9 Stresses

The knowledge of the stresses prevailing in bulk solids, especially when being stored in bins and silos, is extremely important when considering the following topics:

- Silo design for flow (e.g., design procedure according to Jenike)
- Load assumptions for structural silo design (e.g., [9.1–9.3])
- Loads on feeders and inserts
- Limitation of stresses acting in a bulk solid (e.g., to avoid particle damage)
- Adjustment of the stress level of shear tests (or other flow property tests) to the situation in the application under consideration.

Furthermore, the possibility exists to calculate stresses for other applications, for example, consolidation of powders in dies, or movement of bulk solids plugs in tubes. Following some basic considerations of stress states in bulk solids, methods for assessment of stresses in silos and factors influencing the stresses are explained in the present chapter.

9.1 Stress states in silos

9.1.1 Ratio of horizontal to vertical stress

Previously in Sect. 2.3 it was explained that within a bulk solid different stresses can act in different directions, and in Sect. 5.1 the influence of deformation on stresses was discussed. One specific case of deformation, namely uniaxial compression, which represents, for example, deformation in the vertical section of a silo, is presented in an idealized manner in Fig. 9.1.a where an element of bulk solid is confined in a cylinder with frictionless, perfectly rigid walls. In the vertical direction the stress $\sigma_z = \sigma_v$ is applied to the bulk solid causing the horizontal stress, $\sigma_x = \sigma_h$. The ratio of horizontal to vertical stress, σ_h/σ_v, is called the lateral (or horizontal) stress ratio, K, being first introduced in soil mechanics and as well being used in the new European silo code [9.2,9.3] where it is called "lateral pressure ratio” (in the former version of DIN 1055 part 6 [9.1] the lateral pressure ratio was designated as λ). The special case of a frictionless wall (as shown in Fig. 9.1) is indicated by index “0”:

$$K_0 = \lambda_0 = \frac{\sigma_h}{\sigma_v} \tag{9.1}$$

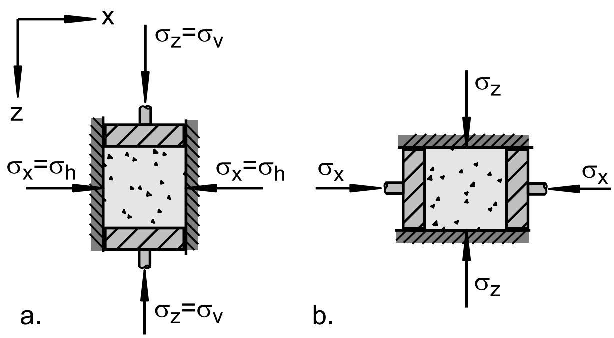

Fig. 9.1. Bulk solid element confined by parallel, frictionless walls; a. vertical compression; b. horizontal compression

Generally the lateral stress ratio, K, prevails whenever a bulk solid is consolidated uniaxially, i.e., when the bulk solid is compressed in one direction while no deformation occurs in the perpendicular direction. The stress perpendicular to the direction of consolidation can be assessed by multiplying the consolidation stress by the stress ratio, K. The latter is independent of the direction of consolidation although the term “horizontal stress ratio” seems to contradict this. Thus, it follows for the horizontal consolidation shown in Fig. 9.1.b (frictionless walls; stresses σ_x and σ_z are principal stresses; gravity force neglected):

$$K_0 = \lambda_0 = \frac{\sigma_z}{\sigma_x} \tag{9.2}$$

As already explained in Chap. 2, in bulk solids technology the bulk solid is mostly regarded as being a continuum. If differently inclined cutting planes within the bulk solid element are considered, one can show that different shear and normal stresses act on those planes. An indication of this fact is figure 9.1 showing that stresses are not the same in all directions. As outlined in Sect. 2.3.1, stresses acting in different cutting planes are represented by the Mohr stress circle (Fig. 9.2). For every Mohr stress circle with a radius greater than zero one can find a maximum normal stress. This stress is called the major principal stress, σ_1. The smallest normal stress, called the minor principal stress, σ_2, is acting perpendicular to σ_1.

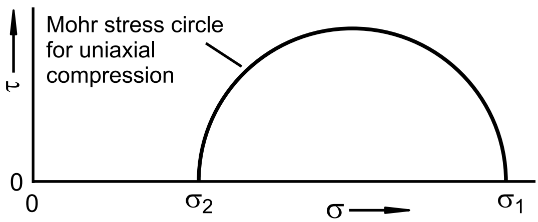

Fig. 9.2. Mohr stress circle (upper half) for uniaxial consolidation

The principal stresses act on the principal stress planes. The shear stresses on these planes are zero (intersections of the Mohr stress circle with the σ-coordinate, Fig. 9.2). Also the stresses presented in Fig. 9.1 are principal stresses, for example, in Fig. 9.1.a the consolidation stress, σ_v, is identical to σ_1, and σ_h acting in the perpendicular direction is identical to σ_2. Thus, the stress ratio, K_0, in Eq. 9.1 is the ratio of the minor to the major principal stress.

