin{bmatrix} F_{11} & 0 & F_{13} & F_{14} & 0 \\ F_{21} & -\gamma_{ii} & F_{23} & F_{24} & \gamma_{ip} \\ \beta_{\varepsilon^m}(\rho_\theta + f_\theta) & -\beta_{\varepsilon^m} & -\beta_{\varepsilon^m}(1-\rho_\varepsilon - f_\varepsilon) & \beta_{\varepsilon^m} f_l & \beta_{\varepsilon^m} \\ 0 & 0 & -i_1 l_0 & 0 & 0 \\ \beta_{\pi^m} f_\theta & 0 & \beta_{\pi^m} f_\varepsilon & \beta_{\pi^m} f_l & 0 \end{bmatrix}, \tag{A.11}$$

where

$$\begin{aligned} F_{11} &= \frac{-1}{1-\kappa_w\kappa_p}[(2-\kappa_p-\kappa_w)\beta_{w\omega} + (1-\kappa_w)\beta_{pu} \underset{(+)}{u_\theta}] < 0, \\ F_{13} &= \frac{1}{1-\kappa_w\kappa_p}[(1-\kappa_p)\beta_{we} \underset{(+)}{u_\varepsilon} - (1-\kappa_w)\beta_{pu} \underset{(+)}{u_\varepsilon}], \\ F_{14} &= \frac{1}{1-\kappa_w\kappa_p}[(1-\kappa_p)\beta_{we} \underset{(-)}{e_l}] < 0, \\ F_{21} &= \gamma_p \underset{(?)}{f_\theta} + \gamma_u \underset{(+)}{u_\theta}, \\ F_{23} &= \gamma_p \underset{(+)}{f_\varepsilon} + \gamma_u \underset{(+)}{u_\varepsilon} > 0, \\ F_{24} &= \gamma_p \underset{(-)}{f_l} < 0. \end{aligned}$$

The sign pattern of the matrix J becomes as follows

$$sign\, J = \begin{bmatrix} - & 0 & ? & - & 0 \\ ? & - & + & - & + \\ - & - & ? & - & + \\ 0 & 0 & - & 0 & 0 \\ ? & 0 & + & - & 0 \end{bmatrix}. \tag{A.12}$$

The characteristic equation of this system can be written as

$$\Gamma(\lambda) = \lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0, \tag{A.13}$$

where coefficients $a_i, i = 1, \ldots, 5$ are given as follows.

$$a_1 = -trace\,J = -\underset{(-)}{F_{11}} + \gamma_{ii} + \beta_{\varepsilon^m}(1 - \underset{(+)}{\rho_\varepsilon} - \underset{(+)}{f_\varepsilon}) = a_1(\beta_{\varepsilon^m}), \tag{A.14}$$

