
Contents

1 Introduction to FPU

<i>G. Gallavotti</i>	1
1.1 The FPU Experiment and Its Ramifications	1
1.2 A Guided Tour of This Volume	2
References	7
1.3 Appendix: Reprint of Fermi, Pasta, Ulam – Studies of Nonlinear Problems (1955)	8

2 Dynamics of Oscillator Chains

<i>Allan J. Lichtenberg, Roberto Livi, Marco Pettini and Stefano Ruffo</i>	21
2.1 Historical Perspective and Background Theory	22
2.2 Formulations: Types of Oscillator Chains	37
2.3 Formulations: Methods of Numerical Analysis	42
2.4 Formulations: Analytic, Low-Energy and Short-Time Results	47
2.5 Numerical Results: Relaxation to Equilibrium from Low-Frequency Modes	73
2.6 Numerical Results: Relaxation to Equilibrium from High-Frequency Modes	82
2.7 Numerical Results: Stationary Nonequilibrium Properties	89
2.8 Analytical Calculations and Estimates: Scaling Estimates for λ and for T_{eq} from Low Frequencies	99
2.9 Analytical Calculations and Estimates: Scaling Estimates from High Frequencies	107
2.10 Conclusions and Final Comments	109
References	116

3 Role of Chaos for the Validity of Statistical Mechanics Laws: Diffusion and Conduction

<i>Massimo Cencini, Fabio Cecconi, Massimo Falcioni and Angelo Vulpiani</i>	123
3.1 Introduction	123
3.2 On the Microscopic Origin of Macroscopic Diffusion	124

3.3	The Heritage of the Fermi–Pasta–Ulam Problem for the Statistical Mechanics	138
3.4	Concluding Remarks	145
	References	146

4 The Fermi–Pasta–Ulam Problem and the Metastability Perspective

	<i>G. Benettin, A. Carati, L. Galgani and A. Giorgilli</i>	151
4.1	Introduction	151
4.2	The First Phase, 1955–1972: From FPU to Izrailev and Chirikov and to Bocchieri et al.; the Suggestion of a Possible Physical Interpretation	154
4.3	A Voice in the Desert: The Paper of Fucito et al. (1982) and the Proposal of a Metastability Scenario. The Work of Parisi and the Analogy with Glasses. Relations with Turbulence Theory	165
4.4	Other Pathways	174
4.5	The Resurgence of the Metastability Perspective, and Its Compatibility with the Existence of a Specific Energy Threshold: The Natural Packet and the Two Relaxation Times	178
4.6	New Analytical Contributions	182
4.7	Conclusions	183
	References	184

5 Resonance, Metastability and Blow up in FPU

	<i>Dario Bambusi and Antonio Ponno</i>	191
5.1	Introduction	191
5.2	Normal Form	193
5.3	Metastability or Blow up	198
5.4	Rigorous Results	201
	References	205

6 Center Manifold Theory in the Context of Infinite One-Dimensional Lattices

	<i>Guillaume James and Yannick Sire</i>	207
6.1	Introduction	207
6.2	Center Manifold Reduction for Maps	211
6.3	Center Manifold Reduction for Infinite-Dimensional Differential Equations	219
6.4	Breathers and Traveling Breathers in Nonlinear Oscillator Chains	226
	References	237

7 Numerical Methods and Results in the FPU Problem

	<i>Simone Paleari and Tiziano Penati</i>	239
7.1	Introduction	239
7.2	The Fermi-Pasta-Ulam Problem	240
7.3	Numerical Integration of Hamiltonian Systems	244

7.4	Natural Packets and Time Scales	251
7.5	Spectral Entropy	258
7.6	Lyapunov Exponents	262
7.7	Poincaré Sections	276
	References	279

8 An Integrable Approximation for the Fermi–Pasta–Ulam Lattice

	<i>Bob Rink</i>	283
8.1	Introduction	283
8.2	Discrete Symmetry	288
8.3	Quasi-particles	290
8.4	The Birkhoff Normal Form	291
8.5	Nishida’s Conjecture	294
8.6	Near-Integrability	295
8.7	Nondegeneracy	299
	References	300



<http://www.springer.com/978-3-540-72994-5>

The Fermi-Pasta-Ulam Problem

A Status Report

Gallavotti, G. (Ed.)

2008, VII, 301 p., Hardcover

ISBN: 978-3-540-72994-5