2. Shape Classes

2.1. Possible Classes of Shape

The proposed method of shape understanding is based on the concept of shape classes that are understood as the basic perceptual categories. The Shape Understanding System (SUS) perceives the visual object by trying to fit it into one of the shape categories. Although shape is one of the most often perceived "properties" of the visual object, there is no satisfactory classification and definition of shape. An attempt to develop the system of shape classification that is based on the shape classes was made by Les [1]. Shape classes called shape categories (in the context of visual thinking) are used as the "material" of the visual thinking process. The shape classes are represented by the symbolic names and are defined in the context of visual understanding process. Each class is related to each other and based on relationships among classes there is relatively easy to establish the "perceptual similarity" of visual objects.

In this chapter, the description of the shape classes is presented within the framework of shape understanding method. Shape understanding method is based on the concept of possible classes of shape [1]. A member of the class that is defined in terms of its attributes is called an *archetype* of this class. In the case of a digital image, the shape is given as an image region or a set of pixels. A perceived object (phantom) is transformed into a digital representation called a digital object. The proper interpretation of the visual object is obtained during the visual reasoning process. During the visual reasoning the perceived object is transformed into its symbolic description called the symbolic name. The symbolic name is the name of the shape category (shape classes) to which the shape of the perceived object is fitted. The symbolic name is used to find the visual concept and to assign the perceived object to one of the ontological categories. The visual concept is a set of symbolic names obtained in the learning process. The shape class is denoted by symbol Ω^{η}, where η denotes the symbolic

Z. Les and M. Les: *Shape Classes*, Studies in Computational Intelligence (SCI) **86**, 47–99 (2008)
www.springerlink.com

description (the symbolic name) of a given class. A member of the class denoted by symbol ω is called an archetype.

In this book, for simplicity, the symbol of the class Ω is omitted and the class is often described by its symbolic name, e.g., Λ instead of Ω^{Λ} or $Q^n_{\Im}[\Lambda](n\Im_{\Gamma})$ instead of $\Omega^{Q^n_{\Im_{\Lambda}}(n\Im_{\Gamma})}$. Also n classes $\underbrace{\Im_1,\ldots,\Im_n}_{n}$ that are identical $\Im_i \equiv \Im_j$ for all $i = 1,\ldots,n$, $j = 1,\ldots,n$, and $i \neq j$ are denoted as $n \cdot \Im$, whereas n classes $\underbrace{\Im_1,\ldots,\Im_n}_{n}$ that are not identical are denoted as $n\Im$.

The general shape classes are defined based on the general attributes of shape such as homotopy, convexity, or thickness. The general class is split into specific classes based on additional features that represent a priori information about local perceptual and geometrical properties of shape and is incorporated into the a priori model of the shape class. The deepness of the splitting process depends on the base class from which the specific class is derived. In this book, the following general classes are presented: cyclic–acyclic general classes $\Omega^{A} - \Omega^{\Gamma}$, convex–concave general classes $\Omega^{\Lambda} - \Omega^{Q}$, and thick–thin general classes $\Omega^{N} - \Omega^{\Theta}$.

2.1.1. General Classes: A Priori Classes

The homotopy measure that is based on the computation of a number of holes is applied to derive the cyclic–acyclic general classes $\Omega^{A} - \Omega^{\Gamma}$. An element of the shape class, called an archetype, is called acyclic, if its homology groups $H_i(X)$ coincides with homology groups of a point, i.e., the Betti numbers b_1, b_2 are equal to 0, and $b_0 = 1$, where $b_i = \operatorname{rank} H_i(X)$. The 0th Betti number b_0 stands for a number of components, while b_1 denotes a number of wholes in shape. The derivation rule for the cyclic general class is given as follows: where $[a^H = 0] \Rightarrow \Omega \prec \Omega^A$, a^H denotes an attribute called homotopy and symbol $\prec$ denotes that class Ω^A is derived from the class Ω.

The convexity coefficient that is given as the ratio of the area of the object A_ω to the area of the convex hull $A_{\aleph}$, $a^{\aleph} = A_{\aleph} / A_\omega$ is used to derive the convex Ω^{Λ} class and the concave Ω^{Q} class. The convex hull of a set of points X in the plane is the smallest convex polygon P that encloses X, smallest in the sense that there is no other polygon P' such that $P \supset P' \supseteq X$. The computation of the convex hull of a finite set of points in the plane has been studied extensively and some of the algorithms as well as discussion of the complexity of the convex hull algorithms can be found in [2, 3]. The derivation rule of the convex class is given as follows: $[a^{\aleph} = 0] \Rightarrow \Omega \prec \Omega^{\Lambda}$, where $a^{\aleph}$ is an attribute of the class. The convex general class Ω^{Λ} is related to the notion of a convex set (see, e.g., [4]). A set X in E^2 is convex if for any two points $x, y \in X$, the (closed) segment $\overline{xy}$ is wholly contained in this set ($\overline{xy} \subseteq X$) or, in another way, a set X is called the convex set if for any two points of this set the following relation takes place: $\lambda x + (1-\lambda) y \in X$, for each $\lambda \in [0,1]$.

The thin class Ω^{Θ} is a class whose members are thin objects. The description of the object in terms of thickness can be obtained utilizing a distance transformation. The distance transformation is a mapping of a set of points into a set of predefined distances (see, e.g., [5]). The distance transformation (the thickness measure) is the image transformation points-number $\Theta^{\hbar}$ described in [6] that assigns the number to each point $u_i^F \in \coprod^F$ based on the local properties and is given as follows:

$$\left[\forall u_i^F \in \coprod^F, \exists \sigma_i^{\rho} \in \Re : \sigma_i^{\rho} = \hbar_{\rho}\left(u_i^F\right)\right] \Rightarrow u_i \triangleright \sigma_i^{\rho},$$

where the local transformation $\hbar_{\rho}\left(u_i^F\right)$ is determined by the selected neighborhood. In the case of a distance transformation the local transformation is given as

$$\hbar_{\rho}\left(u_i^F\right) \equiv \min_{u_k^F \in \coprod^F} | u_k^F u_i^F |,$$

where $| u_k^F u_i^F |$ denotes distance between a point u_i^F and an arbitrary point $u_k^F \in \coprod^F$. The detail description of the image transformations $\Theta^{\hbar}$ is given in Chap. 3. The thin general class Ω^{Θ} is derived based on the thickness measure which is the attribute of this class. The derivation rule for the thin class Ω^{Θ} is given as follows: $[a^{\rho} \le \theta] \Rightarrow \Omega \prec \Omega^{\Theta}$, where a^{ρ} denotes a thickness measure and θ is the threshold.

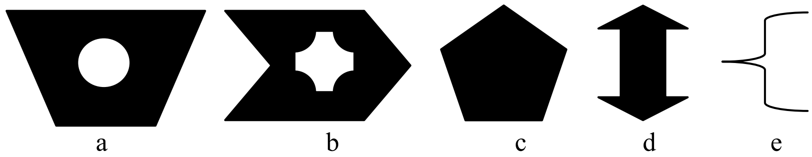


Fig. 2.1. Examples of exemplars of the selected general classes (**a**–**b**) cyclic, (**c**) convex, (**d**) concave, (**e**) thin

In Fig. 2.1 exemplars of the four general classes, the cyclic class (Fig. 2.1a, b), the convex class (Fig. 2.1c), the concave class (Fig. 2.1d), and the thin class (Fig. 2.1e), are shown.

In the further parts of this book the description of the selected shape classes is presented. The a priori classes such as the convex polygon class or the concave polygon class are derived from the general class. The a posteriori classes such as the star class or the spade class are derived from the specific a priori classes.

2.1.1.1. Convex Classes

2.1.1.1.1. Convex Polygon Class and Its Subclasses

The convex polygon class Ω^L consists of elements that are called the convex polygons. A polygon is a simple closed plane figure that is bounded by a finite number of intersecting line segments (at least three segments are required). The polygon $p:[0,1] \to R^2$ is a piecewise linear continuous function. The convex polygon class Ω^L is derived from the convex general class Ω^A by assigning the value 0 to the curvature $\kappa(t)$ of the border curve. The curvature κ at P_0 for a continuous function Γ is defined as the instantaneous rate of change tangent angle with respect to the arc length

$$\kappa = \lim_{P_0 \to P_1} \frac{\alpha(P_1) - \alpha(P_0)}{\bar{P_0 P_1}},$$

where $\alpha(P_1)$ is the angle between the positive x-axis and the direction of the tangent line at a point P_0 and $\bar{P_0 P_1}$ is the arc length between P_0 and P_1. The detail description of the curvature in the context of the concepts of the differential geometry can be found in [7]. In the case when curve is given by the parameterized form $g(t)=\{x(t), y(t)\}$ with parameter $t \in \Theta \subset \Re$, the curvature is expressed in terms of derivatives of the curve as follows

$$\kappa = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}.$$

Several methods of the curvature computation were proposed. For example, curvature as the change of cosine over a region of support is given in [8], the curvature as the rate of change of slope expressed as a function of length is described in [9], or the curvature as a convolution with a Gaussian kernel is described in [10].

The convex polygon class Ω^L is given by the following derivation rule: $[\forall t \in [t_i, t_{i+1}] : \kappa(t) = 0] \Rightarrow \Omega^\Lambda \succ \Omega^L$. Here, $[t_i, t_{i+1}]$ is an interval where the first derivative of the polygon curve given by the equations $x = x(t), y = y(t)$ exists.

The convex polygon class Ω^L is split into base convex polygon classes based on the derivation rules

$$\left[\exists n \in N : n = a^v\right] \Rightarrow \Omega^L \succ \Omega^{L^n},$$

where $a^v = |V|$ denotes the attribute of the class (the cardinality of the set of vertices V). A mathematical object is a cardinal number (cardinality of a set) if and only if it is a power of a set [11]. For the set V of vertices, its cardinality is denoted as $|V|$. The classes with n = 3, 4, 5, and 6 (number of sides) are denoted by the symbolic class description L^n as follows: L^3 (triangle class), L^4 (quadrilateral class), L^5 (pentagon class), and L^6 (hexagon class).

The class L^n is split into specific classes against the relations between selected attributes $\left(a_i^d, a_i^\alpha\right)$. For example, the right triangle class L_R^3 is the class whose archetypes are triangles with one interior angle that is equal to 90°. The derivation of the right triangle class L_R^3 from the triangle class L^3 is given by the following rule

$$\left[\exists a_i^\alpha \in A^G : a_i^\alpha = \frac{\pi}{2}\right] \Rightarrow L^3 \succ L_R^3.$$

2.1.1.1.2. Convex Curve-Polygon Class and Its Subclasses

The convex curve-polygon class Ω^M consists of the geometrical figures, which have curvilinear parts as well as linear segments. The curve-polygon class Ω^M is defined against the value of the curvature $\kappa(t)$ as follows

$$[\exists i : \forall t \in (t_i, t_{i+1}), \kappa(t) = 0] \Rightarrow \Omega^A \succ \Omega^M,$$

where t_i ($i = 1,\ldots,N$) is the value of a parameter for which the curvature $\kappa(t)$ does not exist.

Splitting of the convex curve-polygon class M into the base classes is based on a number of straight line segments and a number of curvilinear segments *m* of archetypes of the class M. The description of the base convex curve-polygon class is related to the generic polygon class L^n. Archetype of the generic polygon class L^n is constructed by joining vertices of the straight line segments as shown in Fig. 2.2. Archetype shown in Fig. 2.2a is a member of the curve-linear class $M^1[L^4]$, where 1 denotes one curvilinear segment and L^4 denotes the generic polygon (rectangle Fig. 2.2b). Examples of the archetypes of the base convex curve-polygon classes are shown in Fig. 2.2. The symbolic names for archetypes shown in Fig. 2.2 are as follows: $M^1[L^3]$ (Fig. 2.2c), $M^1[L^4]$ (Fig. 2.2d), $M^2[L^4]$ (Fig. 2.2e), $M^1[L^5]$ (Fig. 2.2f), and $M^1[L^6]$ (Fig. 2.2g). Construction of the generic polygon is presented in Fig. 2.2a–b. The generic polygon is obtained by joining straight line segment vertices.

The class $M^m[L^n]$ is split into specific classes based on the type of the curvilinear segment and the description of the specific curve-polygon class is given in the form $M^m[L^n](m\theta_H^f)$, where L^n is a generic polygon class,

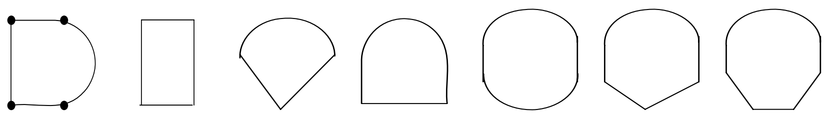
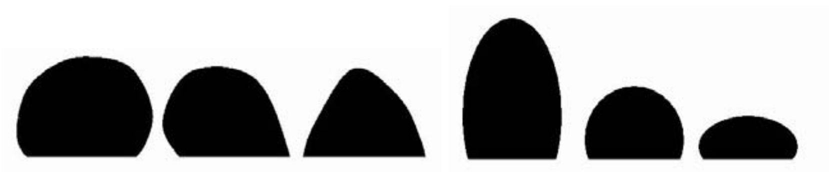


Fig. 2.2. Construction of the generic polygon: (**a**) an archetype of the convex curve-polygon class, (**b**) the generic polygon obtained by joining straight line segment vertices. Examples of archetypes of the convex polygon-curve class (**c–g**)

m is a number of curvilinear segments, and θ_H^f denotes a type of the curvilinear segment. Each symbol of the type of the curvilinear segment θ_H^f has its meaning: θ denotes convexity of the curvilinear segment $\theta \in [c, w]$, where *c* is a convex curvilinear segment and *w* is a concave curvilinear segment; *f* denotes the curvilinear segment $f \in [0,1,2]$, where 0 denotes a "function," 1 denotes a "nonfunction" only on one side, and 2 denotes a "nonfunction" on both sides. The "function" is a curvilinear segment that is the graphical representation of any function $y = f(x)$. *H* denotes the height of the curvilinear segment $H \in [0,1,2]$, where 0 indicates a low height segment, 1 indicates a medium height segment, and 2 indicates a high segment. The height is the perpendicular distance from the chord connecting the endpoints of a curvilinear segment to the farthest point on the curvilinear segment. The symmetrical curvilinear segment is denoted as $\overline{\theta}_H^f$.

Archetypes of the class M^1 possess only one straight line segment and one curvilinear segment. The description of the specific class derived from the class M^1 is given in the form $M^1\left(\theta_H^f\right)$. The examples of exemplars generated from the class M^1 are given in Fig. 2.3. The symbolic names of the exemplars shown in Fig. 2.3 are as follows: $M^1\left(c_1^2\right)$ (Fig. 2.3a), $M^1\left(c_1^1\right)$ (Fig. 2.3b), $M^1\left(c_1^0\right)$ (Fig. 2.3c), $M^1\left(\overline{c}_2^2\right)$ (Fig. 2.3d), $M^1\left(\overline{c}_1^2\right)$ (Fig. 2.3e), and $M^1\left(\overline{c}_0^2\right)$ (Fig. 2.3f).

Archetypes of the class $M^1[L^3]$ possess two straight line segments and one curvilinear segment. The description of the specific class derived from



a b c d e f

Fig. 2.3. Exemplars generated from the class Ω^{M_1} : (**a**) $M^1\left(c_1^2\right)$, (**b**) $M^1\left(c_1^1\right)$, (**c**) $M^1\left(c_1^0\right)$, (**d**) $M^1\left(\overline{c}_2^2\right)$, (**e**) $M^1\left(\overline{c}_1^2\right)$, (**f**) $M^1\left(\overline{c}_0^2\right)$

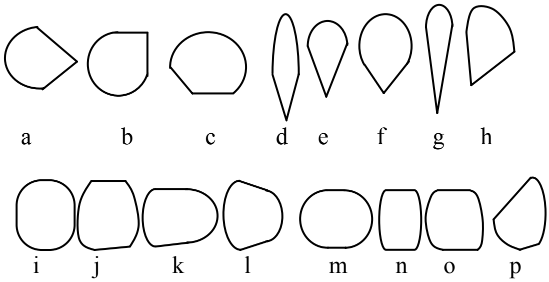


Fig. 2.4. Archetypes of the class $M^1[L^3]\left(\theta_H^f\right)$ (**a**–**h**), $M^2[L^4]\left(\theta_H^f\theta_H^f\right)$ (**i**–**p**)

the class $M^1[L^3]$ is given in the form $M^1[L^3]\left(\theta_H^f\right)$, where θ_H^f denotes the type of the curvilinear segment. Archetypes of the class $M^1[L^3]\left(\theta_H^f\right)$ in Fig. 2.4(a–h).