$$\begin{aligned}
a_2 &= \text{sum of all principal second-order minors of } J \\
&= \begin{vmatrix} F_{11} & 0 \\ F_{21} & -\gamma_r \end{vmatrix} + \beta_{\varepsilon^m} \begin{vmatrix} F_{11} & F_{13} \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) \end{vmatrix} + \begin{vmatrix} F_{11} & F_{14} \\ 0 & 0 \end{vmatrix} \\
&\quad + \begin{vmatrix} F_{11} & 0 \\ \beta_{\pi^m} f_\theta & 0 \end{vmatrix} + \beta_{\varepsilon^m} \begin{vmatrix} -\gamma_{ii} & F_{23} \\ -1 & \rho_\varepsilon + f_\varepsilon - 1 \end{vmatrix} + \begin{vmatrix} -\gamma_{ii} & F_{24} \\ 0 & 0 \end{vmatrix} \\
&\quad + \begin{vmatrix} -\gamma_{ii} & \gamma_{ip} \\ 0 & 0 \end{vmatrix} + \beta_{\varepsilon^m} \begin{vmatrix} -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ -il_0 & 0 \end{vmatrix} \\
&\quad + \beta_{\varepsilon^m}\beta_{\pi^m} \begin{vmatrix} -(1-\rho_\varepsilon - f_\varepsilon) & 1 \\ f_\varepsilon & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ \beta_{\pi^m} f_l & 0 \end{vmatrix}, \\
&= -\underset{(-)}{F_{11}}\gamma_r + \beta_{\varepsilon^m}\{-\underset{(-)}{F_{11}}(1-\rho_\varepsilon - f_\varepsilon) - \underset{(?)}{F_{13}}(\rho_\theta + \underset{(-)}{f_\theta}) \\
&\quad + \gamma_r(1-\rho_\varepsilon - f_\varepsilon) + \underset{(+)}{F_{23}} + il_0 \underset{(-)}{f_l} - \beta_{\pi^m} \underset{(+)}{f_\varepsilon}\} \\
&= a_2(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
a_3 &= -(\text{sum of all principal third-order minors of } J), \\
&= -\beta_{\varepsilon^m} \begin{vmatrix} F_{11} & 0 & F_{13} \\ F_{21} & -\gamma_{ii} & F_{23} \\ \rho_\theta + f_\theta & -1 & -(1-\rho_\varepsilon - f_\varepsilon) \end{vmatrix} - \begin{vmatrix} F_{11} & 0 & F_{14} \\ F_{21} & -\gamma_r & F_{24} \\ 0 & 0 & 0 \end{vmatrix} \\
&\quad - \begin{vmatrix} F_{11} & 0 & 0 \\ F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \beta_{\pi^m} f_\theta & 0 & 0 \end{vmatrix} - \beta_{\varepsilon^m} \begin{vmatrix} F_{11} & F_{13} & F_{14} \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ 0 & -i_1 l_0 & 0 \end{vmatrix} \\
&\quad - \beta_{\varepsilon^m}\beta_{\pi^m} \begin{vmatrix} F_{11} & F_{13} & 0 \\ \rho_\theta + f_\theta & -(1-\rho_\varepsilon - f_\varepsilon) & 1 \\ f_\theta & f_\varepsilon & 0 \end{vmatrix} - \begin{vmatrix} F_{11} & F_{14} & 0 \\ 0 & 0 & 0 \\ \beta_{\pi^m} f_\theta & \beta_{\pi^m} f_l & 0 \end{vmatrix} \\
&\quad - \beta_{\varepsilon^m} \begin{vmatrix} -\gamma_{ii} & F_{23} & F_{24} \\ -1 & -(1-\rho_\varepsilon - f_\varepsilon) & f_l \\ 0 & -i_1 l_0 & 0 \end{vmatrix}
\end{aligned}$$

$$
-\beta_{\varepsilon^m}\beta_{\pi^m}\begin{vmatrix} -\gamma_{ii} & F_{23} & \gamma_{ip} \\ -1 & -(1-\rho_\varepsilon-f_\varepsilon) & 1 \\ 0 & f_\varepsilon & 0 \end{vmatrix} - \begin{vmatrix} -\gamma_{ii} & F_{24} & \gamma_{ip} \\ 0 & 0 & 0 \\ 0 & \beta_{\pi^m} f_l & 0 \end{vmatrix}
$$

$$
-\beta_{\varepsilon^m}\beta_{\pi^m}\begin{vmatrix} -(1-\rho_\varepsilon-f_\varepsilon) & f_l & 1 \\ -i_1 l_0 & 0 & 0 \\ f_\varepsilon & f_l & 0 \end{vmatrix},
$$

$$
\begin{aligned}
&= \beta_{\varepsilon^m}[-\underset{(-)}{F_{11}}\,\gamma_{ii}(1-\rho_\varepsilon-f_\varepsilon) + \underset{(?)}{F_{13}}\,\underset{(?)}{F_{21}} - \underset{(?)}{F_{13}}\,\gamma_{ii}\underset{(-)}{(\rho_\theta+f_\theta)} - \underset{(-)}{F_{11}}\,\underset{(+)}{F_{23}} \\
&\quad + \underset{(-)}{F_{14}}\,i_1 l_0\underset{(-)}{(\rho_\theta+f_\theta)} - \underset{(-)}{F_{11}}\,i_1 l_0\,\underset{(-)}{f_l} - \underset{(-)}{F_{24}}\,i_1 l_0 + \gamma_{ii} i_1 l_0\,\underset{(-)}{f_l} \\
&\quad + \beta_{\pi^m}\{-\underset{(?)}{F_{13}}\,\underset{(+)}{f_\theta} + \underset{(-)}{F_{11}}\,\underset{(+)}{f_\varepsilon} + \gamma_{ip}\,\underset{(+)}{f_\varepsilon} - \gamma_{ii}\,\underset{(+)}{f_\varepsilon} + \underset{(-)}{f_l}\,i_1 l_0\}], \\
&= a_3(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned} \tag{A.16}
$$