The lateral stress ratio is used in silo technology to describe the ratio of the wall normal stress to the mean vertical stress in the silo's vertical section. The bulk solid is compressed vertically by gravity, but strain in the horizontal direction is prevented. Thus, the deformation in the vertical section is comparable to that in Fig. 9.1.a. Since the silo walls are not frictionless as the walls in Fig. 9.1.a, the vertical and horizontal stresses are principal stresses only along the silo axis, but not at the silo walls. Therefore, the lateral stress ratio, K, in a silo's vertical section is dependent on the wall friction angle and, thus, somewhat different from K_0 [9.4].

The value of the lateral stress ratio differs with the bulk solid. While the stress ratio of a perfectly rigid, inelastic solid body would be equal to zero, and that of a Newtonian fluid equal to one, the values for bulk solids at rest (e.g., in a silo's vertical section) are usually between 0.3 and 0.6 [9.4], very rarely beyond these limits.

It has to be mentioned that the above statement is valid for bulk solids which had been compressed in only one direction starting from a very loose state of packing. If a bulk solid is, for example, compressed in two steps, first in z-direction and second in x-direction, a ratio σ_z/σ_x will be achieved after the second step which is not identical to the ratio that would have been achieved without the first compression step. The reason for this is the fact that any deformation causes a specific particle arrangement. Depending on the following deformation this arrangement is more or less preserved and has an influence on the stress relations (see Sect. 5.1.1). Measurements of the deformation behavior and approaches to describe it can be found in [9.5,9.6].

9.1.2 Stresses in silos

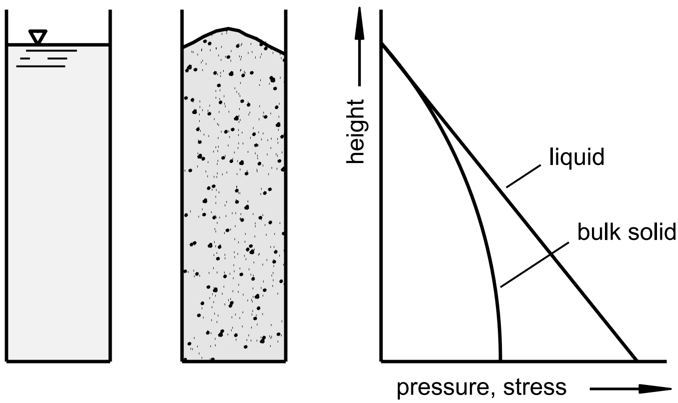

Fig. 9.3. Pressure and stress, respectively, in liquids and bulk solids

As it is known from the friction between solid bodies, bulk solids at rest can also transmit shear stresses (contrary to Newtonian fluids). While pressure increases linearly with depth in a liquid container (Fig. 9.3), the shear stress exerted from the bulk solid onto the container wall – i.e. the friction at the container wall – carries part of the bulk solids weight. As a consequence the stress in a container filled with a bulk solid is increasing less and less in downwards direction.

A typical silo consists of a vertical section (vertical section, cylinder section) and a hopper. The stress conditions in the hopper are more complex than in the vertical section. If a previously empty silo is filled with a bulk solid, a stress distribution results as shown in Fig. 9.4.a [9.7]. In the vertical section both the wall normal stress, σ_w, and the mean vertical stress, σ_v, are increasing in the downward direction and tend to approach an asymptotic value. The ratio of the wall normal stress to mean vertical stress is given by the lateral stress ratio, K (Sect. 9.1). The major principal stress, σ_1, is oriented vertically along the silo axis and deviates more and more from vertical towards the silo walls. This state of stress is called "active state of stress" or "active stress field". The direction of the major principal stress is represented in Fig. 9.4 by major principal stress lines.

At the transition to the hopper the wall normal stress has a discontinuity caused by the sudden change of wall inclination. Further downwards in the hopper both the vertical stress and the wall normal stress are decreasing and approach zero at the hopper apex (the outlet is assumed as infinitely small), but depending on the vertical stress at the transition, the silo shape, and the bulk solid's properties the stresses in the hopper either increase in the first instance and then decrease, or decrease continuously from the transition to the apex as in Fig. 9.4.a (the stress distributions plotted in Fig. 9.4 shall be regarded as qualitative examples). In general the stresses in vertical direction are larger than those in horizontal direction. Along the hopper axis the major principal stress is oriented vertically, i.e., an active state of stress (active stress field) prevails as in the vertical section. The stress field in the hopper prevailing after filling is also referred to as “filling state of stress” or just “filling conditions”.