Archetypes of the class $M^2[L^4]$ possess two straight line segments and two curvilinear segments (see Fig. 2.4(i–p)). The specific class derived from the class $M^2[L^4]$ is given in the form $M^2[L^4]\left(\theta_H^f\theta_H^f\right)$, where θ_H^f denotes the type of the curvilinear segment and L^4 denotes the generic polygon.

2.1.1.1.3. Convex Curve Class and Its Subclasses

The convex curve class Ω^K consists of convex curves. A convex curve in $E2$ can be described in many different forms: an implicit equation $F(x,y,z) = 0$, a parametric equation $(x(t),y(t))$, the parametric Fourier equations, parametric B-splines, or wavelets. The approximated forms of curve representation, such as Fourier series, cubic-splines, B-splines, β-splines, and wavelets, are often used in geometric modeling (e.g., [12]) and are most promising as a model for the convex curve class. The Fourier series can be seen also as a definition of a curve in the parametric form. The curve Γ can be expressed in the form of its truncated Fourier series as follows:

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}\right),$$

$$y(t) = c_0 + \sum_{n=1}^{\infty}\left(c_n \cos\frac{2\pi nt}{T} + d_n \sin\frac{2\pi nt}{T}\right).$$

The coefficients a_n, b_n, c_n, and d_n are computed as described in Brigham [13].

The equation for a single parametric cubic spline segment is given by

$$P(t) = \sum_{i=1}^{4} B_i t^{i-1}, \quad t_1 \le t \le t_2, t_1 \le t \le t_2$$

where t_1 and t_2 are the values of parameters at the beginning and at the end of the segment. $P(t)$ is the position vector of any points on their cubic spline segment. The curve can be computed as

$$C_x^k(t) = \sum_{i=0}^{3} A_{ik} t^i, \quad C_y^k(t) = \sum_{i=0}^{3} B_{ik} t^i, \quad t_1 \le t \le t_2.$$

The constant coefficients A_{ik} and B_{ik} are determined by specifying four boundary conditions for the spline segment [12].

B-splines are given by the parametric equation

$$f(t) = \sum_{i=0}^{n} b_{i,k}(t)\overline{q}_i,$$

where $\overline{q}_0, \overline{q}_1, \ldots, \overline{q}_n$ are $n + 1$ control points. The index $k = 2,3,...$, determines the number of control points that have influence on the points of the curve [14].

Discrete parameter wavelet transform DPWT [15] can be used to represent curve $f(t)$ as follows:

$$f(t) = c \sum_m \sum_n \mathrm{DPWT}(m,n) \Psi_{m,n}(t),$$

where c is some constant dependent on $\Psi(t)$. The discrete parameter wavelet transform is given by $\mathrm{DPWT}(m,n) = \int f(t)\Psi_{m,n}(t)dt$, where $\Psi_{m,n}(t) = a_0^{-m/2}\Psi\left(a_0^{-m} t - n\tau_0\right)$, $\Psi_{0,0}(t) = \Psi(t)$, and a_0 and τ_0 are constants that determine the sampling intervals.

It can be proved (see, e.g., [7]) that a unit-speed curve $\hat{f} : (a,b) \to \Re^2$ whose curvature is given as a piecewise continuous function $\kappa : (a,b) \to \Re^2$ is given by

$$\begin{cases} \hat{f}(s) = \left(\int \cos\theta(s)\mathrm{d}s + c \int \sin\theta(s)ds + d \right) \\ \theta(s) = \int \kappa(s)ds + \theta_0 \end{cases}, \tag{1}$$

where *c*, *d*, and θ_0 are integration constants. The curvature given as a piecewise continuous function κ is characteristic for the curvilinear segment. Equation (1) can be used as a model of the curve class. However to derive the specific classes the heuristic rules are applied that make it possible to define the classes based on more perceptually oriented approach. The curve class Ω^K is defined by using a curvature and is given by the derivation rules $[\forall t \in [t_1, t_2] : \kappa(t) > 0] \Rightarrow \Omega^A \succ \Omega^K$, where parameter *t* varies over a given range $t \in [t_1, t_2]$.

The class K^1 is the class whose archetypes are regular curves. The regular curve is a curve that is convex and symmetrical. Archetypes of the convex curve class K^1 are defined by the ellipse equation

$$\frac{x^2}{a} + \frac{y^2}{b} = 1,$$

which is parameterized by two parameters *a* and *b*. The curvature of the ellipse is given by the equation

$$\frac{ab}{\left(b^2 \cos^2 t + a^2 \sin^2 t\right)^{3/2}}.$$

From the convex curve class K^1 the specific classes, the circle class K_C^1 (Fig. 2.5a) and the ellipsis class K_E^1 (Fig. 2.5b, c), are derived.

The class K^2 is a class for which curvature of each archetype has one clear maximum and each archetype is symmetrical. The maximum of the curvature is the point $t \in [t_1, t_2]$ for which the first derivative of the curvature $\kappa'(t) = 0$. The maximum of the curvature is denoted as $\kappa_{\max}$ and $\kappa_{\max}$ is the attribute of the class K^2. The derivation rule of the convex class K^2 is given as

$$\left[a^{\kappa_{\max}} > \theta_h^{\kappa} \right] \Rightarrow \Omega^K \succ \Omega^{K_2},$$

where θ_h^{κ} is the threshold. Archetypes generated from convex curve class K^2 are shown in Fig. 2.5d, e.

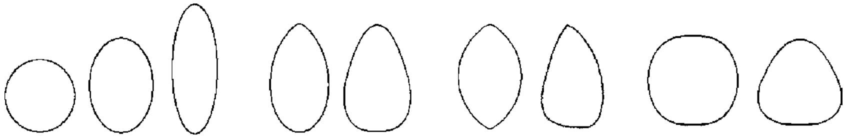


Fig. 2.5. Archetypes of the convex curve class

The class K^3 is described as a class for which curvature of each archetype is in the range $\theta_l^\kappa < \kappa(t) < \theta_h^\kappa$, where θ_l^κ and θ_h^κ are thresholds. The derivation rules of the convex class K^3 are given in the form

$$\left[a_{\max}^\kappa < \theta_h^\kappa \wedge a_{\min}^\kappa > \theta_l^\kappa\right] \Rightarrow \Omega^K \succ \Omega^{K_3},$$

where $a_{\max}^\kappa$, $a_{\min}^\kappa$ are attributes of the convex curve class K^3, and $\theta_l^\kappa, \theta_h^\kappa$ are thresholds. Archetypes generated from convex curve class K^3 are shown in Fig. 2.5f, g.

The convex curve class K^4 is derived from convex class based on derivation rules given in the form

$$\left[a_{\max}^\kappa < \theta_l^\kappa \wedge a^L > \theta_l^L\right] \Rightarrow \Omega^K \succ \Omega^{K_4},$$

where curvature $a_{\max}^\kappa$, and elongation a^L are attributes of the convex curve class Ω^K and $\theta_l^\kappa, \theta_l^L$ are thresholds. Archetypes generated from the convex curve class K^4 are shown in Fig. 2.5h, i. Elongation L is defined as $L = \lambda_1 / \lambda_2$, where λ_1 and λ_2 are the first and second eigenvalues of the matrix of the first and second moments

$$\begin{bmatrix} m_{20} & m_{11} \\ m_{11} & m_{02} \end{bmatrix}, \; where\, m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q dx\, dy, \; and \; p,q \in [0,1,2].$$

2.1.1.2. Concave Classes

In the section "Convex Curve Class and Its Subclasses" the specific convex classes were described. In this section the specific concave classes, derived from the concave general class, are presented. The process of derivation of the concave general class Q was described in the previous chapters. The archetype of the concave class Q consists of elements that can be decomposed into subregions (residuals) iteratively. In decomposition scheme the concave object is broken down into very simple primitives called residuals.

Fig. 2.6. Process of decomposition of the archetype of the concave class: (**a**) an archetype of the concave class $Q^1[L^4](L^4)$, (**b**) the generic convex class L^4, (**c**) residual L^4

At first the convex hull is used as a base for the decomposition of the object into the concave regions and residuals and next each residual is examined in the process called the first level of iteration (see Fig. 2.6). In the case when some residuals are concave they are examined in the process called the second level of iteration. The description of the concave class depends on the level of iteration and is given by a symbolic name $Q^n[\Im_\Lambda](n\Im_\Gamma)$, where n is the number of residuals, $\Im_\Gamma$ is a type of the residuals, $\Im_\Lambda$ is a type of the generic classes, $\Im_\Lambda$ is one of the convex classes $\Im_\Lambda \equiv \{L,K,M\}$, and $\Im_\Gamma$ is one of the acyclic generic classes $\Im_\Gamma = \{\Lambda, Q, \Theta\}$.

The convex hull shown in Fig. 2.6b is used as a base for the decomposition of the object into the concave regions and residuals and is called the generic convex object. The generic convex object is a member of the convex rectangular class L^4. As it was described in decomposition scheme, the concave object is broken down into very simple primitives called residuals. Figure 2.6 shows the process of decomposition of the concave object (a) an archetype of the concave class $Q^1[L^4](L^4)$ (Fig. 2.6a), (b) the generic convex class L^4 (Fig. 2.6b), and (c) residual member of the rectangular class L^4 (Fig. 2.6c).

2.1.1.2.1. Levels of Iterations

As it was described, the description of the concave class depends on the level of iteration, the number of residuals n, type of the residuals $\Im_\Gamma$, and type of the generic class $\Im_\Lambda$. The description of the concave class at the first level of iteration is given by $Q^n[\Im_\Lambda](n\Im_\Gamma)$, where $\Im_\Lambda$ is one of the convex classes $\Im_\Lambda = \{L,K,M\}$ and $\Im_\Gamma$ is one of the acyclic general classes $\Im_\Gamma = \{\Lambda, \Theta\}$. Depending on the number of residuals n, and type of classes $\Im_\Lambda$ and $\Im_\Gamma$ the following concave classes are possible: $Q^n_\Lambda(n\Lambda)$, $Q^n_\Lambda(n\Theta)$, or $Q^n_\Lambda(k\Lambda m\Theta)$, where $k + m = n$.

In the case when the generic class is the convex polygon class L the class that is derived is given by $Q^n_L(n\Lambda)$. The symbol $Q^n_L(n\Lambda)$ denotes the concave class Q whose generic class is the convex polygon class L and archetypes of this class have *n* residuals. All residuals are archetypes of one of the convex classes (the polygon class *L*, the convex polygon-curve class *M*, or the convex curve class *K*). The following concave classes are possible: $Q^n_L(nL)$, $Q^n_L(nM)$, $Q^n_L(nK)$, $Q^n_L(kLmM)$, $Q^n_L(kLmK)$, $Q^n_L(kMmK)$, or $Q^n_L(hLkMmK)$, where $k + m = n$ and $h + k + m = n$. Similarly, the possible classes whose generic class is the convex curve-polygon class $Q^n_M(n\Lambda)$ or the convex curve class $Q^n_K(n\Lambda)$ can be obtained. The symbol $Q^2[L^5](2 \cdot L^3)$ denotes that the concave class Q whose generic class is the convex polygon class (pentagon) L^5 has two residuals. Both residuals are archetypes of the triangle class L^3. Examples of the concave class at the first level of iteration are given in Fig. 2.7.

Archetypes in Fig. 2.7a–c are members of the class $Q^n[L](n\Lambda)$, where the generic class is given by the convex polygon class *L*. The residuals are members of the convex polygon class L, the convex curve-polygon class M, or the convex class K. Archetypes are given by the following symbolic names: $Q^2[L^5](2 \cdot L^3)$ (Fig. 2.7a), $Q[L^4](M)$ (Fig. 2.7b), and $Q[L^4](K^1)$ (Fig. 2.7c).

Archetypes in Fig. 2.7d–f are members of the class $Q^n_M(n\Lambda)$, where the generic class is given by the convex curve-polygon class M. The residuals are members of the convex polygon class L, the convex curve-polygon class M, and the convex class K. Archetypes are given by following symbolic names: $Q[M](L^3)$ (Fig. 2.7d), $Q[M](M)$ (Fig. 2.7e), and $Q[M](K^1)$ (Fig. 2.7f).

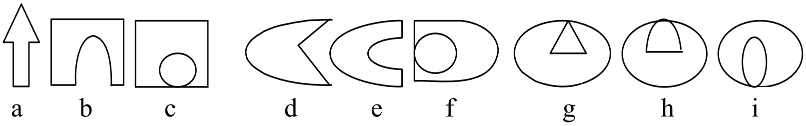


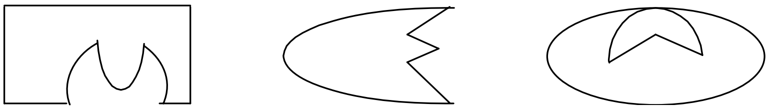
Fig. 2.7. Archetypes of the concave classes at the first level of iteration

Archetypes in Fig. 2.7g–i are members of the class $Q_K^n(n\Lambda)$ where the generic class is given by the convex curve class K. The residuals are members of the convex polygon class L, the convex curve-polygon class M, and the convex class K. Archetypes are given by the following symbolic names: $Q[K](L^3)$ (Fig. 2.7g), $Q[K](M)$ (Fig. 2.7h), and $Q[K](K^1)$ (Fig. 2.7i).

Similarly, at the second level of iteration the description of the concave class is given by $Q_{\Im_\Lambda}^n(n\Im_\Gamma)$, where $\Im_\Lambda$ is one of the convex classes $\Im_\Lambda = \{L, K, M\}$ and $\Im_\Gamma$ is one of the acyclic general classes $\Im_\Gamma = \{\Lambda, Q, \Theta\}$. Depending on the number of residuals *n*, type of the class $\Im_\Gamma$ and type of the generic convex class $\Im_\Lambda$ the following classes are possible: $Q^n[\Lambda](nQ)$, $Q^n[\Lambda](kQm\Lambda)$, $Q^n[\Lambda](kQm\Theta)$, or $Q^n[\Lambda](hQk\Lambda m\Theta)$, where $k+m=n$ and $h+k+m=n$. Examples of the archetypes of the concave classes at the second level of iteration are given in Fig. 2.8.

Archetypes in Fig. 2.8 are members of the class $Q[\Lambda](Q[\Lambda](\Lambda))$, where the generic class of each archetype is one of the following classes: the convex polygon class L, the convex curve-polygon class M, and the convex class K. The residuals are members of the concave class Q. Archetypes shown in Fig. 2.8 are given by following symbolic names: $Q^1[L^4](Q^1[M](M))$ (Fig. 2.8a), $Q^1[M](Q^1[L^4](L^3))$ (Fig. 2.8b), and $Q^1[K](Q^1[M](L^3))$ (Fig. 2.8c).

In the case when a number of iteration levels and the number of residuals are growing an archetype of the concave class can be described as an archetype of the thin class. Also for the class $Q_{L^m}^1(L^i)$ the following expression is true $\lim_{i\to\infty} Q_{L^m}^1(L^i) = Q_{L^m}^1(K)$, where K denotes the curvilinear class. For the convex polygon class $Q^n[L^m](nL^k)$, when *n* is large enough, the convex polygon class is called a noisy class and is denoted as



a b c

Fig. 2.8. Archetypes of the concave classes at the second level of iteration

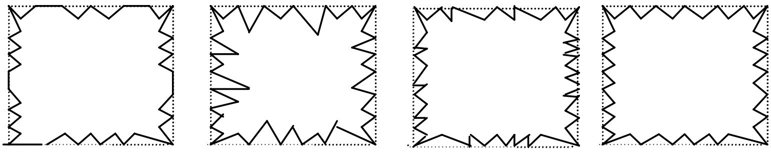


Fig. 2.9. Archetypes of the noisy polygon class

$\aleph[L^m](L^k) \equiv \lim_{n\to\infty} Q^n_{L^m}(nL^k)$. When all residuals are triangles ($k = 3$) the noisy class is denoted as $\aleph[L^m](n \cdot L^3)$ [16]. Examples of archetypes of the noisy class $\aleph[L^4](nL^3)$ are given in Fig. 2.9.

