

Introduction: Some Aspects of Financial Mathematics

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The articles in this volume summarize the lectures given by the authors on February 1, 2005, at an open conference held at the Académie des Sciences, quai Conti, on Financial Mathematics, a field in constant development since the end of the 1970s.

The aim is to make these articles accessible to a wide audience belonging to scientific horizons; indeed, this half-day conference was attended by many young postgraduates in disciplines such as economics, management, and applied mathematics, along with many members of the different sections of the Académie.

Among the reasons to get interested in financial mathematics, the following is one: who has never wondered, looking at the financial pages of a newspaper, displaying the erratic evolutions of quotations on the Stock Exchange, if these were not “governed” by some models, likely to be probabilistic?

This question was at the heart of the studies conducted by Louis Bachelier, particularly in his famous thesis (1900), and he answered the above question in terms of Brownian Motion. His remarkable results, however, remained in a kind of scientific *limbo* for almost 75 years, until Samuelson “corrected” Bachelier (in 1965) by replacing the Brownian Motion by its exponential, and the famous Black-Scholes formula began (in 1973) to play an essential role in the computation of option prices.

Since the 1980s, we have witnessed the explosion of probabilistic models, along with financial products, each in turn becoming more and more complex.

All this technology, which now forms an important part of financial engineering, exists only because some mathematical concepts both simple and universal allow building a “theory of the laws of markets,” based on principles such as *the prices across time of an uncertain asset having the probabilistic structure of a fair game, that is to say of a martingale*. From this concept, little by little was built the entire theory of stochastic processes, which is a pillar on which the mathematical theory of “arbitrage” was developed by Delbaen and Schachermayer.

The six articles in this volume present the following main topics.

H. Föllmer introduces the main probabilistic notions: martingales, martingale measures, risk measures, etc., which have been used and developed during the last 30 years to describe and explain the behavior of financial markets.

W. Schachermayer gives an introduction to the theory of “arbitrage,” proceeding progressively to a mathematical sophistication of his arguments.

P. Barrieu and N. El Karoui present a global overview of the existing interactions between mathematics and finance by emphasizing the different aspects of research in financial risk management.

H. Geman shows that even if the Brownian model (as used by Bachelier (1900) or Black–Scholes (1973), or ever since) is the object of many critical statements, it is nevertheless present in several models, up to a time change, which may even present time jumps. These time changes can be related to different changing speeds between certain financial markets.

D. Lamberton shows how partial differential equations intervene in the study of financial models and presents the numerical methods used to compute the pricing and hedging of options.

E. Gobet, G. Pagès, and M. Yor present a historical evaluation, corresponding to the last 30 years of the evolution of financial markets. Then they explain how applied mathematics, and probability theory in particular, come into play in this evolution. They finally describe in detail the existing qualifying curricula available in French universities for future “quants” in the banking and insurance industries.

These six contributions can be read independently, although they deal with the same subject. As a whole, they present a fairly complete overview of the existing interactions between mathematics and finance during the last 30 years.

This booklet, in particular the English version, benefited from the help of Monique Jeanblanc and Dilip Madan, to whom I am very grateful. Special thanks to Kathleen Qechar for her translation of some of the articles from French to English.

I personally, and also together with the other authors, thank Mr Jean Dercourt, Secrétaire perpétuel de l’Académie des Sciences, as well as Mrs. Bonfils, Martin, and Morand for their warm support and all the efforts they made in order that this Conference day of February 1, 2005 be a success.

I also salute the memory of Mr. Hubert Curien, who so kindly agreed to take the chair of this session, and who passed away a few days later. This book is gratefully dedicated to him.



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