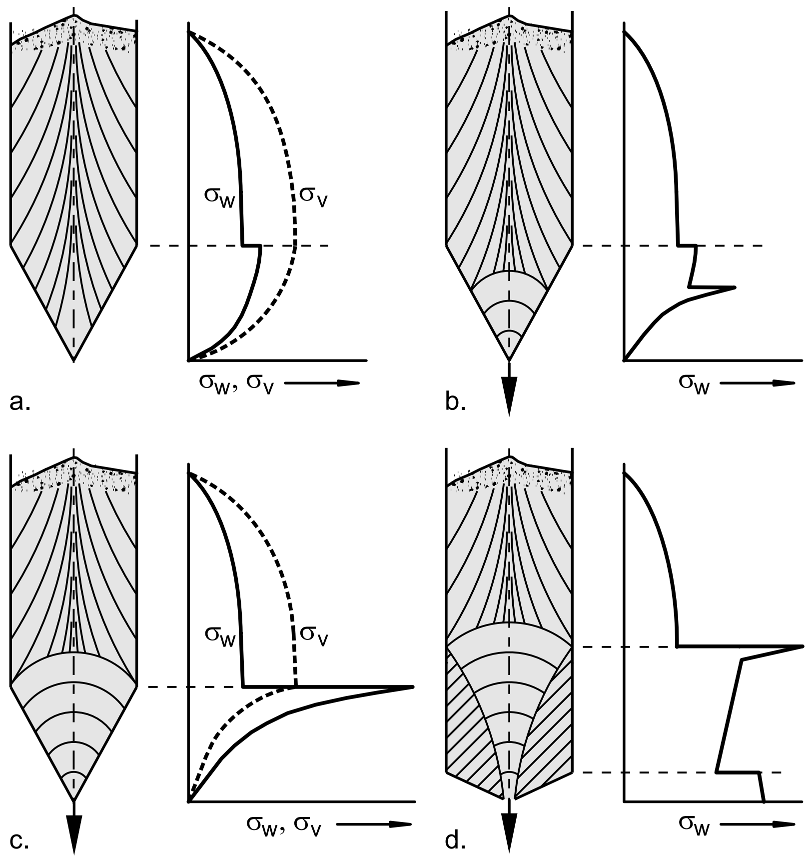

Fig. 9.4. Qualitative distributions of wall normal stress, σ_w, and mean vertical stress, σ_v (dashed, only in Figs. a and c), vs. the vertical coordinate (within the silos lines of major principal stress, σ_1, are plotted; stagnant zones are hatched [9.7,9.8]; the hopper outlet in Figs.9.4.a to c is assumed to be infinitely small). a. Filling conditions prevailing during initial filling of an empty silo; b. emptying conditions in the lower part of the hopper; c. emptying conditions in the entire hopper; d. emptying conditions in a funnel flow silo (mixed flow according to [9.3]).

When material is discharged from a mass flow silo the first time after it has been filled, after a short transition period the entire contents of the silo moves downward. Due to the convergent flow zone in the hopper the bulk solid is compressed horizontally, while it dilates in the vertical direction due the downwards flow. As a result the larger stresses act in the horizontal direction, and the major principal stress along the hopper axis is oriented horizontally. This stress field is called "arched" (according to the shape of the lines of major principal stress in the hopper, which are increasingly inclined towards the wall; see Fig. 9.4.c) or "passive" stress field. Other designations are "passive state of stress", "emptying state of stress" or just "emptying conditions". Figure 9.4.b shows the situation a very short time after the onset of discharge, where the passive stress field has developed only in the lower part of the hopper. A bit later compared to the situation in Fig 9.4.b, the passive stress field is fully developed (Fig. 9.4.c). Here the stresses in the hopper decrease remarkably towards the apex. In the lower part of the hopper the so-called "radial stress field" develops where the local stress is nearly proportional to the distance from the hopper apex. In the emptying state the stresses close to the outlet are independent of the stresses in the upper part of the hopper and, therefore, also independent of the silo's dimensions or level of filling.