2.1.1.2.2. Concave Polygon Class

In the previous section the specific concave classes, derived from the general concave class, were described. In this section the subspecific concave classes, derived from the concave polygon class, are presented. The concave polygon class is the class archetypes of which are concave polygons. The concave polygon class at the first level of iteration is described as $Q^n[L^m](nL^k)$. For the concave polygon class $Q^n[L^m](nL^k)$ the generic class is the convex polygon class L and all residuals are archetypes of one of the convex polygon classes. The concave polygon class at the second level of iteration is described as $Q^n[L^m](nQ^h[L^m](hL^p))$ and the concave polygon class at the third level of iteration is given by symbolic name $Q^n[L^m](nQ^h[L^m](hQ^w[L^u](wL^s)))$. Example of the archetype generated from the concave polygon class at the third level of iteration given by symbolic name $Q^3[L^6](Q^3[L^5](Q^1[L^4](L^4), L^3, L^3), Q^1[L^4](L^4), Q^1[L^4](L^3))$ is shown in Fig. 2.10. The symbol $Q^3[L^6](Q^3[L^5](Q^1[L^4](L^4), 2L^3)$, $Q^1[L^4](L^4)$, $Q^1[L^4](L^3))$ denotes the archetype of the concave class Q whose generic class is the archetype of the convex polygon class (hexagon) L^6 and the concave class is described at three levels of iteration. At the first level of iteration there are three residuals, archetypes of the concave classes $Q^3[L^5](Q^1[L^4](L^4),\ 2L^3)$, $Q^1[L^4](L^4)$, and $Q^1[L^4](L^3)$. At the second level of iteration each residual $Q^3[L^5](Q^1[L^4](L^4),\ 2L^3)$, $Q^1[L^4](L^4)$, and $Q^1[L^4](L^3)$ is considered as an archetype of the concave class whose generic classes are archetypes of the convex polygon classes L^5, L^4, and

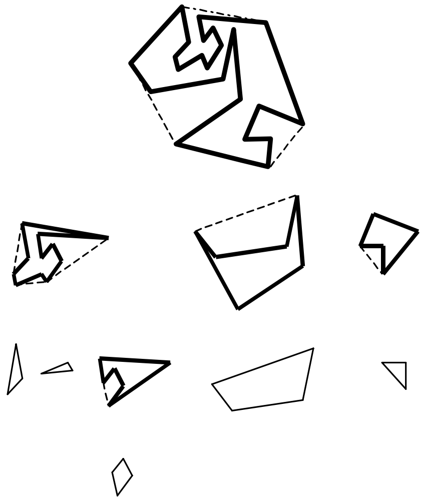
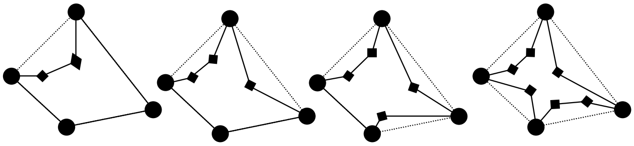


Fig. 2.10. The archetype of the class $Q^3[L^6](Q^3[L^5](Q^1[L^4](L^4), L^3, L^3)Q^1[L^4](L^4), Q^1[L^4](L^3))$

L^4. The archetype of the class $Q^3[L^5](Q^1[L^4](L^4), 2L^3)$ has three residuals $Q^1[L^4](L^4)$, L^3, and L^3. At the third level of iteration the residual $Q^1[L^4](L^4)$ is decomposed into the generic class L^4 and one residual L^4.

The concave polygon class can be described by applying the different symbolic descriptions. One of the descriptions is based on the computation of the convex and concave vertices. Let *m* denote the number of vertices of the generic convex polygon (convex vertices) of the archetype of the concave class $Q^n[L^m]$. Let *n* denote a number of residuals and h_i $(i = 1,\ldots,n)$ denotes a number of concave vertices w_j^i between two convex vertices v_i and v_{i+1}. To obtain description of this class in a more convenient way, let $v_i \equiv a$ denotes a convex vertex and $k_i \equiv \left\{w_1^i, w_2^i,\ldots,w_{h_i}^i\right\}$ denotes a set of concave vertices between two adjacent vertices v_i and v_{i+1} so as the class description can be represented by the string in the form $\text{Ł}_m^n[ak_1ak_2,\ldots,ak_i,\ldots,ak_n]$. The description given by the concave vertices string can be transformed into the description given by the iterative model $Q^n[L^m]\left(L_1^k,\ldots,L_n^k\right)$. Examples of the transformations of the description given by the concave vertices string into the description given by



a b c d

Fig. 2.11. Archetypes of the class defined by the description given by the concave vertices string

the iterative model for the archetype shown in Fig. 2.11c is as follows: $\text{Ł}_4^4[a4a3a4a3] \equiv Q^4[L^4](L^4, L^3, L^4, L^3)$. Examples of archetypes defined by the description given by the concave vertices string are shown in Fig. 2.11. Those archetypes are given by the following symbolic names: $\text{Ł}_4^1[a4aaa]$ (Fig. 2.11a), $\text{Ł}_4^2[a4a3aa]$ (Fig. 2.11b), $\text{Ł}_4^3[a4a3a3a]$ (Fig. 2.11c), and $\text{Ł}_4^4[a4a3a4a3]$ (Fig. 2.11d).

The archetype of the complex polygon class C is obtained as the result of a certain type of topological operation called a complex polygon addition. The addition operation defines the way in which polygons are joined together. One of the addition operations that make the complex polygon object by joining two polygons along the common edge is the edge-sum. The edge-sum is defined as follows. Let $\omega^{L^n} \in \Omega^{L^n}$ and $\omega^{L^k} \in \Omega^{L^k}$, where $\omega^{L^k}, \omega^{L^n}$ are archetypes of the polygon class. The sum $\omega^{L^n} \otimes \omega^{L^k}(v_i)$ at the edge $E_i(\omega^{L^n})$ is defined to be a polygon resulted from adding ω^{L^n} with ω^{L^k} by translating, rotating, and scaling ω^{L^k} so that $E_j(\omega^{L^k})$ coincides with $E_i(\omega^{L^n})$. The edge $E_i(\omega^{L^n})$ given by vertices $v_i(\omega^{L^n})$ and $v_{i+1}(\omega^{L^n})$ describes the bounding rectangle of the sum $\omega^{L^n} \otimes \omega^{L^k}(v_i)$. The bounding rectangle is given by a line passing through vertices $v_i(\omega^{L^n})$, $v_{i+1}(\omega^{L^n})$ and perpendicular to the line given by the edge $E_i(\omega^{L^n})$. The archetype of the complex polygon class can consist with more than two parts. The complex polygon class is denoted as $C(nL^i)$, where n is a number of polygonal parts L^i. There is a conversion from the notation of the complex class into the notation given by the iterative model. Figure 2.12 shows an archetype of the complex polygon class represented by four different symbolic representations in the form of the

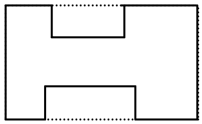


Fig. 2.12. An archetype of the concave polygon class

iterative model $Q^2[L^4](2L^4)$, the complex model $C(L^4, L^4, L^4)$, the subtracting model $\not{L}_4^2[v4vv4v]$, and the cyclic model $\perp^{12}\{(12\cdot\pi)(12d)$ described in [17].

2.1.1.2.3. Concave Curve-Polygon Class

The concave curve-polygon class is a class archetypes of which are the concave curve-polygons. The concave curve-polygon is the class archetypes of which need to have at least one curvilinear segment. At the first level of iteration the following concave curve-polygon classes are possible: $Q^k[M](kM)$, $Q^k[L](kM)$, $Q^k[M](kL)$, $Q^K[L](k_1Mk_2L)$, or $Q^K[M](k_1Mk_2L)$. The description of the specific concave curve-polygon classes can be given using the concave vertices form $W^{i,j,k}[L^m](i\cdot\theta, j\cdot g, k\cdot l^n)$, where θ is a type of the concave curvilinear segment, g is the concave straight-curvilinear segment, and l^n is the concave n-gon. There is a conversion from the notation of the concave vertices form into the notation given by the iterative model. For example, archetypes shown in Fig. 2.13a–c are given by description both in a concave vertices form and by an iterative model:

$Q^2[M[L^3]](M^1, L^3) \equiv W^2[M[L^3]](cl^3w)$ (Fig. 2.13a)

$Q^2[M[L^4]](M^1, L^4) \equiv W^2[M[L^4]](cwcl^4)$ (Fig. 2.13b)

$Q^3[M[L^4]](M^1[L^3], M^1, L^3) \equiv W^3[M[L^4]](cwl^3g^1)$ (Fig. 2.13c)

The regular concave curve-polygon class is the class given by symbolic name $Q^k[M](kM)$. For this class the generic class and all residuals are members of the curve-polygon class M. Examples of archetypes generated from class $Q^k[M](kM)$ are shown in Fig. 2.13d–f. Those archetypes are given by the following symbolic names: $Q^2[M[L^3]](2M^1)$ (Fig. 2.13d), $Q^2[M^2[L^4]](2M^1)$ (Fig. 2.13e), and $Q^2[M^1[L^4]](2M^1)$ (Fig. 2.13f).

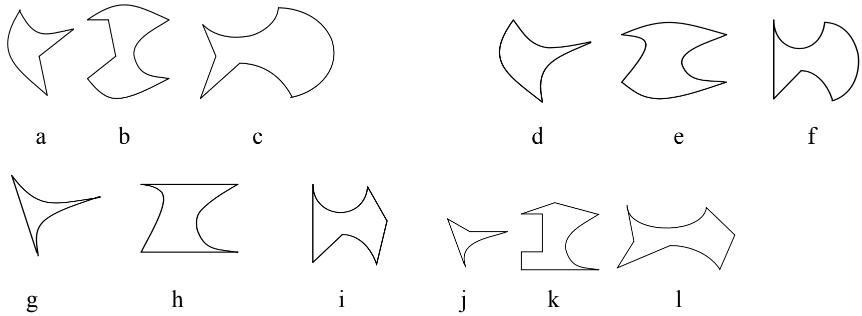


Fig. 2.13. Examples of archetypes whose descriptions are given both in a concave vertices form and by an iterative model

The archetypes of the concave curve-polygon class whose generic class is a member of the polygon class are given by the symbolic name $Q^k[L](kM)$ or $Q^K[L](k_1Mk_2L)$, where $K = k_1 + k_2$. Examples of archetypes generated from the class $Q^k[L](kM)$ are shown in Fig. 2.13g–i. Archetypes shown in Fig. 2.13g–i are given by the following symbolic names: $Q^2[L^3](2M_1)$ (Fig. 2.13g), $Q^2[L^4](2M_1)$ (Fig. 2.13h), and $Q^2[L^5](2M_1)$ (Fig. 2.13i). Examples of archetypes generated from the class $Q^K[L](k_1Mk_2L)$ are shown in Fig. 2.13j–l. Archetypes shown in Fig. 2.13j–l are given by the following symbolic names: $Q^2[L^3](M_1L^3)$ (Fig. 2.13j), $Q^2[L^5](M_1L^4)$ (Fig. 2.13k), and $Q^3[L^5](M_1M_2^1L^3)$ (Fig. 2.13l).

The concave curve-polygon class, for which archetypes of the generic class are members of a convex polygon class L, is called the concave curve-polygon star class and is given by the symbolic name $Q^k[L^k](mMnL)$, $m + n = k$. The concave curve-polygon star class whose all residuals are members of the curve-polygon class is called the regular concave curve-polygon star class and is denoted as $Q^k[L^k](kM)$. Examples of archetypes generated from the curve-polygon star class are shown in Fig. 2.14. Archetypes shown in Fig. 2.14 are given by the following

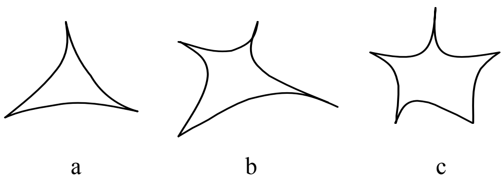


Fig. 2.14. Archetypes of the regular concave curve-polygon star classes

symbolic names: $Q^3[L^3](3M)$ (Fig. 2.14a), $Q^4[L^4](4M)$ (Fig. 2.14b), and $Q^5[L^5](5M)$ (Fig. 2.14c).

2.1.1.3. Thin Classes

As it was described in the section "Concave Curve-Polygon Class," the thin class is a class whose members are thin objects. In this book the term the thin class is used to denote the acyclic-thin class. The thin class is represented by the acyclic graph called a tree. The undirected graph $G = (V, E)$, where V is the set of nodes and $E \subseteq V \times V$ is the set of edges, is called a tree if it satisfies two conditions: the graph is connected and the graph contains no cycles. It can be shown that in the case of the thin acyclic shape class a tree is a spanning tree. An edge of a spanning tree is called a branch and a spanning tree with H vertices consists of H-1 branches. The spanning tree represents an archetype of the thin class. The archetype of the thin class consists of edges and vertices. The two types of vertices are distinguished: the endpoint v^{ξ} and the branching-point v^{ζ}.

The thin class, the archetype of which has a branch $v_i^{\zeta} v_j^{\zeta}$ connecting only the branching points, is called the thin bridge class and the branch $v_i^{\zeta} v_j^{\zeta}$ is called a bridge. Depending on the curvilinearity of the branch, two types of branches can be distinguished: the straight branch and the curvilinear branch. The class whose archetypes have all straight branches is called the straight thin class. For the straight thin class a set of angles and distances called the set of attributes of the straight thin class is computed. The set of attributes is denoted as $A^{\Theta} = \left\{\left(a_1^d, a_1^{\alpha}\right), \left(a_2^d, a_2^{\alpha}\right), \ldots, \left(a_N^d, b_N^{\alpha}\right)\right\}$, where a_i^d is a distance computed as $d_k^{\xi} = \left|\overline{v_i^{\xi} v_j^{\zeta}}\right|$ for two different types of the vertices and $d_k^{\zeta} = \left|\overline{v_i^{\zeta} v_j^{\zeta}}\right|$ for this same type of vertices, and a_i^{α} is an angle computed as $\alpha_k^{\xi} = \angle v_i^X v_k^{\zeta} v_j^X$, where X denotes vertices type ξ or ζ, and $k = 1,\ldots,$H-1, $m = 1,\ldots,M$, and $\alpha_k^{\zeta} = \angle v_i^X v_k^{\zeta} v_j^X$, where X denotes vertices type ζ, and $k = 1,\ldots,$H-1, $m = 1,\ldots,M$.

Depending on the type of branches the thin class is split into three classes: the 1-D class archetypes of which have only isolated branches $v_1^{\xi} v_2^{\xi}$, the star class $\oplus$ archetypes of which have only external branches $v_i^{\xi} v_j^{\zeta}$, and the thin bridge class Θ_k^1 archetypes of which have both external $v_i^{\xi} v_j^{\zeta}$ and internal $v_i^{\zeta} v_j^{\zeta}$ branches. Examples of the archetypes from the thin class are shown in Fig. 2.15. Archetypes from the Θ^2 class are shown

a b c d e f g h

Fig. 2.15. Archetypes of the thin class (**a**–**c**) the Θ^2 class, (**d**, **e**) the star class $\oplus$, (**f**–**h**) the bridge class Θ_k^1

in Fig. 2.15a–c, archetypes from the star class $\oplus$ are shown in Fig. 2.15d, e, and archetypes from the bridge class Θ_k^1 are shown in Fig. 2.15f–h.