$$
a_4 = \text{sum of all fourth-order minors of } J,
$$

$$
\begin{aligned}
&= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \left\{ \begin{vmatrix} -\gamma_{ii} & F_{23} & F_{24} & \gamma_{ip} \\ -1 & -(1-\rho_\varepsilon-f_\varepsilon) & f_l & 1 \\ 0 & -1 & 0 & 0 \\ 0 & f_\varepsilon & f_l & 0 \end{vmatrix} \right. \\
&\quad + \begin{vmatrix} F_{11} & F_{13} & F_{14} & 0 \\ \rho_\theta+f_\theta & -(1-\rho_\varepsilon-f_\varepsilon) & f_l & 1 \\ 0 & -1 & 0 & 0 \\ f_\theta & f_\varepsilon & f_l & 0 \end{vmatrix} \\
&\quad + (1/i_1 l_0)\begin{vmatrix} F_{11} & 0 & F_{13} & 0 \\ F_{21} & -\gamma_{ii} & F_{23} & \gamma_{ip} \\ \rho_\theta+f_\theta & -1 & -(1-\rho_\varepsilon-f_\varepsilon) & 1 \\ f_\theta & 0 & f_\varepsilon & 0 \end{vmatrix} \\
&\quad \left. + \begin{vmatrix} F_{11} & 0 & F_{13} & F_{14} \\ F_{21} & -\gamma_{ii} & F_{23} & \gamma_{ip} \\ \rho_\theta+f_\theta & -1 & -(1-\rho_\varepsilon-f_\varepsilon) & f_l \\ 0 & 0 & -1 & 0 \end{vmatrix} \right\},
\end{aligned}
$$

$$
\begin{aligned}
&= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \left\{ \begin{vmatrix} -\gamma_{ii} & F_{24} & \gamma_{ip} \\ -1 & f_l & 1 \\ 0 & f_l & 0 \end{vmatrix} + \begin{vmatrix} F_{11} & F_{13} & F_{14} \\ 0 & -1 & 0 \\ f_\theta & f_\varepsilon & f_l \end{vmatrix} \right. \\
&\quad +(1/i_1 l_0)F_{11} \begin{vmatrix} -\gamma_{ii} & F_{23} & \gamma_{ip} \\ -1 & 1-\rho_\varepsilon - f_\varepsilon & 1 \\ 0 & f_\varepsilon & 0 \end{vmatrix} \\
&\quad \left. +(1/i_1 l_0)F_{13} \begin{vmatrix} F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & 1 \\ f_\theta & 0 & 0 \end{vmatrix} + \begin{vmatrix} F_{11} & 0 & F_{14} \\ F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & f_l \end{vmatrix} \right\}, \\
&= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \{ -f_l(\gamma_{ip}-\gamma_{ii}) - F_{11} f_l + F_{14} f_\theta - (1/i_1 l_0) F_{11} f_\varepsilon (\gamma_{ip}-\gamma_{ii}) \\
&\quad +(1/i_1 l_0) F_{13} f_\theta (\gamma_{ip}-\gamma_{ii}) - F_{11}\gamma_{ii} f_l - F_{14}F_{21} + F_{14}\gamma_{ii}(\rho_\theta + f_\theta) \\
&\quad +F_{11}\gamma_{ip} \}, \\
&= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 [- \underset{(-)}{f_l} \{(\gamma_{ip}-\gamma_{ii}) + \underset{(-)}{F_{11}}(1+\gamma_{ii})\} \\
&\quad + \underset{(-)}{F_{14}} \{ - \underset{(?)}{F_{21}} + \underset{(-)}{\rho_\theta}\, \gamma_{ii} + (1+\gamma_{ii}) \underset{(?)}{f_\theta} \} \\
&\quad + \{ - \underset{(-)}{F_{11}} \underset{(+)}{(f_\varepsilon / i_1 l_0)} + \underset{(?)}{(F_{13}/i_1 l_0)} \underset{(?)}{f_\theta} \} (\gamma_{ip}-\gamma_{ii}) + \underset{(-)}{F_{11}} \gamma_{ip}] \\
&= a_4(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned} \tag{A.17}
$$

