

# Preface

Research on the human brain has become a truly interdisciplinary enterprise that no longer belongs to medicine, neurobiology and related fields alone. In fact, in our attempts to understand the functioning of the human brain, more and more concepts from physics, mathematics, computer science, mathematical biology and related fields are used. This list is by no means complete, but it reflects the aim of the present book. It will show how concepts and mathematical tools of these fields allow us to treat important aspects of the behavior of large networks of the building blocks of the brain, the neurons.

This book applies to graduate students, professors and researchers in the above-mentioned fields, whereby I aimed throughout at a pedagogical style. A basic knowledge of calculus should be sufficient. In view of the various backgrounds of the readers of my book, I wrote several introductory chapters. For those who have little or no knowledge of the basic facts of neurons that will be needed later I included two chapters. Readers from the field of neuroscience, but also from other disciplines, will find the chapter on mathematical concepts and tricks useful. It shows how to describe spiking neurons and contains material that cannot easily be found in conventional textbooks, e.g. on the handling of  $\delta$ -functions. Noise in physical systems – and thus also in the brain – is inevitable. This is true for systems in thermal equilibrium, but still more so in active systems – and neuronal systems are indeed highly active. Therefore, I deal with the origin and effects of noise in such systems.

After these preparations, I will deal with large neural networks. A central issue is the spontaneous synchronization of the spiking of neurons. At least some authors consider it as a basic mechanism for the binding problem, where various features of a scene, that may even be processed in different parts of the brain, are composed to a unique perception. While this idea is not generally accepted, the problem of understanding the behavior of large nets, especially with respect to synchronization, is nevertheless a fundamental problem of contemporary research. For instance, synchronization among neurons seems to play a fundamental role in epileptic seizures and Parkinson's disease. Therefore, the main part of my book will be devoted to the synchronization problem and will expose various kinds of integrate and fire models as well as what I called the lighthouse model. My approach seems to be more

realistic than conventional neural net models in that it takes into account the detailed dynamics of axons, synapses and dendrites, whereby I consider arbitrary couplings between neurons, delays and the effect of noise. Experts will notice that this approach goes considerably beyond those that have been published so far in the literature.

I will treat different kinds of synaptic (dendritic) responses, determine the synchronized (phase-locked) state for all models and the limits of its stability. The role of non-synchronized states in associative memory will also be elucidated. To draw a more complete picture of present-day approaches to phase-locking and synchronization, I present also other phase-locking mechanisms and their relation, for instance, to movement coordination. When we average our basic neural equations over pulses, we reobtain the by now well-known Wilson–Cowan equations for axonal spike rates as well as the coupled equations for dendritic currents and axonal rates as derived by Nunez and extended by Jirsa and Haken. For the sake of completeness, I include a brief chapter on the equations describing a single neuron, i.e. on the Hodgkin–Huxley equations and generalizations thereof.

I had the opportunity of presenting my results in numerous plenary talks or lectures at international conferences and summer schools and could profit from the discussions. My thanks go, in particular, to Fanji Gu, Y. Kuramoto, H. Liljenström, P. McClintock, S. Nara, X.L. Qi, M. Robnik, H. Saito, I. Tsuda, M. Tsukada, and Yunjiu Wang. I hope that the readers of my book will find it enjoyable and useful as did the audience of my lectures. My book may be considered complementary to my former book on “Principles of Brain Functioning”. Whereas in that book the global aspects of brain functioning are elaborated using the interdisciplinary approach of synergetics, the present one starts from the neuronal level and studies modern and important aspects of neural networks. The other end is covered by Hugh R. Wilson’s book on “Spikes, Decisions and Actions” that deals with the single neuron and the action of a few of them. While his book provides readers from neuroscience with an excellent introduction to the mathematics of nonlinear dynamics, my earlier book “Synergetics. An Introduction” serves a similar purpose for mathematicians and physicists.

The tireless help of my secretary Ms. I. Möller has been pivotal for me in bringing this book to a good end. When typing the text and composing the formulas she – once again – performed the miracle of combining great speed with utmost accuracy. Most of the figures were drawn by Ms. Karin Hahn. Many thanks to her for her perfect work.

Last but not least I thank the team at Springer-Verlag for their traditionally excellent cooperation, in particular Prof. W. Beiglböck, Ms. S. Lehr and Ms. B. Reichel-Mayer.

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