Based on the relations between attributes the following thin star classes $\oplus_k$ are derived:

- The equilateral-star class: this is a class for which all archetypes have all branches equal
- The equiangular-star class: this is a class for which all archetypes have all angles equal
- The ideal star class: this is a class for which all archetypes have all angles and branches equal

The derivation rules applied for each individual class are as follows:

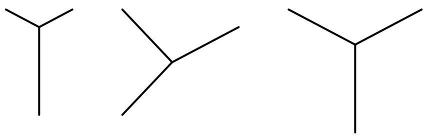
$$\left[\forall a_i^d \in A^T, \exists T_d \in \Re,: a_i^d = T_d\right] \Rightarrow \oplus_k \succ \ddot{\oplus}_k \qquad \text{(the equilateral-star class)}$$

$$\left[\forall a_i^\alpha \in A^T, \exists T_\alpha \in \Re,: a_i^\alpha = T_\alpha\right] \Rightarrow \oplus_k \succ \hat{\oplus}_k \qquad \text{(the equiangular-star class)}$$

$$\left[(\forall a_i^d \in A^T, \exists T_d \in \Re,: a_i^d = T_d) \wedge (\forall a_i^\alpha \in A^T, \exists T_\alpha \in \Re,: a_i^\alpha = T_\alpha)\right] \Rightarrow \oplus_k \succ \overline{\oplus}_k$$

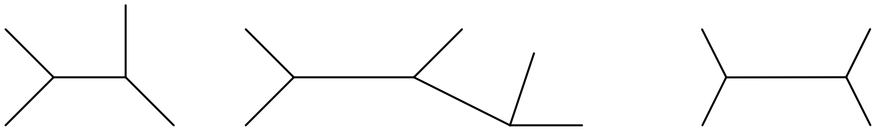
(the ideal star class)

Examples of archetypes of the thin straight star class $\Omega^{\oplus_k}$ are shown in Fig. 2.16. The archetype from the $\ddot{\oplus}^3$ class is shown in Fig. 2.16a, the archetype from the $\hat{\oplus}^3$ class is shown in Fig. 2.16b, and the archetype from the $\overline{\oplus}^3$ class is shown in Fig. 2.16c.



a b c

Fig. 2.16. Archetypes of the straight star class $\Omega^{\oplus_k}$: (**a**) the equiangular-star class $\ddot{\oplus}^3$, (**b**) the equilateral-star class $\hat{\oplus}^3$, (**c**) the ideal star class $\overline{\oplus}^3$



a b c

Fig. 2.17. Archetypes of the bridge thin straight class $\overline{\Theta}_k^1$: (**a**) the equilateral-thin class $\hat{\overline{\Theta}}_4^2$, (**b**) the equilateral-branch thin class $\ddot{\overline{\Theta}}_4^2$, (**c**) the equiangular-branch thin class $\breve{\overline{\Theta}}_4^2$

Similarly, the bridge thin straight class $\overline{\Theta}_k^1$ can be split into specific classes based on a set of attributes A^{Θ} . Examples of archetypes of the bridge thin class are shown in Fig. 2.17. The archetype from the bridge thin straight equilateral-class $\hat{\overline{\Theta}}_4^2$ is shown in Fig. 2.17a, the archetype from the bridge thin straight equilateral-branch class $\ddot{\overline{\Theta}}_4^2$ is shown in Fig. 2.17b, and the archetype from the bridge thin straight equiangular-branch class $\breve{\overline{\Theta}}_4^2$ is shown in Fig. 2.17c.

The special subclass of the thin class is a thin fractal class denoted as Ω^F . The fractal class is described in the form of the thin class as Θ_n^m , where m and n are numbers that characterize the L-system [18]. The thin fractal class defined by the L-system is restricted to the class for which its graph representation is a spanning tree. It imposes the constraints for the level of iteration of the system and a set of parameters of the model. Archetypes of the fractal class are generated by L-systems. L-system uses strings that are interpreted based on the notion of a LOGO-style turtle. For example, the dragon curve can be generated by repetitively substituting line segments by pairs of lines forming either a left or a right turn and is described by the following L-system:

: Fl
p1: Fl→ Fl + Fr+
p2: Fr→Fl – Fr

The symbols Fl, Fr are interpreted by turtle as the “move left” and “move right” commands, and p1, p2 are productions rules [18]. From the thin fractal class the following specific classes are derived: the equiangular-branch thin fractal class $\hat{F}_{\pi}^{8}$, the equiangular-thin fractal class F_{α}^{5} , the thin fractal class F_m^k , the thin curved fractal class F ^{5}, and the thin curved fractal

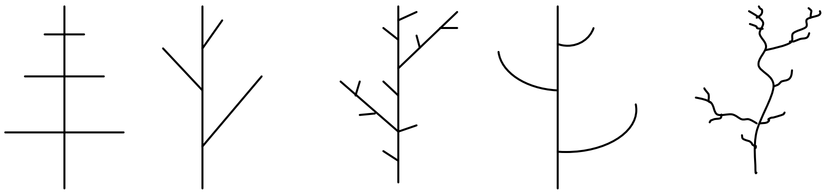


Fig. 2.18. Archetypes of the thin fractal class Ω^F : (**a**) the equiangular-branch thin fractal class $\hat{F}_\pi^8$, (**b**) the equiangular-thin fractal class F_α^5 , (**c**) the thin fractal class F_m^k , (**d**) the thin curved fractal class F^5 , (**e**) the thin curved fractal class F_m^k

class F_m^k. Figure 2.18 shows examples of archetypes generated from the specific fractal classes. These classes are defined in the similar way as the specific classes described in previous sections. The archetype of the class $\hat{F}_\pi^8$ is shown in Fig. 2.18a, the archetype of the class F_α^5 is shown in Fig. 2.18b, the archetype of the class F_m^k is shown in Fig. 2.18c, the archetype of the class F^5 is shown in Fig. 2.18d, and the archetype of the class F_m^k is shown in Fig. 2.18e.

As it was described in the section “Concave Curve-Polygon Class” the 1-D thin class Θ^2 is the class archetypes of which have only isolated branches $v_1^\xi v_2^\xi$. From the 1-D thin class the specific classes are derived based on the properties of the graph function that is representative of the archetype of the class Θ^2 . The function $y = f(x)$ is defined in the closed interval [a,b] and is prescribed by an analytical expression or a formula. It is assumed that the function fulfils the conditions: $f(-a) = f(a) = c$ and $\forall x \in [a,b], f(x) \le c \wedge f(x) \ge d$, where c, d are the greatest and the smallest of all values of the function $f(x)$. The 1-D thin class Θ_F^2 is defined as follows: $[\forall x \in [a,b], \exists y \in [c,d] : y = f(x)] \Rightarrow \Theta^2 \succ \Theta_F^2$. The 1-D thin convex function class derived from Θ^2 is defined as follows: $[\forall x_1, x_2 \in [a,b],$ $\exists \lambda \in (0,1) : f(\lambda x_1 + (1-\lambda)x_2) \le \lambda f(x_1) + (1-\lambda) f(x_2)] \Rightarrow \Theta^2 \succ \Theta_C^2$. The 1-D thin class derived from Θ^2 for which their graph is symmetric with respect to the vertical axis $f(-x) = f(x)$ is called 1-D symmetric class Θ_S^2 . The derivation rules are as follows: $[\forall x \in [a,b] : f(-x) = f(x)] \Rightarrow \Theta^2 \succ \Theta_S^2$.

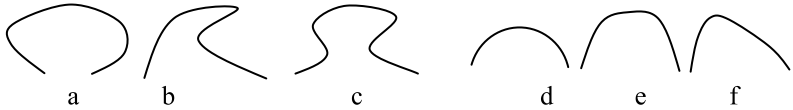


Fig. 2.19. Archetypes of the nonfunction classes $\Omega^{\Im}$ (**a**–**c**) and archetypes of the convex function classes: (**d**) symmetrical $\Omega^{\hat{\hat{\Xi}}}$, (**e** and **f**) nonsymmetrical $\Omega^{\hat{\Xi}}$

Examples of archetypes generated from specific 1-D thin class are shown in Fig. 2.19. Archetypes of the nonfunction classes $\Omega^{\Im}$ are shown in Fig. 2.19a–c, the archetype from the convex symmetrical function class $\Omega^{\hat{\hat{\Xi}}}$ is shown in Fig. 2.19d, and archetypes from the convex nonsymmetrical (NS) function class $\Omega^{\hat{\Xi}}$ are shown in Fig. 2.19e, f.

The 1-D thin class Θ^2, archetypes of which are straight poly-lines, is described in relation to its generic class and is called the thin poly-line class $\otimes$. The archetypes of the generic class are obtained by joining the pseudo-nodes of the archetypes of the class $\otimes$ as shown in Fig. 2.20. The archetype of the class $\otimes[L^4]$ shown in Fig. 2.20a is described in relation to its generic class L^4 (Fig. 2.20b) and the archetype of the class $\otimes[Q^1[L^4](L^3)]$ (Fig. 2.20c) is described in relation to its generic class $Q^1[L^4](L^3)$ (Fig. 2.20d).

The bridge tree class is the class derived from the bridge thin class. Archetypes of the bridge tree class are represented by the acyclic graph called a tree. The bridge tree class is described by the bridge notation that is explained in Fig. 2.22. The bridge is denoted by the bracket “[ ],” whereas branch by the bracket “( ).” The notation is based on the decomposition of the tree into branches and bridges. During decomposition the

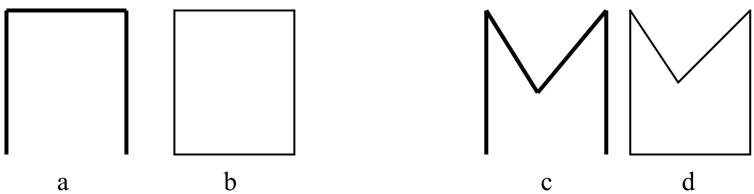
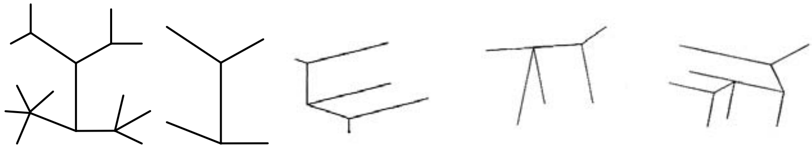


Fig. 2.20. Archetypes of the thin poly-line class $\otimes$ and its generic class: (**a**) the class $\otimes[L^4]$ and (**b**) its generic class L^4, (**c**) the class $\otimes[Q^1[L^4](L^3)]$ and (**d**) its generic class $Q^1[L^4](L^3)$

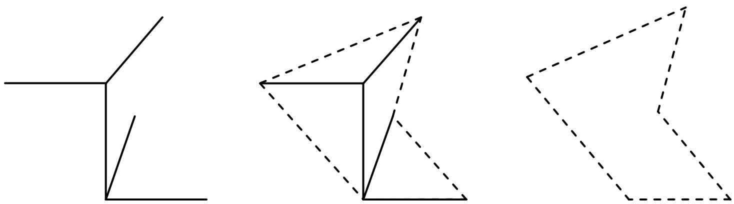


a b c d e

Fig. 2.21. Archetypes of the bridge tree class

branches are removed and the bridge that is left becomes the generic bridge of the tree. For example, the archetype shown in Fig. 2.21a is an archetype from the bridge tree class $\Theta\langle[1]\{[1](2)\}\{[1](2)\}\{[1](3)\}\{[1](4)\}\rangle$. The result of removing branches is the string $\Theta\langle[1]\{[1]\}\{[1]\}\{[1]\}\{[1]\}\rangle$ and finally after renaming bridges into branches the bridge class $\Theta\langle[1](2)(2)\rangle$ is obtained. The bridge class $\Theta\langle[1](2)(2)\rangle$ that is the result of decomposition is shown in Fig. 2.21b. Examples of the archetypes from the bridge tree classes are shown in Fig. 2.21. The archetype from the class $\Theta\langle[1](2)(1)[1](2)\rangle$ is shown in Fig. 2.21c, the archetype from the class $\Theta\langle[1](3)(2)\rangle$ is shown in Fig. 2.21d, and archetype from the class $\Theta\langle[1]\{(2),[1](2)\}\{(1),[1](2)\}\rangle$ is shown in Fig. 2.21e.

As it was described in previous sections each class can be described by applying the different notations. The archetypes in Fig. 2.22 are described by the notation of the bridge tree class $\Theta\langle[1](2)(2)\rangle$, generic bridge tree class $\Theta\langle[1](2)(2)\rangle[Q^1[L^4](L^3)]$, or by notation of the Θ/ρ class as $\Theta/\rho[Q^1[L^4](L^3)]\{3L^3,L^4\}$. The notation of the Θ/ρ class is derived from the notation of the ρ class described in the further part of this chapter. In order to explain the notation of the Θ/ρ class, an example of decomposition of the archetype from the Θ/ρ class is shown in Fig. 2.22. Figure 2.22a shows the archetype from the thin bridge class $\Theta\langle[1](2)(2)\rangle$. The



a b c

Fig. 2.22. Explanation of the notation of the Θ/ρ class

archetype consists of one bridge that has two branches on its ends. The endpoints of this archetype are joined by straight lines as shown in Fig. 2.22b, and as the result the object consisting of the four parts, three triangles L^3, and one quadrilateral L^4, was obtained. The generic polygon, archetype of the class $Q^1[L^4](L^3)$, is shown in Fig. 2.22c.

2.1.1.4. Cyclic Class

As it was described in the section "Thin Classes," the cyclic general class A is defined based on the values of the attribute called a homotopy measure. The cyclic class A consists of elements that can be decomposed iteratively into subregions (holes). The decomposition scheme in which the cyclic object is broken down into very simple primitives, called holes, is similar to the decomposition scheme of the concave object described in previous sections. At first all holes are filled and an object "without holes" is used as a base for the decomposition of the object into the filled regions and holes. Next each hole is examined in the process called the first level of iteration. In the case when some holes are cyclic they are examined in the process called the second level of iteration. The description of the concave class depends on the level of iteration and is given by a symbolic name $A^n[\Im_\Phi](n\Im_A)$, where n is the number of residuals, $\Im_A$ is a type of the holes, and $\Im_\Phi$ is a type of the generic classes. The base cyclic class is denoted as $A^n[\Im_\Phi]$, where $\Im_\Phi$ is one of the acyclic general classes $\Im_\Phi = \{\Lambda, Q\}$ from which the base cyclic class is derived and n is a number of holes. The description of the specific cyclic classes is based on a type of the generic class $\Im_\Phi$ as well as on the type of the holes $\Im_A$. The archetype of a cyclic class derived from the acyclic class can be seen as a result of subtraction of the acyclic region and holes.

At the first level of iteration the symbolic representation of the cyclic class is given as $A^n[\Gamma](n\Im_A)$, where a hole can be a member of the thin or acyclic class $\Im_A \equiv \{\Theta, \Gamma\}$. Depending on the number of holes n, and a type of generic class $\Im_\Gamma$, and a type of holes $\Im_A$, the following specific cyclic classes can be derived: $A^n[\Lambda](n\Gamma)$, $A^n[\Lambda](n\Theta)$, $A^n[Q](n\Gamma)$, and $A^n[Q](n\Theta)$. In the case when there are n holes there are the following classes given by the symbolic names: $A^n_\Lambda(n\Lambda)$, $A^n_\Lambda(nQ)$, $A^n_\Lambda(n\Theta)$,

a b c d e f g h

Fig. 2.23. Examples of exemplars from the cyclic class

$A_Q^n(n\Lambda)$, $A_Q^n(nQ)$, and $A_Q^n(n\Theta)$. Figure 2.23 shows exemplars generated from the cyclic class. The symbol $A^3\,[L^4]\,(3L)$ (Fig. 2.23b) denotes exemplar whose generic class is the convex polygon class L^4 (rectangle) and it has three holes. All holes are archetypes of the rectangle class L^4. Figure 2.23e shows exemplar generated from the cyclic class $A^2[Q^4[L^8](4L^4)]\left(K_E^1, L^4\right)$, whose generic class is the concave polygon class $Q^4\,[L^8\,](4L^4)$, and it has two holes. The first hole is an archetype of the rectangle class L^4 and the second one is the archetype of the curvilinear class (ellipse) K_E^1.