$$
\begin{aligned}
a_5 &= -\det J = -\beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \begin{vmatrix} F_{11} & 0 & F_{13} & F_{14} & 0 \\ F_{21} & -\gamma_{ii} & F_{23} & F_{24} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & -(1-\rho_\varepsilon - f_\varepsilon) & f_l & 1 \\ 0 & 0 & -1 & 0 & 0 \\ f_\theta & 0 & f_\varepsilon & f_l & 0 \end{vmatrix}, \\
&= -\beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \begin{vmatrix} F_{11} & 0 & F_{14} & 0 \\ F_{21} & -\gamma_{ii} & F_{24} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & f_l & 1 \\ f_\theta & 0 & f_l & 0 \end{vmatrix}, \\
&= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 \left\{ -F_{11} \begin{vmatrix} -\gamma_{ii} & F_{24} & \gamma_{ip} \\ -1 & f_l & 1 \\ 0 & f_l & 0 \end{vmatrix} - F_{14} \begin{vmatrix} F_{21} & -\gamma_{ii} & \gamma_{ip} \\ \rho_\theta + f_\theta & -1 & 1 \\ f_\theta & 0 & 0 \end{vmatrix} \right\}, \\
&= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 (\underset{(-)}{F_{11}} \underset{(-)}{f_l} - \underset{(-)}{F_{14}} \underset{(?)}{f_\theta})(\gamma_{ip}-\gamma_{ii}) = a_5(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned} \tag{A.18}
$$

We can define the Routh–Hurwitz terms Δ_j $(j = 1, 2, \ldots, 5)$ as follows

$$\left.\begin{aligned}
&\text{(i)} \quad \Delta_1 = a_1 = a_1(\beta_{\varepsilon^m}),\\
&\text{(ii)} \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix} = a_1 a_2 - a_3 = \Delta_2(\beta_{\varepsilon^m}, \beta_{\pi^m}),\\
&\text{(iii)} \quad \Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = a_3 \Delta_2 + a_1(a_5 - a_1 a_4),\\
&\qquad = a_1 a_2 a_3 - a_1^2 a_4 - a_3^2 + a_1 a_5 = \Delta_3(\beta_{\varepsilon^m}, \beta_{\pi^m}),\\
&\text{(iv)} \quad \Delta_4 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ 1 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \end{vmatrix} = a_4 \Delta_3 - a_5 \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & 1 & a_2 \end{vmatrix},\\
&\qquad = a_4 \Delta_3 + a_5(-a_1 a_2^2 - a_5 + a_2 a_3 + a_1 a_4),\\
&\qquad = a_4 \Delta_3 + a_5(a_1 a_4 - a_5 - a_2 \Delta_2) = \Delta_4(\beta_{\varepsilon^m}, \beta_{\pi^m}),\\
&\text{(v)} \quad \Delta_5 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ 1 & a_2 & a_4 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 1 & a_2 & a_4 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix} = a_5 \Delta_4 = \Delta_5(\beta_{\varepsilon^m}, \beta_{\pi^m}).
\end{aligned}\right\} \tag{A.19}$$

It is well known that the equilibrium point of the five dimensional dynamical system (A.10) is locally stable if and only if the following Routh–Hurwitz conditions for stable roots are satisfied

$$\Delta_j > 0 \text{ for all } j \in \{1,\ 2,\ \ldots,\ 5\}. \tag{RH}$$

It is also well known that the set of conditions RH can be expressed in any of the four following alternative forms, which are called Lienard–Chipart conditions (cf. Gandolfo 1996, p. 223)

$$\left.\begin{aligned}
&\text{(a) } a_5 > 0,\ a_3 > 0,\ a_1 > 0,\ \Delta_3 > 0,\ \Delta_5 > 0,\\
&\text{(b) } a_5 > 0,\ a_3 > 0,\ a_1 > 0,\ \Delta_2 > 0,\ \Delta_4 > 0,\\
&\text{(c) } a_5 > 0,\ a_4 > 0,\ a_2 > 0,\ \Delta_1 > 0,\ \Delta_3 > 0,\ \Delta_5 > 0,\\
&\text{(d) } a_5 > 0,\ a_4 > 0,\ a_2 > 0,\ \Delta_2 > 0,\ \Delta_4 > 0.
\end{aligned}\right\} \tag{LC}$$

It follows from Lienard–Chipart conditions that the following conditions are *necessary* (but not sufficient) conditions for local asymptotic stability.