The deformation of the bulk solid while flowing in the hopper approximately corresponds to steady-state flow (flow at constant stresses and constant bulk density, i.e., compression in one direction and dilation in the perpendicular direction). The ratio of the minor and major principal stresses at steady-state flow cannot be described with the lateral stress ratio, K, which is valid only for uniaxial compression. Figure 9.5 shows an example of a stress circle for steady-state flow and a stress circle for uniaxial consolidation as discussed above. It can be clearly seen that for identical major principal stresses, σ_1, a smaller minor principal stress, σ_2, results for steady-state flow compared to uniaxial consolidation (for further explanations regarding the influence of deformation on stresses refer to Sect. 5.1). Thus, the ratio of the minor to the major principal stress in the hopper axis is similar to that of steady-state flow.

$$\frac{\sigma_2}{\sigma_1} = \frac{1 - \sin \varphi_e}{1 + \sin \varphi_e} \tag{9.3}$$

In the silo's vertical section the active state of stress remains during emptying, as long as no local convergences exist (reduction of the cross-section due to inserts, dents, etc.). The transition from the active to the passive state of stress is called "switch". Considering the arched principal stress lines in the hopper, the switch takes place in an arched region originating from the cylinder/hopper transition (see Fig. 9.4.c). The switch is accompanied by a local peak of the wall normal stress. Since the passive stress field and the stress peak are not a result of dynamic forces, they are preserved even when discharge is stopped.

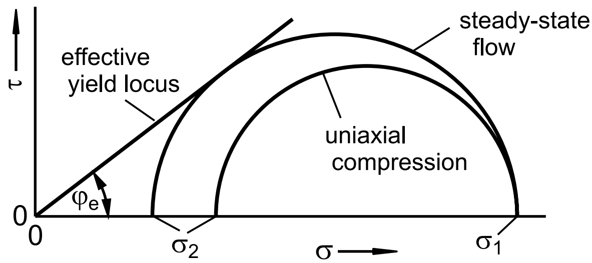

Fig. 9.5. Mohr stress circles representing steady-state flow and uniaxial consolidation

The switch stress peak develops while starting discharge from a silo which has been freshly filled from completely empty (Fig. 9.4.a). From the beginning of discharge the passive stress field develops in the hopper starting at the outlet and, thus, the switch region and the stress peak travel up (Fig. 9.4.b) until they become caught at the cylinder/hopper transition (Fig. 9.4.c). The passive stress field does not establish in the entire hopper immediately after the start of discharge, but only after a certain amount of bulk solid has been discharged (depending, for example, on the bulk solid's properties and hopper dimensions). One reason for this is that a bulk solid dilates somewhat at the onset of flow.

In funnel flow stagnant zones are formed which remain at rest while the material in the flow zone is flowing downwards (Fig. 9.4.d). If the boundary between a stagnant zone and the flow zone meets the silo wall within the vertical section, a stress peak occurs due to the "switch" from active to passive stress field caused by the convergent flow zone beneath the top of the stagnant zone. For the sake of completeness it has to be mentioned that the large wall normal stress along the hopper walls of the funnel flow silo in Fig. 9.4.d results from the shallow slope of the hopper walls.

The stress peak has to be considered when designing a silo for strength [9.1]. Compared to mass flow, this is more difficult in funnel flow, because the stress peak acts in the sensitive vertical section where its position cannot be accurately predicted [9.9]. In addition, the stress peak can be asymmetric with respect to the perimeter, and it can change its position with time. Thus, the complete vertical section has to be designed accordingly [9.1]. Silo damages resulting from unexpected stress peaks are reported in [9.10]. Some bulk solids (e.g., crystalline sugar) have the tendency to form very steep flow zones in funnel flow silos (Fig. 9.6.a). The boundary between stagnant and flow zones does not meet the silo wall, or it meets the silo wall only near to the surface where the stresses are small. Thus, no (or no significant) stress peak develops at the silo wall.

Figure 9.6.b shows a funnel flow silo with eccentric outlet causing an eccentric flow zone (eccentric flow, Sect. 9.4.2). Although a stagnant zone is formed only on the left side, the principal stresses change their direction over the total width of the silo due to the transition from active to passive stress field (switch). Thus, a stress peak is to be expected also at the right side [9.10]. The same would happen if the silo in Fig. 9.6.b were a mass flow silo with a steep hopper wall, i.e., an inclined wall instead of the stagnant zone. Thus it becomes clear that a switch, i.e., the transition from an active to a passive state of stress, results in a pronounced local increase of wall normal stress where the transition zone meets the silo wall. Therefore, a stress peak can occur at the upper edge of a hopper (Fig. 9.4.c), and also along vertical walls without any stagnant zones located directly at these walls (Fig. 9.6.b).