The archetype of the class $A^1[\Lambda](\Gamma)$ for which the hole has common points with the border points is a concave point class $A^1[\Lambda](\Gamma) \equiv \bar{Q}^1[\Lambda](\Gamma)$. Similarly, the archetype of the class $A^1[Q](\Gamma)$ for which the hole has common points with the border points is the archetype of the concave point class $A^1[Q](\Gamma) \equiv \bar{Q}^1[Q](\Gamma)$. Figure 2.24 shows exemplars generated from the concave point class $\bar{Q}^1[L^4](L^4)$ (Fig. 2.24a) and $\bar{Q}^1[L^4](L^3, \bar{L}^3)$ (Fig. 2.24b).

Similarly, at the second level of iteration the symbolic representation of the cyclic class is given as $A^n[\Gamma](n\Im_A)$, where at least one hole from the set $\Im_A$ is a member of the cyclic class A. Examples of exemplars generated from the cyclic classes at the second level of iteration are shown in Fig. 2.25a–d. The symbolic names of these exemplars are as follows:

a b

Fig. 2.24. Exemplars of the concave point class

$$A^1\left[L_R^4\right]\left(A^1\left[L_R^4\right]\left(L_R^4\right)\right) \text{ (Fig. 2.25a)}$$

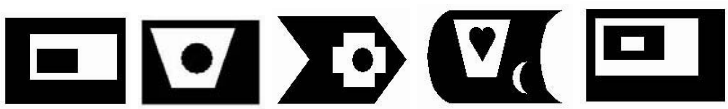
$$A^1\left[L_R^4\right]\left(A^1\left[L_T^4\right](K)\right) \text{ (Fig. 2.25b)}$$

$$A[Q^1[L^5](L^3)](A[Q^4[L^8](4L^3)](K)) \text{ (Fig. 2.25c)}$$

$A^2[Q^1[M^2[L^4]](M^1)](A[L_T^4](Q^1[M^2[L^5]](Q^2[L^3](2M^1))), Q^1[M^1](M^1))$ (Fig. 2.25d). Example of exemplar generated from the cyclic classes at the third level of iteration, whose symbolic name is given as follows $A^1\left[L_R^4\right]\left(A^1\left[L_R^4\right]\left(A^1\left[L_R^4\right]\left(L_R^4\right)\right)\right)$, is shown in Fig. 2.25e. The symbolic name $A^1\left[L_R^4\right]\left(A^1\left[L_R^4\right]\left(L_R^4\right)\right)$ denotes an exemplar generated from the class whose generic class is the convex polygon class L^4 (rectangle) and it has one hole. The hole is an archetype of the cyclic class $A^1\left[L_R^4\right]\left(L_R^4\right)$. The generic class of the hole is the convex polygon class L^4 (rectangle). The hole is an archetype of the rectangle class L^4.

2.1.1.5. Complex Cyclic Class

Archetypes of the complex cyclic class $C(\Theta)$ are obtained as the result of the certain type of topological operation called a complex addition [19], [20]. The complex class is denoted as $C(\Gamma_1, \Gamma_2, \ldots, \Gamma_N)$, where $\Gamma_1, \Gamma_2, \ldots, \Gamma_N$ are classes of the addition operation. In the case when $N = 2$, the complex class is reduced into the class of the two-element operation and denoted as $C(\Gamma_1, \Gamma_2)$. In the case when $\Gamma_1 \equiv \Lambda$, the class is called the complex convex class. In the case when $\Gamma_1 \equiv A$, the class is called the complex cyclic class. Archetype of the complex cyclic class consists of parts, where one of the parts needs to be an archetype of the cyclic class. Examples of the archetypes of the complex cyclic class are given in Fig. 2.26. Symbol $C\left(A^1\left[L_R^4\right](L^4), L^3\right)$ (see Fig. 2.26a) denotes that archetype of the complex class C consists of two parts, one archetype



a b c d e

Fig. 2.25. Exemplars of the complex cyclic class given by symbolic names

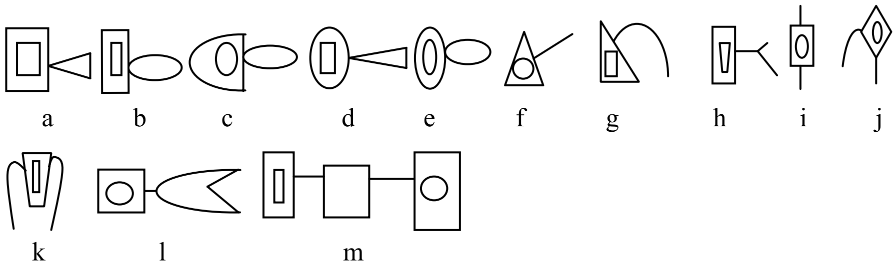


Fig. 2.26. Archetypes of the complex cyclic class

of the cyclic class $A^1\left[L_R^4\right](L^4)$ and the second archetype of the convex class L^3. In the case when $\Gamma_2 \equiv \Theta$, the complex class is defined by the point-sum operation and the class is called the complex convex thin class. Examples of the archetypes of the complex cyclic-thin class are given in Fig. 2.26. Symbol $C(A^1[L^3](K),\Theta)$ (see Fig. 2.26f) denotes that an archetype of the complex class C consists of two parts, one archetype of cyclic class $A^1[L^3](K)$ and the second archetype of the thin class Θ.

Examples of archetypes generated from the complex cyclic-thin class given by symbolic names are shown in Fig. 2.26: $C\left(A^1\left[L_R^4\right](L^4),L^3\right)$ (Fig. 2.26a), $C\left(A^1\left[L_R^4\right]\left(L_R^4\right),K\right)$ (Fig. 2.26b), $C(A^1[M](K),K)$ (Fig. 2.26c), $C\left(A^1[K]\left(L_R^4\right),L^3\right)$ (Fig. 2.26d), $C(A^1[K](K),K)$ (Fig. 2.26e), $C(A^1[L^3](K),\Theta)$ (Fig. 2.26f), $C(A^1[L^3](L^4),\tilde{\Theta})$ (Fig. 2.26g), $C\left(A^1\left[L_R^4\right]\left(L_T^4\right),\Theta^3[L^3]\right)$ (Fig. 2.26h), $C\left(A^1\left[L_R^4\right](K),2\Theta\right)$ (Fig. 2.26i), $C\left(A^1\left[L_O^4\right](K),\Theta\tilde{\Theta}\right)$ (Fig. 2.26j), $C\left(A^1\left[L_T^4\right](L^4),2\tilde{\Theta}\right)$ (Fig. 2.26k), $C\left(A^1\left[L_R^4\right](K),\Theta,Q^1[M^1](L^3)\right)$ (Fig. 2.26l), and $C\left(A^1\left[L_R^4\right]\left(L_R^4\right),\Theta,L_R^4,\Theta,A^1\left[L_R^4\right](K)\right)$ (Fig. 2.26m).

2.1.1.6. Cyclic Thin Class: The G-Class

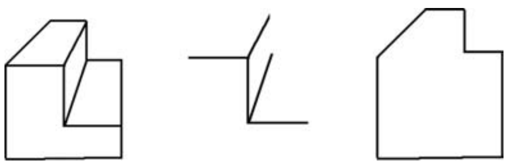
The archetype of the cyclic class $A_\Lambda^1(\Gamma)$ for which the type of the hole and the generic acyclic class is equal and area of the hole is close to the area of the archetype of the generic acyclic class is called the archetype of the cyclic-thin class $A_\Lambda^1(\Gamma) \equiv \rho[\Lambda]\{\Gamma\}$. Examples of exemplars generated from classes given by the symbolic names: (a) $\rho\left[L_R^4\right]\left\{L_R^4\right\}$,

a b c d

Fig. 2.27. Examples of exemplars of the cyclic-thin class

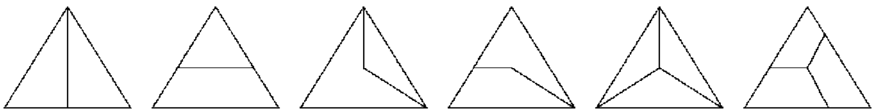
(b) $\rho\left[L_R^4\right]\left\{3L_R^4\right\}$, (c) $\rho\left[L_R^4\right]\left\{3L_R^3, L_R^4, L_T^4\right\}$, and (d) $\rho\left[K_E^1\right]\{2M^1\}$ are shown in Fig. 2.27a–d. The symbolic name $\rho\left[L_R^4\right]\left\{L_R^4\right\}$ denotes an exemplar generated from the class whose generic class is the convex polygon class L^4 (rectangle) and it has one hole. The hole is an archetype of the rectangle class L^4. The symbol ρ denotes that the exemplar is generated from the acyclic class whose area of the hole is close to the area of the archetype of the generic acyclic class.

The archetype of the cyclic-thin class $\rho[\Lambda]\{\Gamma\}$ can be represented by notation of the G-class. In this notation the object is decomposed into the core object and the thin object. Example of this decomposition is shown in Fig. 2.28. Figure 2.28 shows archetypes from the cyclic-thin class $\rho[Q^1[L^6](L^3)]\left\{Q[L^5]\left(L_R^3\right), 3L_T^4\right\}$ that are decomposed according to the convention of the G-class. The archetype in Fig. 2.28a given by the symbolic name $G[Q[L^6](L^3)]\{\Theta^{2(2)2}[Q[L^4](L^3)]\}$ is decomposed into the thin object $\Theta^{2(2)2}[Q[L^4](L^3)]$ (Fig. 2.28b) and the concave core object $Q[L^6](L^3)$ (Fig. 2.28c). This archetype is represented as a member of the cyclic-thin class $\rho[Q^1[L^6](L^3)]\left\{Q[L^5]\left(L_R^3\right), 3L_T^4\right\}$ and is decomposed into the concave core object $Q[L^6](L^3)$ and four objects: one concave $Q[L^5](L_R^3)$ and the three convex L_T^4.



a b c

Fig. 2.28. Archetypes of the cyclic-thin class



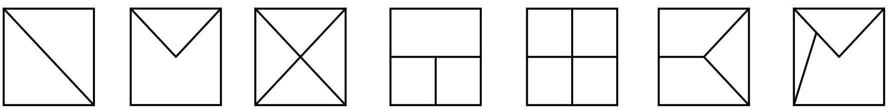
a b c d e f

Fig. 2.29. Archetypes of the class triangle convex thin class

2.1.1.6.1. Convex Cyclic Thin G-Class

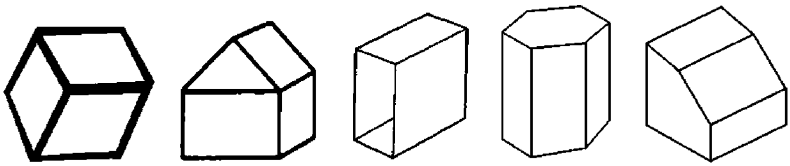
Archetype of the convex cyclic-thin class can be decomposed into the core convex object Λ and holes. Figure 2.29 shows archetypes of the convex cyclic-thin class whose the core convex object is the member of the convex triangle class L^3. Archetypes shown in Fig. 2.29 are represented by the following symbolic names: $\rho[L^3]\left\{2L_R^3\right\}$ (Fig. 2.29a), $\rho[L^3]\{L^3, L^4\}$ (Fig. 2.29b), $\rho[L^3]\{Q[L^3](L^3), L^3\}$ (Fig. 2.29c), $\rho[L^3]\{L^4, Q[L^3](L^3)\}$ (Fig. 2.29d), $\rho[L^3]\{3L^3\}$ (Fig. 2.29e), and $\rho[L^3]\{3L^4\}$ (Fig. 2.29f). Figure 2.30 shows archetypes of the convex cyclic-thin class whose the core convex object is the member of the convex rectangle class L_R^4. Archetypes shown in Fig. 2.30 are represented by the following symbolic names: $\rho\left[L_R^4\right]\left\{2L_R^3\right\}$ (Fig. 2.30a), $\rho\left[L_R^4\right]\left\{L^3, Q\left[L_R^4\right](L^3)\right\}$ (Fig. 2.30b), $\rho\left[L_R^4\right]\left\{4L_R^3\right\}$ (Fig. 2.30c), $\rho\left[L_R^4\right]\left\{2L_Q^4, L_R^4\right\}$ (Fig. 2.30d), $\rho\left[L_R^4\right]\left\{4\cdot L_R^4\right\}$ (Fig. 2.30e), $\rho\left[L_R^4\right]\left\{2L_T^4, L^3\right\}$ (Fig. 2.30f), and $\rho\left[L_R^4\right]\left\{2L^3, Q^1[L^4](L^3)\right\}$ (Fig. 2.30g).

Figure 2.31 shows the archetype of the convex cyclic class. The core convex object of this archetype is the member of the convex class L^n. The symbolic names for these objects are given in the form of the convex thin class $\rho\{L^n\}\{kL^m\}$ and the G-class $G\{L^n\}\{\Theta\}$. The symbolic names are as follows: $\rho[L^6]\left\{3L_O^4\right\}$, $G[L^6]\{\Theta^3[L^3]\}$ (Fig. 2.31a),



a b c d e f g

Fig. 2.30. Archetypes of the convex rectangle cyclic-thin class



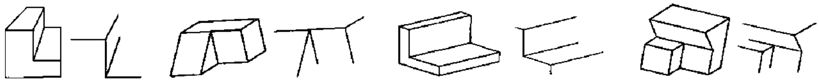
a b c d e

Fig. 2.31. Archetypes of the class $G\{L^n\}\{\Theta\}$

$\rho[L^{6-7}]\left\{2L_O^4, L^3, L_O^4\right\}$, $G[L^{6-7}]\{\Theta^4[L^4]\}$ (Fig. 2.31b), $\rho[L^4]\left\{2L_O^4, L^3, L^4\right\}$, $G[L^4]\{\Theta^{2(2)2}[L^4]\}$ (Fig. 2.31c), $\rho[L^8]\left\{L^6, 3L_O^4\right\}$, $G[L^8]\{\Theta^{2(2)2}[L^4]\}$ (Fig. 2.31d), and $\rho[L^7]\left\{L^5, 3L_O^4\right\}$, $G[L^7]\{\Theta^{2(2)2}[L^4]\}$ (Fig. 2.31e). The symbol L^{6-7} denotes that the archetype is the member of the class L^6 or L^7.