$$a_j > 0 \quad \text{for all} \quad j \in \{1, 2, \ldots, 5\}. \tag{A.20}$$

From the relationships $\gamma_{ii} = \alpha_{ii}$ and $\gamma_{ip} = \alpha_{ii}(1 + \phi_{ip})$ we always have

$$\gamma_{ip} - \gamma_{ii} = \alpha_{ii}\phi_{ip} > 0, \tag{A.21}$$

which means that we have the following expressions

$$\begin{aligned} a_4 = \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 [&- \underset{(-)}{f_l}\{\alpha_{ii}\phi_{ip} + \underset{(-)}{F_{11}}(1 + \alpha_{ii})\} \\ &+ \underset{(-)}{F_{14}}\{- \underset{(?)}{F_{21}} + \underset{(-)}{\rho_\theta}\, \alpha_{ii} + (1 + \alpha_{ii})\, \underset{(?)}{f_\theta}\} \\ &+\{- \underset{(-)}{F_{11}}\, \underset{(+)}{(f_\varepsilon/i_1 l_0)} + \underset{(?)}{(F_{13}/i_1 l_0)}\, \underset{(?)}{f_\theta}\}\alpha_{ii}\phi_{ip} + \underset{(-)}{F_{11}}\, \alpha_{ii}(1 + \phi_{ip})], \end{aligned} \tag{A.22}$$

$$a_5 = \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 (\underset{(-)}{F_{11}}\, \underset{(-)}{f_l} - \underset{(-)}{F_{14}}\, \underset{(?)}{f_\theta})\alpha_{ii}\phi_{ip}, \tag{A.23}$$

where $(f_\varepsilon/i_1 l_0)$ and $(F_{13}/i_1 l_0)$ are *independent* of the parameter $i_1 > 0$.

We can easily see that the following relationships are satisfied

$$\left.\begin{array}{l} \text{(i)}\ \ f_l = 0 \text{ if } \kappa_p = 0 \\ \text{(ii)}\ f_\theta = 0 \text{ and } F_{21} > 0 \text{ if } \kappa_p = \beta_{pu} = \beta_{p\omega} = 0. \end{array}\right\} \tag{A.24}$$

Therefore, we can obtain the following results

$$\begin{aligned} a_4 &= \beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0 [\underset{(-)}{F_{14}}(- \underset{(+)}{F_{21}} + \underset{(-)}{\rho_\theta}\, \alpha_{ii}) + \alpha_{ii}\, \underset{(-)}{F_{11}}\{- \underset{(+)}{(f_r/i_1 l_0)} + 1 + \phi_{ip}\}] \\ &\quad \text{if } \kappa_p = \beta_{pu} = \beta_{p\omega} = 0, \end{aligned} \tag{A.25}$$

$$\begin{aligned} a_5 &= -\beta_{\varepsilon^m}\beta_{\pi^m} i_1 l_0\, \underset{(-)}{F_{14}}\, \beta_{p\omega}\alpha_{ii}\phi_{ip} > 0 \\ &\quad \text{for all } (\beta_{\varepsilon^m}, \beta_{\pi^m}, i_1, \beta_{p\omega}) > (0, 0, 0, 0) \text{ if } \kappa_p = 0. \end{aligned} \tag{A.26}$$

Now, let us assume as follows

Assumption A.3. The parameters $\kappa_p > 0$, $\beta_{pu} > 0$, $\beta_{p\omega} > 0$, and $\gamma_{ii} = \alpha_{ii} > 0$ are sufficiently small.

Lemma A.1. *Under Assumptions A.1–A.3, we have $a_4 > 0$ and $a_5 > 0$ for all $(\beta_{\varepsilon^m}, \beta_{\pi^m}, i) > (0, 0, 0)$.*

Proof. This result follows directly from (A.25), (A.26) and Assumption A.3 by continuity. ■

Lemma A.2. *Under Assumptions A.1–A.3, we have $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $\Delta_2 > 0$, $\Delta_3 > 0$, and $\Delta_4 > 0$ for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i_1 > 0$ are sufficiently small.*

Proof. We can easily see that the following equalities are satisfied

$$\lim_{i_1\to 0} y_\varepsilon = \lim_{i_1\to 0} l^d_\varepsilon = \lim_{i_1\to 0} u_\varepsilon = \lim_{i_1\to 0} e_\varepsilon = \lim_{i_1\to 0} \rho_\varepsilon = \lim_{i_1\to 0} f_\varepsilon = \lim_{i_1\to 0} F_{13} = \lim_{i_1\to 0} F_{23} = 0. \tag{A.27}$$

