Preface

This lecture notes volume has its origins in a course by Husemöller on fibre bundles and twisted K-theory organized by Brano Jurčo for physics students at the LMU in München, summer term 2003. The fact that K-theory invariants, and in particular twisted K-theory invariants, were being used in the geometric aspects of mathematical physics created the need for an accessible treatment of the subject. The course surveyed the book *Fibre Bundles*, 3rd. Ed. 1994, Springer-Verlag by Husemöller, and covered topics used in mathematical physics related to K-theory invariants. This book is referred to just by its title throughout the text.

The idea of lecture notes came up by J. Wess in 2003 in order to serve several purposes. Firstly, they were to be a supplement to the book *Fibre Bundles* providing companion reading and alternative approaches to certain topics; secondly, they were to survey some of the basic results of background to K-theory, for example operator algebra K-theory, not covered in the *Fibre Bundles*; and finally the notes would contain information on the relation to physics. This we have done in the survey following this introduction "Physical background to the K-theory classification of D-branes: Introduction and references" tracing the papers how and where K-theory invariants started to play a role in string theory. The basic references to physics are given at the end of this survey, while the mathematics references are at the end of the volume.

Other lectures of Husemöller had contributed to the text of the notes. During 2001/2002 in Münster resp. during Summer 2002 in München, Husemöller gave Graduate College courses on the topics in the notes, organized by Joachim Cuntz resp. Martin Schottenloher, and in the Summer 2001, he had a regular course on C*-algebras and K-theory in München. The general question of algebra bundles was studied with the support of Professor Cuntz in Münster during short periods from 2003 to 2005. Finally, Husemöller lectured on these topics during a workshop at IPM, Tehran, Iran, September 2005. It is with a great feeling of gratitude that these lecture opportunities are remembered here.

The notes are organized into five parts. The first part on basic bundle theory emphasizes the concept of bundle as one treats the concepts of set, space, homotopy, group, or ring in basic mathematics. A bundle is just a map called the projection from the total space to its base space. As with commutative groups, topological

groups, transformation groups, and Lie groups, the concept of bundle is enhanced or enriched with additional axioms and structures leading to étale bundles, principal bundles, fibre bundles, vector bundles, and algebra bundles. A topic discussed in the first part, which is not taken up in the *Fibre Bundles*, is the Serre–Swan theorem which relates vector bundles on a compact space X with finitely generated projective modules over the ring $C(X)$ of continuous complex valued functions on the space X. This is one of the points where topological, algebraic, and operator K-theory come together.

The second part of the notes takes up the homotopy classification of principal bundles and fibre bundles. Applications to the case of vector bundles are considered and the role of homotopy theory in K-theory is developed. This is related to the fact that K-theory is a representable functor on the homotopy category. The theory of characteristic classes in describing orientation and spin structures on vector bundles is carried out in detail, also leading to the notion of a string structure on a bundle and on a manifold.

There are various versions of topological K-theory, and their relation to Bott periodicity is considered in the third part of the notes. An advanced version of operator K-theory, called KK-theory which integrates K-cohomology and K-homology, is introduced, and various features are sketched.

The fourth part of the notes begins with algebra bundles with fibres that are either matrix algebras or algebras of bounded operators on a separable Hilbert space. The infinite dimensional algebra bundles are classified by only one characteristic class in the integral third cohomology group of the base space along the lines of the classification of complex line bundles with its first Chern class in the integral second cohomology group of the base space. The twisting of twisted K-theory is given by an infinite dimensional algebra bundle, and the twisted K-theory is defined in terms of cross sections of Fredholm bundles related to the algebra bundle describing the twist under consideration.

A fundamental theme in bundle theory centers around the gluing of local bundle data related to bundles into a global object. In the fifth part we return to this theme and study gluing on open sets in a topological space of not just simple bundle data but also data in a more general category where the gluing data may satisfy transitivity conditions only up to an isomorphism. The resulting objects are gerbes or stacks.

August 2007 Dale Husemöller

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