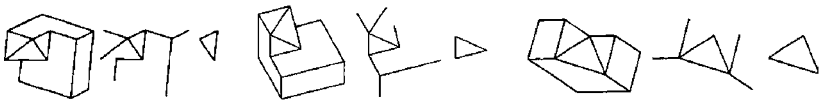
2.1.1.6.2. Concave Cyclic Thin G-Class

Archetype of the concave cyclic-thin class can be decomposed into the core concave object Q and holes. Following the notation of the G-class the archetype is decomposed into the concave core object Q and the thin objects Θ or the complex thin objects $C(\Theta)$. Figure 2.32 shows the archetypes of the concave cyclic-thin class that are decomposed into the core concave object and the thin object Θ. Archetypes are represented by the symbolic names as follows: $\rho[Q[L^6](L^3)]\left\{3L_T^4, Q_{L^5}\left(L_R^3\right)\right\}$, $G[Q[L^6](L^3)]\left\{\Theta^{2(2)2}\left[Q_{L^4}(L^3)\right]\right\}$ (Fig. 2.32a), $\rho\,[Q[L^7](L^3)]\left\{4L_o^4, L^3\right\}$, $G\left[Q_{L^7}(L^3)\right]\left\{\Theta^{3(2)2}\left[Q_{L^4}(L^3)\right]\right\}$ (Fig. 2.32b), $\rho\left[Q_{L^{6-8}}(L^3)\right]\left\{2L_o^4, \hat{L}_o^4, Q_{L^5}(L^3)\right\}$, $G\left[Q_{L^{6-8}}(L^3)\right]\left\{\Theta^{2(2)1(2)2}\left[Q_{L^5}(L^3)\right]\right\}$ (Fig. 2.32c), and $\rho\left[Q_{L^{6-8}}^2(2L^3)\right]\left\{\left((L^6)3L_O^4\right), 3L_O^4, Q_{L^4}(L^3)\right\}$, $G\left[Q_{L^{6-8}}^2(2L^3)\right]\left\{\Theta^{2(2)2(2)2(2)2}\left[Q_{L^5}^2(2L^3)\right]\right\}$ (Fig. 2.32d). Figure 2.33 shows archetypes of the concave cyclic-thin class that are decomposed into the core concave object and the complex thin objects $C(\Theta)$. Archetypes are represented by the symbolic names as follows: $G\left[Q_{L^6}(L^3)\right]\left\{C\left(L^3, 3\Theta^2, \Theta_{L^3}^3\right)\right\}$ (Fig. 2.33a), $G\left[Q_{L^7}(L^3)\right]\left\{C\left(L^3, 3\Theta^2, \Theta_{L^3}^3\right)\right\}$ (Fig. 2.33b), and $G\left[Q_{L^{6-7}}(L^3)\right]\{C(L^3, 4\Theta^2)\}$ (Fig. 2.33c).



a b c d

Fig. 2.32. Archetypes of the thin concave G-class $G\{Q\}\{\Theta\}$



a b c

Fig. 2.33. Archetypes of the thin concave G-class $G\{Q\}\{C(\Theta)\}$

2.1.1.7. Colored Classes

The class for which an archetype can be seen as consisting of adjacent regions of the different uniform colors is called the colored class $\aleph$. An archetype of the colored class $\aleph$ can be decomposed into the regions of the different colors and assigned to one of the specific classes. The decomposition of the archetype is shown in Fig. 2.34.

The colored class is the class archetypes of which have their parts marked by the different colors. The description of the convex colored class can be reduced into the description of the cyclic class $A^n[\Im_\Phi](n\Im_A)$. The archetype of the colored class $\aleph^2\left[L_T^4(g)\right]\left(L_R^4(y), L_T^4(b)\right)$ is shown in Fig. 2.35. The symbol $\aleph^2\left[L_T^4(g)\right]\left(L_R^4(y), L_T^4(b)\right)$ denotes that the convex

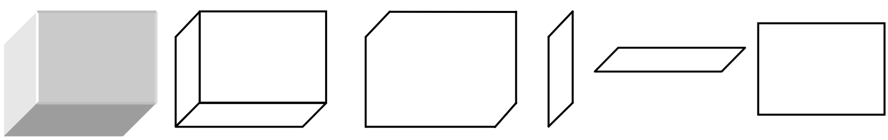


Fig. 2.34. Decomposition of the archetype consisting of adjacent regions of the different uniform colors

Fig. 2.35. Archetype of the convex colored class $\aleph^2\left\lceil L_T^4(g)\right\rceil\left(L_R^4(y), L_T^4(b)\right)$

a b

Fig. 2.36. Archetypes of the concave complex colored class

colored class $\aleph$, whose generic class (the convex polygon class – quadrilateral $L_T^4(g)$ – color green) called background, has two regions of the different colors. Both regions are archetypes of the quadrilateral class L^4. The first region $L_R^4(y)$ is marked by the letter (y) denoting the yellow color whereas the second region $L_T^4(b)$ is marked by the letter (b) denoting the blue color.

The archetype of the concave complex colored class can be decomposed into the parts of the different colors. The concave complex colored class is denoted as $\wp(\Gamma_1, \Gamma_2, \ldots, \Gamma_N)$, where $\Gamma_1, \Gamma_2, \ldots, \Gamma_N$ are general classes of shape. Archetypes of the concave complex colored class are given in Fig. 2.36. The symbol $\wp\left(L_R^4(y), Q\left[L_R^4\right]\left(L_R^4\right)(o)\right)$ denotes that the archetype of the concave complex colored class $\wp$, can be decomposed into two regions – the convex polygon class (rectangle) L_R^4 and the concave polygon (rectangle with the one concavity) $Q\left[L_R^4\right]\left(L_R^4\right)$. The archetype shown in Fig. 2.36b is represented by the symbolic name $\wp\left(\aleph[L^4(g)]\left(L_T^4(b)\right), \aleph\left[L_R^4(y)\right](K^1(r))\right)$.

2.1.2. The a Posteriori Classes

The shape classes described in the previous chapters were established based on the geometrical properties of the figure. The derivation of the specific classes was based on constraining the values of selected attributes of the general classes. These classes are called a priori classes because derivation of the specific class is based on geometrical properties of archetypes generated from the selected class. In this book shape is interpreted as the basic perceptual category to which the perceived object is fitted. Shape as the perceptual category is used during the learning of the visual concept of the different ontological categories such as a letter, a sign, or a real-world object. During categorical learning the specific shape classes that are good representative of the shape of the given ontological category need to be derived from the existing a priori classes. The classes of shape that are derived as the result of "specialization" of the existing a priori classes are called the a posteriori classes.

2.1.2.1. The Star Class

As it was described in Sect. 2.1.2 the a priori classes are established based on the geometrical properties of the visual object. The a posteriori classes are derived from the a priori classes based on the specialization of the selected shape classes. Specialization means that the a posteriori classes are established to match shape of the sign or the real-world object. Example of the class that is established based on the existing meaningful objects called a sign is the star class. The star class is defined based on generalization of the most often used visual representations of the star signs. The star class is a class derived from the concave class $Q^n[\Lambda](n\Lambda)$, where n>2. The polygon star class is a class derived from the concave polygon class and is given by the symbolic name $Q^n[L^n](nL^3)$. The curvilinear star class is a class derived from the concave class where all residuals are archetypes of the curve-polygon class $Q^n[L^n](nM)$. The concave star class is a class derived from the concave class where all residuals are archetypes of the concave class $Q^n[L^n](nQ)$. The concave I-star class is a class derived from the concave class where all residuals are archetypes of the concave class, residuals of which are archetypes of the concave class $Q^n[L^n](nQ(mQ))$. In similar way the concave II-star class $Q^n[L^n](nQ(mQ(kQ)))$ or the concave III-star class $Q^n[L^n](nQ(mQ(kQ(hQ))))$ can be defined.

The concave polygon star class is a class derived from the concave polygon class where all residuals are archetypes of the concave polygon class $Q^n[L^n](nQ^h[L^k](L^l))$, where indexes h, k, and l denote: h-the number of residuals, the generic k-polygon, and the residual l-polygon. The concave polygon I-star class is a class derived from the concave polygon class where all residuals are archetypes of the concave polygon class, residuals of which are archetypes of the concave polygon class $Q^n[L^n](nQ^h[L^k](mQ^b[L^c](L^d)))$.

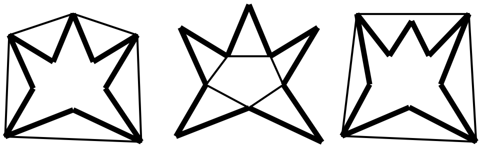
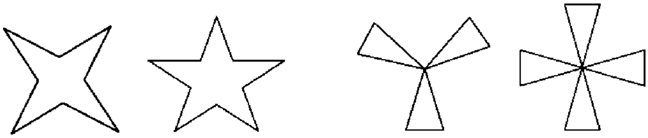


Fig. 2.37. Explanation of the different notations of the star class

The star class can be described by using the notation of the complex-core class. Objects shown in Fig. 2.37 explain the differences in the description of the object in terms of the concave class $Q^5[L^5](5L^3)$ (Fig. 2.37a) and the complex-core class $\Delta^5[L^5]\{L^5\}(5L^3)$ (Fig. 2.37b). The symbol in the bracket "[ ]" denotes the generic polygon, for example, $[L^5]$ (see Fig. 2.37a), whereas the symbol in the bracket "{ }" denotes the core of the archetype of the complex class, for example, $\{L^5\}$ (see Fig. 2.37b). The advantage of the second approach is such that the object is interpreted as an object having the "arms." Based on this interpretation we can establish the proper similarity relations among objects. For example, the objects from Fig. 2.37b, c that look very similar, have the same complex-core class description given by the symbolic name $\Delta^5[L^5]\{L^5\}(5L^3)$ but the different convex class description given by the symbolic names $Q^5[L^5](5L^3)$ (Fig. 2.37b) or $Q^4[L^4](Q^1[L^4],3L^3)$ (Fig. 2.37c). It seems that the complex-core class description is more perceptually oriented.

The archetype generated from the n-star class is represented by the symbolic name $Q^n[L^n](nL^3)$, where the $2n$-star class is a class derived from the concave polygon class $Q^n[L^{2n}](nL^3)$. The class $R^n[L^{2n}](nL^3)$ derived from the class $Q^n[L^{2n}](nL^3)$, where all residuals have the common point, can be given by notation of the complex class $S^n[L^{2n}](nL^3)$. In the case when there is no common point the class will have description $S^n[L^{2n}]\{L^h\}(nL^k)$. By generalization, the class $S^n[L^{2n}](nL^3)$ can be extended to the c-class $S^n_\Gamma\{n\Im\}$, where $\Im$ is a general class. Examples of the n-star class are given in Fig. 2.38a–b, and the $2n$-star class in Fig. 2.38c–d.

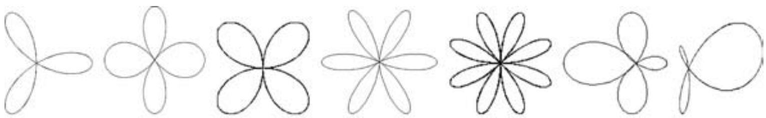


a b c d

Fig. 2.38. Simple I-star classes (**a**, **b**) and simple II-star classes (**c**, **d**)

The curvilinear star class $Q^n[A](nQ^2[L^3](2M^1))$, where A denotes one of the classes $A=\{L^n,\widehat{L}^n,M^n(L^{2n})\}$, is the specific class derived from the concave star class. In the case where all residuals have the common point the class can be given by the complex class description. The complex class description interpret the object in terms of petals (the parts of the object that are "glued" in one point) and described by complex class description as the *c*-complex class $S^n[\Gamma]\{n\Im\}$. When all petals are archetypes of the curve class $\Im\equiv K$ the class is the regular curve c-class $S^n[A]\{nK\}$. In the case where *n* is big enough *n*>*M*, the generic class becomes the polygon class and the regular curve *c*-class is given as $S^n[L^n]\{nK\}$. In the case where petals are different (members of the archetypes of the different curve classes) $\Im\equiv K_i$, the c-class is called the nonregular curve c-class and is given as $S^n[A]\{n*K\}$. Examples of archetypes of the regular curve c-class are shown in Fig. 2.39a–e and archetypes of the nonregular curve c-class are shown in Fig. 2.39f–g.

The c-class $S^n[\Gamma](n\Im)$, for which all petals are archetypes of the thin class $\Im\equiv\Theta$, is the regular thin c-class $S^n[L^n](n\Theta)$. In the case where $\Im\equiv\Theta^2$, the class is reduced to the thin star class $\oplus^n[L^n]$. In the case when the thin star class has different sizes of the "rays" $S^n[L^m](n\Theta), m<n$ the class is the thin para-star class $\oplus^n[L^m]$. Example of the archetype



a b c d e f g

Fig. 2.39. Examples of archetypes of the regular curve c-class and the nonregular curve c-class

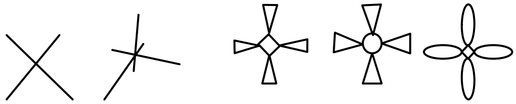


Fig. 2.40. Archetypes of the thin star class, exemplar of the concave c-class

generated from the thin star class $\oplus^4[L^4]$ is shown in Fig. 2.40a and the archetype generated from the thin para-star class $\oplus^6[L^4]$ is shown in Fig. 2.40b.

The complex star point class is the class that has the nucleus and petals that are joined in one point with nucleus. This class is given by the notation of the complex class $S^n[\Gamma]\{\Gamma\}(n\Im)$. The complex polygon star point class is the class whose nucleus and petals are polygons. The complex polygon star point class is given by the symbolic name $S^n[L^h]\{L^k\}(nL^m)$. Examples of the archetypes of the complex star point class are given in Fig. 2.40c $S^4[L^8]\{L^4\}(4L^3)$, Fig. 2.40d $S^4[L^8]\left\{K_C^1\right\}(4L^3)$, and Fig. 2.40e $S^4[\widehat{L}^4]\{L^4\}\left(4K_E^1\right)$.

2.1.2.2. The Spade Class

The spade class is the a posteriori class that is established based on the properties of the real-world object. The spade class denoted as $('S')C(\Gamma,\Theta)$ is derived from the complex symmetrical thin class $C(\Gamma,\Theta)$ archetypes of which consist of two parts, one called the blade and the other one called handle. The handle is a member of the thin class Θ. The members of the a posteriori spade class are used as the structural archetypes of the real-world object called spade. The spade class $('S')C(\Gamma,\Theta^2)$ is the class archetypes of which are obtained by joining the straight line with the object called the core that is a member of one of the classes: the convex, the concave, or the cyclic in such a way that the straight line has one common point with one of the sides of the core and the whole figure is symmetrical. Examples of the spade class are shown in Fig. 2.41a $('S')C\left(L_R^4,\Theta^2\right)$, Fig. 2.41b–c $('S')C\left(L_T^4,\Theta^2\right)$, Fig. 2.41d $('S')C\left(L_T^5,\Theta^2\right)$, Fig. 2.41e $('S')C\left(L_M^5,\Theta^2\right)$, Fig. 2.41f $('S')C(L^6,\Theta^2)$, Fig. 2.41g $('S')C\left(M_{L^4}^1,\Theta^2\right)$, Fig. 2.41h $('S')C\left(K_C^1,\Theta^2\right)$, Fig. 2.41i

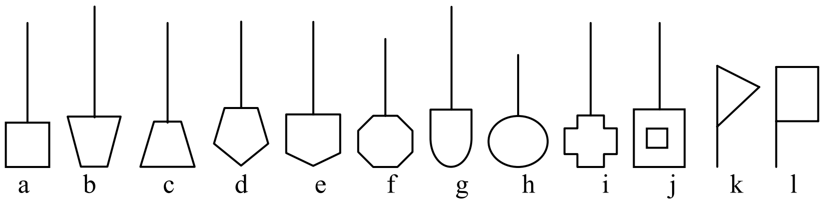


Fig. 2.41. Archetypes of the "spade" class (**a–j**) and archetypes at the classes similar to the spade class (**k–l**)

$('S')C(Q^4[L^8](4 \cdot L^3), \Theta^2)$, and Fig. 2.41j $('S')C(A^1[L^4](L^4), \Theta^2)$. The notation of the spade class can be expressed in the form of the Θ/ρ class. For example, the symbolic name of the archetype in Fig. 2.41a is $\Theta/\rho\left[[L^5]\left\{2 \cdot L_R^3, L_R^4\right\}\right]$, the archetype in Fig. 2.41b is $\Theta/\rho\left[[L^5]\left\{2 \cdot L_R^3, L_T^4\right\}\right]$, and the archetype in Fig. 2.41c is $\Theta/\rho\left[[L^5]\left\{2 \cdot Q[L^3](L^3), L_T^4\right\}\right]$. This notation makes it possible to find the difference between the archetype shown in Fig. 2.41b and the archetype shown in Fig. 2.41c. The archetypes shown in Fig. 2.41k $\Theta/\rho[[L^3]\{2L^3\}]$ and in Fig. 2.41l $\Theta/\rho\left[[L^4]\left\{L_R^3, L_R^4\right\}\right]$ are similar to archetypes shown in Fig. 2.41a–j and are not members of the spade class.