Therefore, we have the following inequalities from (A.14)–(A.19)

$$\lim_{i_1\to 0} a_1(\beta_{\varepsilon^m}) = -\underset{(-)}{F_{11}} + \gamma_{ii} + \beta_{\varepsilon^m} > 0 \quad \text{for all } \beta_{\varepsilon^m} \geq 0, \tag{A.28}$$

$$\lim_{i_1\to 0} a_2(\beta_{\varepsilon^m}, 0) = -\underset{(-)}{F_{11}}\gamma_{ii} + \beta_{\varepsilon^m}(-\underset{(-)}{F_{11}} + \gamma_{ii}) > 0 \quad \text{for all } \beta_{\varepsilon^m} \geq 0, \tag{A.29}$$

$$\lim_{i_1\to 0} a_3(\beta_{\varepsilon^m}, 0) = -\beta_{\varepsilon^m}\underset{(-)}{F_{11}}\gamma_{ii} > 0 \quad \text{for all } \beta_{\varepsilon^m} > 0, \tag{A.30}$$

$$\lim_{i_1\to 0} \Delta_2(\beta_{\varepsilon^m}, 0) = (\gamma_{ii} - \underset{(-)}{F_{11}})\{\beta^2_{\varepsilon^m} + (\gamma_{ii} - \underset{(-)}{F_{11}})\beta_{\varepsilon^m} - \underset{(-)}{F_{11}}\gamma_{ii}\} > 0 \quad \text{for all } \beta_{\varepsilon^m} \geq 0, \tag{A.31}$$

$$\lim_{i_1\to 0} \Delta_3(\beta_{\varepsilon^m}, 0) = \lim_{i_1\to 0}\{a_3(\beta_{\varepsilon^m}, 0)\Delta_2(\beta_{\varepsilon^m}, 0)\} > 0 \quad \text{for all } \beta_{\varepsilon^m} > 0. \tag{A.32}$$

These inequalities imply that we have $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $\Delta_2 > 0$, and $\Delta_3 > 0$ for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i > 0$ are sufficiently small (by continuity).

Next, let us turn to the analysis of the term Δ_4. Substituting (A.19)(iii) into (A.19)(iv), we obtain

$$\begin{aligned}\Delta_4(\beta_{\varepsilon^m}, \beta_{\pi^m}) &= a_4(a_3\Delta_2 + a_1a_5 - a_1^2a_4) + a_1a_4a_5 - a_5^2 - a_2a_5\Delta_2,\\ &= (a_3a_4 - a_2a_5)\Delta_2 + a_1a_4(2a_5 - a_1a_4) - a_5^2,\\ &= \beta_{\varepsilon^m}\beta_{\pi^m}i_1l_0[(a_3\tilde{a}_4 - a_2\tilde{a}_5)\Delta_2(\beta_{\varepsilon^m}, \beta_{\pi^m})\\ &\quad + a_1\tilde{a}_4(2a_5 - a_1a_4) - \tilde{a}_5a_5],\\ &= \beta_{\varepsilon^m}\beta_{\pi^m}i_1l_0\phi(\beta_{\varepsilon^m}, \beta_{\pi^m}),\end{aligned} \tag{A.33}$$

where $\tilde{a}_j = a_j/\beta_{\varepsilon^m}\beta_{\pi^m}i_1l_0$ $(j = 4, 5)$. We can easily see that

$$\lim_{i_1\to 0}\phi(\beta_{\varepsilon^m}, 0) = [\lim_{i_1\to 0}\{a_3(\beta_{\varepsilon^m}, 0)\tilde{a}_4 - a_2(\beta_{\pi^m}, 0)\tilde{a}_5\}]\{\lim_{i_1\to 0}\Delta_2(\beta_{\varepsilon^m}, 0)\}, \tag{A.34}$$

where $\lim_{i_1\to 0}\Delta_2(\beta_{\varepsilon^m}, 0) > 0$ is satisfied for all $\beta_{\varepsilon^m} \geq 0$ because of (A.31).