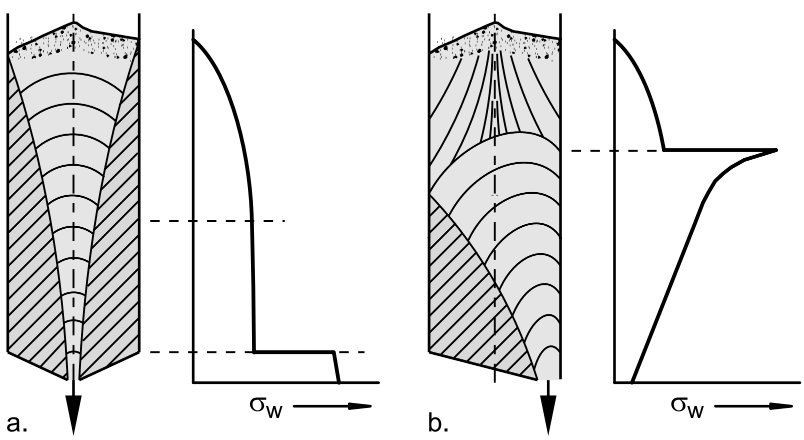

Fig. 9.6. Wall normal stresses σ_w (at the right silo wall) during discharge; lines of major principal stress in the flowing bulk solid; stagnant zones are hatched; a. funnel flow with steep flow zone (pipe flow according to [9.3]), b. funnel flow with eccentric flow zone (eccentric pipe flow according to [9.3]).

The stress peak can be explained by the orientations of the principal stresses (Fig. 9.7): In the vertical section the major principal stress, σ_1, is oriented more or less vertically (see lines of principal stress in Fig. 9.4.c). Thus the vertical stress is greater than the horizontal stress. The principal stresses, σ_1 and σ_2, acting along the silo axis are plotted in Fig. 9.7 as arrows (see bulk solid elements right to the silo), with the length of an arrow being a qualitative measure of the magnitude. When flowing in the hopper the bulk solid is compressed in the horizontal direction, but it can dilate vertically. Thus, the horizontal stress is larger than the vertical stress in the hopper, and along the hopper axis the minor principal stress, σ_2, acts vertically and the major principal stress, σ_1, horizontally. Since at the transition from active to passive stress field the vertical stresses from above have to be in equilibrium with the vertical stresses from below, a significant increase of the horizontal stress occurs as can be seen from the peak in the wall normal stress (e.g., Fig. 9.4.c). Even though the principal stresses are increasingly inclined to the vertical and the horizontal, respectively, with increasing distance from the axis, this does not change anything in the principal validity of this consideration.

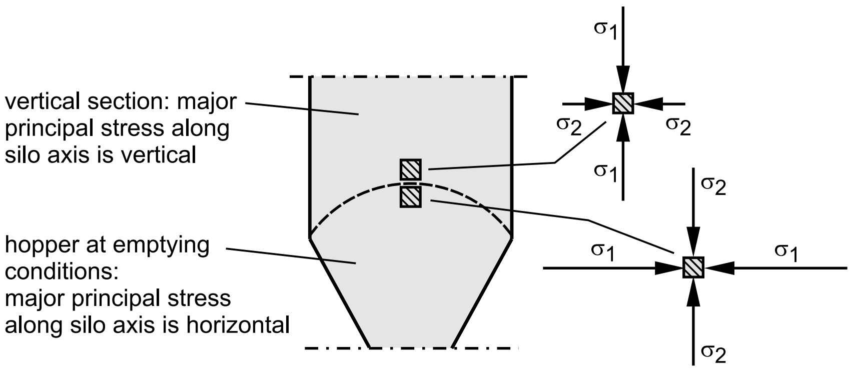

Fig. 9.7. Principal stresses at the transition from the vertical section to the hopper of a mass flow silo (major principal stresses acting along the silo axis above and below the transition from active to passive stress field are plotted as arrows with the length of an arrow being a qualitative measure of the magnitude).

An estimation of the magnitude of the stress peak is in principle possible, for example, with the approaches derived by Enstad [9.11], Walters [9.12,9.13], and Jenike [9.14]. Silo codes such as as the German code DIN 1005 part 6 or the present Eurocode [9.1–9.3] contain equations for calculation of the stress peak; however, it is reasonable to assume that the stress-strain behavior of the bulk solid has an influence on the stress peak. A stiff, incompressible bulk solid will lead to a more pronounced local peak stress than a fine-grained, compressible material. The latter needs more deformation for reorientation of the major principal stresses (active → passive) and, thus, must flow further downwards in the hopper until the passive state of stress is fully attained. Therefore it is to be expected that the stress peak is less sharp and has a smaller maximum, but will act over a larger area.

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