The a posteriori T-spade class is derived from the complex thin class (spade class). The archetype of the T-spade class $('S')C\left(\Gamma, (s)\otimes_1^3\right)$ instead of the handle that is a member of the thin straight class has the handle that is a member of the *s*-star class $(s)\otimes_1^3$. The s-star class $(s)\otimes_1^3$ is the thin star class whose archetypes are symmetrical and have one 1 branch that is significantly longer from other branches. Examples of archetypes from the s-star are shown in Fig. 2.42a–c. Archetypes shown in Fig. 2.42a, c have the symbolic name $(s)\otimes_1^3[L^3]$, whereas the archetype in Fig. 2.42b has the symbolic name $(s)\otimes_1^3[L^4]$. Examples of the archetypes from the T-spade class are shown in Fig. 2.42d–f. The symbolic names of archetypes from the T-spade class shown in Fig. 2.42 are as follows: $('S')C\left(L_T^4, (s)\otimes_1^3[L^3]\right)$ (Fig. 2.42d), $('S')C\left(L_R^4, (s)\otimes_1^3[L^3]\right)$ (Fig. 2.42e), and $('S')C\left(L_R^4, (s)\otimes_1^3[L^4]\right)$ (Fig. 2.42f).

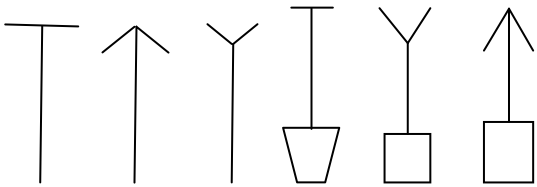


Fig. 2.42. Archetypes of the T-spade class

The a posteriori C-spade class is derived from the complex thin class (spade class). The archetype of the C-spade class consists of three parts: the blade, the handle, and the small handle. The C-spade class $('S')C(\Gamma,\Theta^2,(\varepsilon)\Lambda)$ is the class archetypes of which are obtained by joining the straight line with the object called the core and the other object called the small handle in such a way that the whole object is symmetrical. The core can be a member of the convex, the concave, or the cyclic classes. Examples of the C-spade class are shown in Fig. 2.43. The symbolic names of the archetypes shown are as follows: $('S')C\left(L_R^4,\Theta^2,(\varepsilon)L^3\right)$ (Fig. 2.43a), $('S')C\left(L_R^4,\Theta^2,(\varepsilon)M^1\right)$ (Fig. 2.43b), and $('S')C\left(L_R^4,\Theta^2,(\varepsilon)K^1\right)$ (Fig. 2.43c).

Similarly like archetypes of the spade class, the archetypes of the R-spade class are members of the complex symmetrical classes $('S')C(\Gamma,\overset{+}{\Gamma})$ consisting of two parts: one called the blade and the other one called the handle. The handle is a member of the elongated class $\overset{+}{\Gamma}$, whereas the blade is a member of one of the classes: the convex class, the concave class, or the cyclic class. In the case when both the handle and the blade are members of the convex class we have convex R-spade class

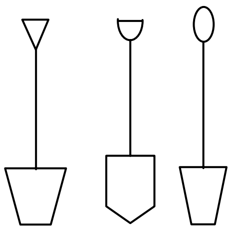
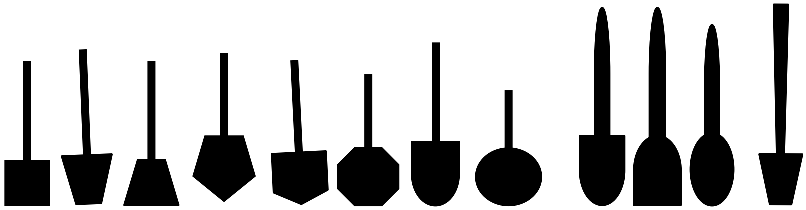


Fig. 2.43. Examples of archetypes generated from the C-spade class

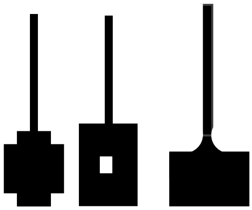


a b c d e f g h i j k l

Fig. 2.44. Exemplars generated from the convex R-spade class

$('S')C(\Lambda,\overset{+}{\Lambda})$. Examples of the convex R-spade class are shown in Fig. 2.44. The symbolic names of exemplars shown in Fig. 2.44 are as follows: $('S')C\left(L_R^4,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44a), $('S')C\left(L_T^4,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44b, c), $('S')C\left(L_T^5,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44d), $('S')C\left(L_M^5,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44e), $('S')C\left(L^6,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44f), $('S')C\left(M_{L^4}^1,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44g), $('S')C\left(K_C^1,\overset{+}{L}{}_R^4\right)$ (Fig. 2.44h), $('S')C(M^1,\overset{\times}{M}{}^1)$ (Fig. 2.44i, j), $('S')C\left(K_E^1,\overset{\times}{M}{}^1\right)$ (Fig. 2.44k), and $('S')C\left(L_T^4,\overset{+}{L}{}_T^4\right)$ (Fig. 2.44l). The notation of the convex R-spade class can be expressed in the notation of the concave class. For example, the exemplar generated from the R-spade class shown in Fig. 2.44a–b has its symbolic name $Q^2[\widehat{L}^5]\left(2\cdot L_R^3\right)$, where symbol $\widehat{L}^5$ denotes an archetype with a small side.

In the case when the handle is a member of the convex class and the blade is a member of the concave class, we have Q-spade class $('S')C(Q,\overset{+}{\Lambda})$. In the case when the handle is a member of the convex class and the blade is a member of the cyclic class we have the A-spade class $('S')C(A,\overset{+}{\Lambda})$. In the case when both the handle and the blade are members of the concave class we have the Q-q-spade class $('S')C(Q,\overset{+}{Q})$. In the case when both the handle and the blade are members of the cyclic



a b c

Fig. 2.45. Exemplars generated from (**a**) the Q-spade class, (**b**) the A-spade class, (**c**) q-spade class

class we have the A-a-spade class $('S')C(A,\overset{+}{A})$. Example of exemplar from the Q-spade class $('S')C\left(Q^4[L^8](4L^3),\overset{+}{L}{}_R^4\right)$ is shown in Fig. 2.45a, example of exemplar from the A-spade class $('S')C\left(A^1[L^4](L^4),\overset{+}{L}{}_R^4\right)$ is shown in Fig. 2.45b, and exemplar from the q-spade class $('S')C\left(L^4,Q^2[\overset{+}{L}{}_T^4](2M^1)\right)$ is shown in Fig. 2.45c.

The spade-pike class is derived from the complex class $C(\Gamma,\overset{\times}{\Gamma})$, where $\overset{\times}{\Gamma}$ is the elongated pike class. The archetypes of the spade-convex pike class are complex classes $('\hat{S}')C(\Gamma,\overset{\times}{\Lambda})$ consisting of two parts, where one part called the handle is a member of the convex elongated pike class $\overset{\times}{\Lambda}$. The convex elongated pike class $\overset{\times}{\Lambda}$ consists of archetypes that have at least one sharp corner. Examples of exemplars generated from the convex elongated pike class are shown in Fig. 2.46. Symbolic names of exemplars shown in Fig. 2.46 are as follows: $\overset{\times}{L}{}^3$ (Fig. 2.46a), $\overset{\times}{M}{}^1$ (Fig. 2.46b), and $\overset{\times}{K}{}^1$ (Fig. 2.46c).

a b c

Fig. 2.46. Archetypes of convex elongated pike class Λ_E

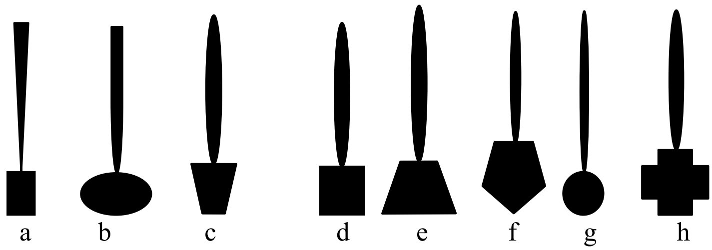


Fig. 2.47. Archetypes of the spade-pike class

Examples of exemplars generated from the spade-convex pike class are shown in Fig. 2.47. Symbolic names of exemplars shown in Fig. 2.47 are as follows: $('\hat{S}')C\left(L_R^4,\overset{\times}{L^3}\right)$ (Fig. 2.47a), $('\hat{S}')C\left(K_E^1,\overset{\times}{M^1}\right)$ (Fig. 2.47b), $('\hat{S}')C\left(L_T^4,\overset{\times}{K^1}\right)$ (Fig. 2.47c), $('\hat{S}')C\left(L_R^4,\overset{\times}{K_E^1}\right)$ (Fig. 2.47d), $('\hat{S}')C\left(L_T^4,\overset{\times}{K_E^1}\right)$ (Fig. 2.47e), $('\hat{S}')C\left(L^5,\overset{\times}{K_E^1}\right)$ (Fig. 2.47f), $('\hat{S}')C\left(K_E^1,\overset{\times}{K_E^1}\right)$ (Fig. 2.47g), and $('\hat{S}')C\left(Q^4[L^8](4L^3),\overset{\times}{K_E^1}\right)$ (Fig. 2.47h).

2.1.2.3. The Letter Class

The a posteriori classes described in this section are derived based on the specialization of the a priori shape classes that means these classes are established to match shape of the letter. In this section the class that is derived from the thin class, which is established based on the properties of the letters, is presented. The letter class is defined based on generalization of the most often used visual representation of the letters. The archetypes of this class represent the structural archetype of the letter.

To represent a letter, the descriptions of the specific classes need to include the specific parameters that refer to the straightness of the segments, the length of the segment, the angle between segments, type of thinness, as well as the orientation of the object. The attributes such as the length are expressed by applying the graded values: $a_i^d \in \{\varepsilon, s, m, L\}$, where ε denotes a "very small," s denotes a "small," m denotes a "medium," and L denotes a "large" value. The attribute such as the angle can be expressed by applying the graded values: $a_i^\alpha \in \{\varepsilon, R, O, A\}$, where ε denotes a "very small," R denotes a "right," O denotes an "obtuse," and A denotes an “acute” angle. The orientation of the object is expressed by a selected type of the letter and a type of the transformation M – a mirror transformation and a rotation O^R in a clockwise direction by the angle $a_i^\alpha \in \{R, O, A\}$. Figure 2.48 shows archetypes of the specific class Θ^2. The letters “L,” “Γ,” “Λ,” “V,” “J,” “˥,” “˥,” “ك” and the mathematical symbols $<$, $\vee$, $\rangle$, $\angle$, $\neg$ are described by the symbolic names of the specific thin class shown in Figs. 2.48 and 2.49. For example, the letter “L” is given by the symbolic name $\otimes\left[L_R^3\right][l,s]$ or by adding the letter “L” in bracket “[ ]” into the name of the class $["L"]\otimes\left[L_R^3\right]$.

The symbolic names of the letter classes show similarities of the objects from these classes. This property of the symbolic name is used in process of generalization (abstraction). Archetypes shown in Fig. 2.48 are represented by the symbolic names as follows $\otimes\left[L_R^3\right][l,s]$ (Fig. 2.48a), $\otimes\left[L_R^3\right][l,m]$ (Fig. 2.48b), $\otimes\left[L_R^3\right][m,m]$ (Fig. 2.48c), $\otimes\left[L_O^3\right][m,m]$ (Fig. 2.48d), and $\otimes\left[L_A^3\right][m,m]$ (Fig. 2.48e). The generalization process shows that all objects shown in Fig. 2.49 are members of the class $\otimes[L^3]$. In order to find the proper archetype that matches a given letter the subspecific class that includes the spatial orientation of the object needs to be introduced. Figure 2.49 shows archetypes of the subspecific letter class that is established to differentiate among the different letters that are members of the same specific class $\otimes\left[L_R^3\right][l,s]$. The symbolic names of the subspecific classes are as follows: $\otimes\left[L_R^3\right][l,s]\{'L'\}$ (Fig. 2.49a), $\otimes\left[L_R^3\right][l,s]\{'L'(M)\}$ (Fig. 2.49b), $\otimes\left[L_R^3\right][l,s]\{'L'(MO^{2R})\}$ (Fig. 2.49c), $\otimes\left[L_R^3\right][l,s]\{'L'(O^{2R})\}$ (Fig. 2.49d), $\otimes\left[L_R^3\right][l,s]\{'L'(O^R)\}$ (Fig. 2.49e), $\otimes\left[L_R^3\right][l,s]\{'L'(MO^R)\}$ (Fig. 2.49f), $\otimes\left[L_R^3\right][l,s]\{'L'(MO^{3R})\}$ (Fig. 2.49g), and $\otimes\left[L_R^3\right][l,s]\{'L'(O^{3R})\}$ (Fig. 2.49h).

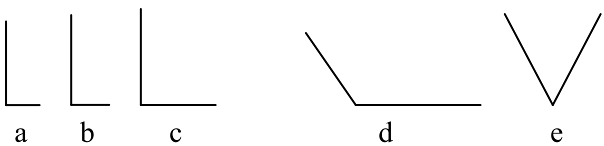
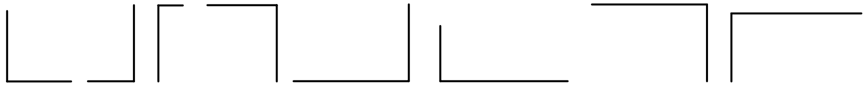


Fig. 2.48. Archetypes of the specific thin class Θ^2



a b c d e f g h

Fig. 2.49. Archetypes of the specific thin class

Understanding of the letter requires identifying the similar objects in order to be able to predict a new font or to recognize a letter that is subjected to one of many distortions. The shape classes convey information about the similarities between archetypes of the members of the different classes. For example, from the function class the specific classes are derived in order to represent the difference among letters that looks very similar. Figures 2.50 and 2.51 show examples of the archetypes of the convex function class. The concept of the function class is explained in the section "The Spade Class." The letters "V" and "U" and the mathematical symbols $<, \vee, \rangle, \angle, \neg, \succ, \cup, \bigcup, \subset, \prod, \coprod$ are described by the symbolic names of the symmetrical convex function class. Figure 2.50 shows archetypes of the subspecific letter class that represent symbols $<, \vee, \rangle, \angle, \neg, \cup, \bigcup, \subset$ and letters "V" and "U." The symbolic names of the subspecific classes for archetypes shown in Fig. 2.50 are as follows: $\otimes\left[M^1\left[K_O^1\right]\right]$ (Fig. 2.50a), $\otimes\left[L_A^3\right]$ (Fig. 2.50b), $\otimes\left[M^1\left[K_E^1\right]\right]$ (Fig. 2.50c), $\otimes\left[M^1\left[L_T^4\right]\right]$ (Fig. 2.50d), $\otimes\left[M^1\left[K_S^1\right]\right]$ (Fig. 2.50e), and $\otimes[M^1[K^2]]$ (Fig. 2.50f). Figure 2.51 shows archetypes of the subspecific letter class that represent symbols $\subset, \prod, \coprod$ and letter "U." The symbolic names of the subspecific classes for archetypes shown in Fig. 2.51 are as follows: $\otimes\left[L_R^4\right]$ (Fig. 2.51a), $\otimes\left[L_T^4\right]$ (Fig. 2.51b), $\otimes\left[L_T^4\right]$ (Fig. 2.51c), $\otimes[L^4]$ (Fig. 2.51d), $\otimes[M^1[L^3]]$ (Fig. 2.51e, f), $\otimes[M^1[L^4]]$ (Fig. 2.51g), and $\otimes[M^1[K^4]]$ (Fig. 2.51h). The mathematical symbol "$\subset$" is interpreted as the rotated version of the letter "U." Archetypes in Fig. 2.51b–d can be interpreted as the representatives of the distorted version of the symbols $\prod, \coprod$.

a b c d e f

Fig. 2.50. Archetypes of the symmetrical convex function class

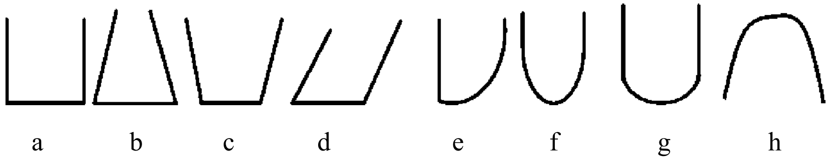


Fig. 2.51. Archetypes of the nonsymmetrical convex function class

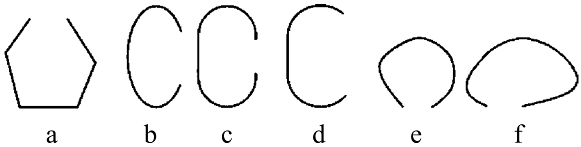


Fig. 2.52. Archetypes of the nonfunction class

Archetypes of the nonfunction class are shown in Fig. 2.52. The letter "U" can be described by the symbolic names of the nonfunction class $\otimes[L^6]$ (Fig. 2.52a). The letter "C" can be described by the symbolic names $\otimes\left[M^1\left[K_E^1\right]\right]$ (Fig. 2.52b), $\otimes[M^2[L^4]]$ (Fig. 2.52c, d), $\otimes[M^1[K^4]]$ (Fig. 2.52e), and $\otimes[M^1[K^3]]$ (Fig. 2.52f).