From (A.24), (A.25), (A.26), and (A.30) we can obtain the following result if $\kappa_p = \beta_{pu} = \beta_{p\omega} = 0$:

$$\lim_{i_1\to 0}\{a_3(\beta_{\varepsilon^m},0)\tilde{a}_4 - a_2(\beta_{\varepsilon^m},0)\tilde{a}_5\} = -\beta_{\varepsilon^m} \underset{(-)}{F_{11}}\,\alpha_{ii}[\underset{(-)}{F_{14}}(-\underset{(+)}{F_{21}} + \underset{(-)}{\rho_\theta}\,\alpha_{ii}) \\ +\alpha_{ii}\underset{(-)}{F_{11}}\{-\underset{(+)}{(f_i/i_1 l_0)} + 1 + \phi_{ip}\}], \tag{A.35}$$

which will be positive for all $\beta_{\varepsilon^m} > 0$ if $\gamma_{ii} = \alpha_{ii} > 0$ is sufficiently small. From (A.33), (A.34), and (A.35) we have $\Delta_4 > 0$ for all $\beta_{\varepsilon^m} > 0$ if $\beta_{\pi^m} > 0$ and $i_1 > 0$ are sufficiently small under Assumptions A.1–A.3 by continuity reasons. This completes the proof of Lemma A.2. ∎

The following two propositions are our main results.

Proposition A.1. (i) *Under Assumptions A.1–A.3, the equilibrium point of the system (A.10) is locally asymptotically stable for all* $\beta_{\varepsilon^m} > 0$ *if* $\beta_{\pi^m} > 0$ *and* $i > 0$ *are sufficiently small.*

(ii) *Suppose that* $\beta_{\varepsilon^m} > 0$*. Then, the equilibrium point of the system (A.10) is locally unstable for all sufficiently large values of* $\beta_{\pi^m} > 0$*.*

Proof. (i) Lemma A.1 and Lemma A.2 imply that all of the conditions (LC)(b) (or alternatively, all of the conditions (LC)(d)) are satisfied for all $\beta_{\varepsilon^m} > 0$ under Assumptions A.1–A.3 if $\beta_{\pi^m} > 0$ and $i_1 > 0$ are sufficiently small.

(ii) Suppose that $\beta_{\varepsilon^m} > 0$. Then, we have $a_2 < 0$ for all sufficiently large values of $\beta_{\pi^m} > 0$. In this case, one of the necessary conditions for local stability (20) is violated. ∎

Proposition A.2. *We posit Assumptions A.1–A.3 and assume that* $i_1 > 0$ *is sufficiently small. Furthermore,* β_{ε^m} *is fixed at an arbitrary positive value, and we select* $\beta_{\pi^m} > 0$ *as a bifurcation parameter. Then, there exists at least one bifurcation point* $\beta^0_{\pi^m}$ *at which the local stability of the equilibrium point of the system (A.10) is lost as the parameter* β_{π^m} *is increased. At the bifurcation point, the characteristic equation (A.13) has at least one pair of pure imaginary roots, and there is no real root* $\lambda = 0$*.*

Proof. Existence of the bifurcation point $\beta^0_{\pi^m}$, at which the local stability of the system is lost, is obvious from Proposition A.1 by continuity. By the very nature of the bifurcation point, the characteristic equation (A.13) must have at least one root with zero real part at $\beta_{\pi^m} = \beta^0_{\pi^m}$. But, we can exclude a real root $\lambda = 0$, because we have $\Gamma(0) = a_5 > 0$. ∎

Remark. In general, the following two cases are possible.

(A.1) At the bifurcation point, the characteristic equation (A.13) has a pair of purely imaginary roots and three roots with negative real parts.

(A.2) At the bifurcation point the characteristic equation (A.13) has two pairs of purely imaginary roots and one negative real root.

The case (A.1) corresponds to the so called "Hopf bifurcation", and in this case we can establish the existence of the closed orbits at some parameter values β_{π^m} which are sufficiently close to the bifurcation value (cf. Gandolfo 1996, Chap. 25 and in Asada et al. 2003, the mathematical appendix). On the other hand, in the case (A.2) one of the conditions for Hopf bifurcations is not satisfied. The case (A.1) will be more likely to occur than the case (A.2), and the case (A.2) will occur only by accident. Even in the case (A.2), however, the existence of the cyclical fluctuations is ensured at some range of the parameter values β_{π^m} which are sufficiently close to the bifurcation value, because of the existence of two pairs of the complex roots.

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