The letters "M" and "Σ" can have their curvilinear versions. The specific classes that can be used for description of these letters are derived from the thin polygon-curve class $\otimes$. Because there is a big range of shapes that can be used as representatives of the letters type M, the M-letter classes has to be established during learning process described in Chap. 5. In this section, examples of the archetypes from the selected M-letter classes are presented. The poly-line version of the letter type M is described in Chap. 5. The symbolic names of some of the possible curvilinear versions of the letters are given by the following notations: $\otimes[Q^1[L^4](M^1)]$ (Fig. 2.53a), $\otimes[Q^1[M^2[L^4]](M^1)]$ (Fig. 2.53b), $\otimes[Q^3[L^4](3M^1)]$ (Fig. 2.53c), $\otimes[Q^1[M^2[L^4]](Q^2[L^3](2M^1))]$ (Fig. 2.53d), $\otimes[Q^1[M^2[L^4]](Q^2[L^3](2M^1))]$ (Fig. 2.53e), $\otimes[Q^1[M^2[L^6]](Q^2[L^3](2M^1))]$ (Fig. 2.53f), and $\otimes\left[Q^1[M^2[L^6]]\left(Q^2[M^1\left[L_T^4\right]](2M^1)\right)\right]$ (Fig. 2.53g).

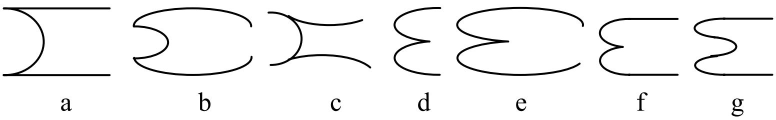


Fig. 2.53. Archetypes of the nonfunction classes

2.1.3. String Form: Type of the Class

Archetypes of the shape classes are described in the form of the symbolic names. For the purpose of the visual reasoning the symbolic name is transformed into the string form. The string consists of combination of the selected letters, numbers, and the symbol "|." The string has a following form: B1|…|Bi|...|Bn|, where Bi denotes the symbolic name of the class. There is a conversion from the notation of the symbolic name into the string notation. For example, the convex class L^3 is expressed as L3 in the string form.

The string notation is used to introduce the type of the class. The string without symbol "|" is denoted as the type P. It represents exemplars of the convex classes. For example, exemplars of the convex classes given in Fig. 2.54 (L3A, L4R, M1L3A, M1L4R, and M2L4R) are all of the type P.

Examples of exemplars that represent the different types of string forms are shown in Figs. 2.55–2.67. The type S that represents cyclic and concave classes, is given in the form Sn|A|1X|…|iX|…|nX|. The type Sq (the concave type) is given in the form Qn|G|1R|…|iR|…|nR|, whereas the type Sa (the cyclic type) is given as An|C|1W|…|iW|…|nW|. Examples of the exemplars type Sn|A|1X|…|iX|…|nX| are given in Figs. 2.55–2.57. The type S1|A|1_S1|1_A|1_X| and the type S1|A|1_S1|1_A|2_S1|2_A|2_X| both represent the exemplar *o* of the concave or cyclic classes on the first and the second level of iteration. The concave class $Q^4_{L^4}(4L^3)$ is expressed as Q4|L4|L3|L3|L3| in the string form. For example, an exemplar shown in Fig. 2.59a given as $A^1_{L^4_R}\left(A^1_{L^4_R}\left(A^1_{L^4_R}\left(L^4_R\right)\right)\right)$ is transformed into the string form as A1|L4R|1_A1|1_L4R|2_A1|2_L4R|2_L4R|.

Examples of the general type string forms Sn|A|1X|…|iX|…|nX| that generate the following patterns are as follows:

Q1|G|R|, A1|C|W|, Q2|G|1R|2R|, Q3|G|1R|2R|3R|, A3|C|1W|2W|3W|
A1|Q1|G|R|W|, A1|Q3|G|1R|2R|3R|W|, A2|Q1|G|R|1W|2W|
A1|Q1|G|1_Q1|1_G|R|W|, A1|Q2|G|1_Q1|1_G|1_R|R|W
A1|Q3|G|1_Q1|1_G|1_R|1R|2R|W

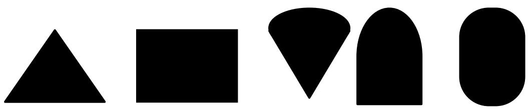


Fig. 2.54. Exemplars of the type P

Examples of general type string form S1|A|1_S1|1_A|1_X| that generates the following patterns are as follows:

Q1|G|1_Q1|1_G|R|, A1|C|1_A1|1_C|W|, Q2|G|R1|1_1Q1|1_1G|1_2R|.

Examples of the exemplars of the complex types are shown in Figs. 2.63 and 2.64.

Fig. 2.55. Exemplars of the type Q1|G|R|

Fig. 2.56. Exemplars of the type A1|G|W|

Fig. 2.57. Exemplars of the type Q2|G|1R|2R|

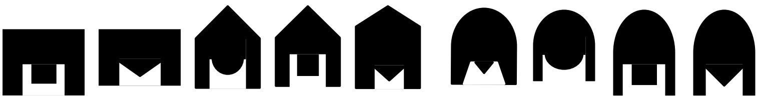


Fig. 2.58. Exemplars of the type Q1|G|1_Q1|1_G|R|

a b c

Fig. 2.59. Exemplars of the types A1|G|1_A1|1_G|W| and Q3|G|1R|2R|3R|

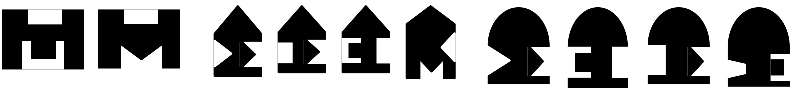


Fig. 2.60. Exemplars of the type Q2|G|R1|1_1Q1|1_1G|1_2R|

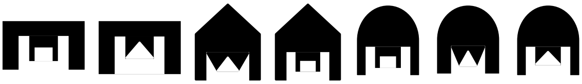


Fig. 2.61. Exemplars of the type Q1|G|1_Q1|1_G|2_Q1|2_G|2_R|

Fig. 2.62. Exemplars of the types Q3|G|1R|2R|1_1Q1|1_1G|1_R| and A1|Q2|G|1_Q1|1_G|1_R|R|W

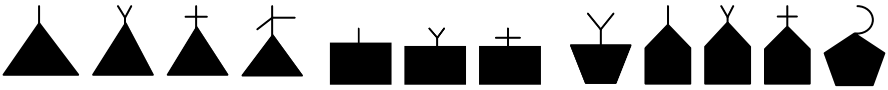


Fig. 2.63. Exemplars of the type C2|K|T|

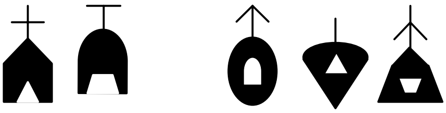


Fig. 2.64. Exemplars of the types C2|Q1|G|R|T| and C2|A1|G|W|T|

Fig. 2.65. Exemplars of the types A1|Q1|G|R|W| and A1|Q3|G|1R|2R|3R|W|

Fig. 2.66. Exemplars of the type A2|Q1|G|R|1W|2W|

Fig. 2.67. Exemplars of the types A1|Q1|G|1_Q1|1_G|R|W and A1|Q3|G|1_Q1|1_G|1_R|1R|2R|W

2.1.4. Generalization

In the shape understanding system (SUS) a symbolic name is given in the form of the SUS representation. It is easy to translate the SUS representation into the form of the symbolic names. For example, the SUS representation C[L3,L3] is translated into the symbolic name $C(L^3, L^3)$ and the SUS representation <Q><L4>|{<L3>[O]}{<L3>[O]} is translated into the symbolic name $Q^2_{L^4}(L^3_O, L^3_O)$. Figure 2.68 illustrates the meaning of the symbols used by the SUS. The complex class is described by a symbolic name, the type of vertices, the normalized size of the sides, and the type of angles as follows: C[L3,L3], [vvvqvq], and L{ mmslle}{apaoao}. The symbolic name C[L3,L3] ($C(L^3, L^3)$) denotes an archetype of the complex class (two triangles). The term [vvvqvq] denotes the convex *v* and concave *q* vertices. The term L{mmslle} denotes the normalized size of the sides (l – large, m – medium, s – small, and e – very small). The term {apaoao} denotes angles (a – acute, o – obtuse, and p – right).

The concave class is described by the symbolic name, the type of the sides (straight or curvilinear), and the symmetry and elongatedness as follows:

<Q><L4>|{<L3>[O]}{<L3>[O]}|[AAAA][NS][El]|[AAA][NS][El]|[AAA][NS][El].

The symbolic name <Q><L4>|{<L3>[O]}{<L3>[O]}, $\left(Q^2_{L^4}\left(L^3_O, L^3_O\right)\right)$ denotes an archetype of the concave polygon class with L4 as a generic polygon and two residuals L3[O] – the obtuse triangles. The symbol [AAAA][NS][E1] denotes the polygon (straight lines – A), nonsymmetrical (NS), and medium elongated (E1).

The translation of the symbolic name into a string form requires including all details of the symbolic name. The level of details is marked by introducing the symbol "_." The symbolic name is translated into the form L0_L1_...Ln, where the level Ln denotes the level of the detailed description of the archetype of the class. For example, the triangle class L^3_O

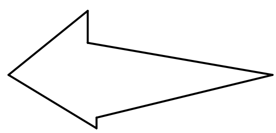


Fig. 2.68. The archetype of the complex class

Fig. 2.69. Exemplars of the class $Q^{2}_{L^4}\left(L^3_O, L3_A\right)$

(m,m,m) is translated into the form L_3_A_mmm. An exemplar of the concave class (Fig. 2.69a) is described by the symbolic name, the type of vertices, the normalized size of the sides, and the type of angles: $Q^{2}_{L^4}\left(L^3_O, L3_A\right)$, [vaqavvqv], and L{lmmsllml}{paaapaoa}. The symbolic names of exemplars of the concave class $Q^{2}_{L^4}\left(L^3_O, L3_A\right)$ and all detail descriptions are translated into the string form as follows:

(Fig. 2.69a) Q_1|L_4_R_mlml_1010|L_3_A_mmm_2|L_3_O_llm_0|
(Fig. 2.69b) Q_1|L_4_R_mlml_1100|L_3_A_mmm_2|L_3_O_llm_0|

During generalization the symbol is dropped from the right to the left, e.g., for the symbol L_3_A, the two generalizations are possible: L_3 and L, where “L_3_A” is any acute triangle, “L_3” is any triangle, and “L” is any polygon. In the case of the concave polygon Q_1|L_4_R|L_3_A_2| the generalization involves dropping the letters in the “ordered” manner or in the “combinatorial” manner.

An ordered manner takes into account the structural feature of the exemplar, for example, for the concave class the generic class is treated differently than residuals. The ordered manner required to compare only the “known” features of the shape.

The combinatorial manner does not distinguish between the types of the class description treating all elements of the string as the symbols of the type L0_L1_...Ln. The generalization means to drop any combination of the letters. The final step of the combinatorial manner is interpretation of the final string (the string where selected combination of the letters was removed).

Example of the string obtained during generalization performed in the “ordered” manner:

Q_1|L_4_R|L_3_A_2|Q_1|L_4_R|L_3_A|, Q_1|L_4 |L_3|, Q_1|L |L|, Q

Example of the strings obtained during generalization performed in the “combinatorial” manner:

Q_1|L_4_R|L_3_A_2|, Q_1|L_4_R|L_3_2|, Q_1|L_4|L_3_A_2|, Q_1|L_4|L_3_2|, Q_1|L_4_R|L_3_A|, Q_1|L_4_R|L_3|, Q_1|L_4|L_3_A|, Q_1|L_4|L_3|, Q_1|L_4|L|, Q_1|L|L_3|, Q_1|L|L|, Q.

References

1. Les, Z., Shape understanding. Possible classes of shapes. *International Journal of Shape Modelling*, 2001. **7**(1): pp. 75–109
2. Preparata, F.P., and Shamos, M.I., *Computational Geometry: An Introduction.* 1985, Berlin Heidelberg New York: Springer
3. O'Rourke, J., *Computational Geometry in C.* 1998, New York: Cambridge University Press
4. Valentine, F.A., *Convex Sets*. 1964, New York: McGraw-Hill
5. Borgefors, G., Distance transformation in digital images. *Computer Vision Graphics and Image Processing*, 1986. **34**: pp. 344–371
6. Les, Z., The processing method as a set of image transformations in shape understanding. *An International Journal Computers and Graphics*, 2001. **25**(2): pp. 223–233
7. Gray, A., *Modern Differential Geometry of Curves and Surfaces*. 1992, Boca Raton, FL: CRC Press
8. Rosenfeld, A., and Johnston, E., Angle detection in digital curves. *IEEE Transactions on Computing*, 1973. **22**: pp. 875–878
9. Ansari, N., and Huang, K., Non-parametric dominant point detection. *Pattern Recognition*, 1991. **24**(9): pp. 849–862
10. Mokhtarian, F., and Mackworth, A.K., Scale-based description and recognition of planar curves and two-dimensional shapes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1986. **8**: pp. 34–43
11. Slupecki, J., and Borkowski, L., *Elements of Mathematical Logic and Set Theory*. 1967, Oxford: Pergamon Press
12. Farin, G., *Curves and Surfaces for Computer Aided Geometrical Design.* 1993, Boston: Academic Press
13. Brigham, E.O., *The Fast Fourier Transform.* 1974, Englewood Cliffs: Prentice-Hall
14. Bartels, R., Beatty, J., and Barsky, B., *An Introduction to Splines for Use in Computer Graphics and Geometric Modeling.* 1987, Los Altos, CA: Morgan Kaufmann
15. Meyer, Y., and Ryan, R.D., *Wavelets. Algorithms and Applications*. 1993, Philadelphia, PA: Society for Industrial and Applied Mathematics
16. Les, Z., and Les, M., Shape understanding system: the noisy class, in *International Conference on Information Systems Analysis and Synthesis and World Multiconference on Systemics, Cybernetics and Informatics, July 23–26*. 2001, Orlando
17. Les, Z., and Les, M., Shape understanding system: understanding of the complex object. *The Journal of Electronic Imaging*, 2005. **14**(2): pp. 023015-1–023015-13

18. Prusinkiewicz, P., and Lindenmayer, A., *The Algorithmic Beauty of Plants*. 1990, Berlin Heidelberg New York: Springer
19. Les, Z., and Les, M., Understanding of the concave-complex object, in *IASTED Conference Visualization, Imaging, and Image Processing*. 2003, Benalmadena, Spain: ACTA Press, Anaheim
20. Les, Z., and Les, M., Shape understanding system: understanding of the complex thin object, in *IASTED Conference Computer Graphics and Imaging*. 2004, Kauai, Hawaii

http://www.springer.com/978-3-540-75768-9

Shape Understanding System
The First Steps toward the Visual Thinking Machines
Les, Z.; Les, M.
2008, XIII, 399 p. With online files/update., Hardcover
ISBN: 978-3-540-75768-9