

N Flow Acoustics

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Sound propagation in a flowing medium is treated also in the chapters “J. Duct Acoustics” and “K. Acoustic Mufflers”, but there mostly with simplifying assumptions.

This chapter uses throughout the “double subscript summation rule”, i.e., terms in an expression in which a subscript (for example i) appears twice represent a sum over that term with the multiple subscript as summation index. Terms containing x_i^2 , for example, are also cases of the summation rule.

The general convention to symbolise the density of air with ρ_0 and sound velocity with c_0 must be suspended in this chapter, because these quantities may be used in other than standard conditions. These conditions will always be defined in the context.

N.1 Concepts and Notations in Fluid Mechanics, in Connection With the Field of Aeroacoustics

► See also: Morfey (2001), Lauchle (1996), Douglas (1986), Roger (1996).

N.1.1 Types of Fluids

Ideal fluids:	$\mu = 0, \lambda = 0$	μ : dynamic viscosity λ : thermal conductivity
Newtonian fluid:	$\mu = \text{constant}$	
Non-Newtonian fluid:	$\mu \neq \text{constant}$	
	The relationship between shear stress τ and velocity gradient $\partial v / \partial n$ is non-linear.	

N.1.2 Properties of Fluids

Density:	ρ , mass per volume, $[\rho] = \frac{\text{kg}}{\text{m}^3}$	
Pressure	p , normal force pushing against a plane area divided by the area $[p] = \frac{\text{N}}{\text{m}^2} = \text{Pa}$	
Viscosity:	dynamic viscosity μ , $[\mu] = \frac{\text{N} \cdot \text{s}}{\text{m}^2} = \text{Pa} \cdot \text{s}$ kinematic viscosity ν , $[\nu] = \frac{\text{m}^2}{\text{s}}$	
Gas constant	R , $[R] = \frac{\text{J}}{\text{kg} \cdot \text{K}}$	
Specific heats	at constant volume $c_v = T \left(\frac{\partial s}{\partial T} \right)_p = \left(\frac{\partial u}{\partial T} \right)_p$	(1)
	at constant pressure $c_p = T \left(\frac{\partial s}{\partial T} \right)_p = \left(\frac{\partial h}{\partial T} \right)_p$	(2)

$$[c_p, c_v] = \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

with: s specific entropy
 u specific internal energy
 h specific enthalpy.

Specific heat ratio $\kappa = c_p/c_v$, ratio of the specific heat at constant pressure to that at constant volume. (3)

Speed of sound $c, [c] = \frac{\text{m}}{\text{s}}$

Bulk modulus K , expresses the compressibility of a fluid, $[K] = \text{Pa}$,
 adiabatic or isentropic bulk modulus: $K_s = \rho \left(\frac{\partial p}{\partial \rho} \right)_s$ (4)

isothermal bulk modulus: $K_T = \rho \left(\frac{\partial p}{\partial \rho} \right)_T$ (5)

The reciprocal $1/K_s$ or $1/K_T$ is the adiabatic or isothermal compressibility.

Coefficient of expansion $\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$ $[\beta] = \frac{1}{\text{K}}$ (6)

Thermal conductivity $\lambda, [\lambda] = \frac{\text{W}}{\text{m} \cdot \text{K}}$

Shear stress $\tau, [\tau] = \text{Pa}$

N.1.3 Models of Fluid Flows

Real flow: flow without any assumptions;
 ideal flow: flow without viscosity and thermal conductivity.

Inviscid flow: flow without viscosity;
 viscous flow: $\mu \neq 0$.

Incompressible flow: $\rho = \text{constant}$;
 compressible flow: $\rho \neq \text{constant}$.

Adiabatic flow: flow without heat transfer;
 isentropic flow: $\frac{Ds}{Dt} = 0$, the specific entropy of each fluid particle along its path is constant, but may vary from a particle to another (ROGER);
 inviscid and non heat-conducting gas flow, also frictionless
 adiabatic flow

homotropic flow: $s = \text{constant}$ throughout the flow, uniform specific entropy;
 isothermal flow: $T = \text{constant}$.

Steady flow: no time dependence for v, p, ρ, T, \dots ; thus $\frac{\partial \dots}{\partial t} = 0$;

stationary flow: $\frac{\partial \bar{A}}{\partial t} = 0$, with $\bar{A} = \bar{v}, \bar{p}, \bar{\rho}, \bar{T}, \dots$ and $\bar{A} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T A dt$; (7)

unsteady flow: $\frac{\partial A}{\partial t} \neq 0$, possibly also $\frac{\partial \bar{A}}{\partial t} \neq 0$.

Uniform flow: $\frac{\partial \bar{v}}{\partial s} = 0$;

non-uniform flow: $\frac{\partial \bar{v}}{\partial s} \neq 0$.

Rotational flow: $\vec{\omega} = \text{rot} \vec{v} = \text{curl} \vec{v} = \nabla \times \vec{v} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \neq 0 \quad (8)$

with: $v_x = u, v_y = v, v_z = w$.

Vorticity: $\vec{\omega} = \text{rot} \vec{v} = \nabla \times \vec{v}$ is a measure of local fluid rotation.

Irrotational flow: $\vec{\omega} = \text{rot} \vec{v} = \nabla \times \vec{v} = 0. \quad (9)$

Comment:

From Crocco's form of the momentum equations it follows that (stationary flow with constant stagnation enthalpy) $\vec{\omega} \times \vec{v} = T \cdot \text{grad } s$.

Consequences:

a rotational flow cannot exist with uniform entropy;

a homentropic flow must be irrotational (except when vorticity field and velocity field are parallel).

Laminar flow: viscous or streamline flow, without turbulence; the particles of the fluid moving in an orderly manner and retaining the same relative positions in successive flow cross-sections.

Turbulent flow, Turbulence: a random, non-deterministic motion of eddying fluid flow; characterized by (MORFEY):

- three-dimensional velocity fluctuations field;
- unsteady flow;
- viscous flow;
- rotational flow;
- flow with viscous dissipation of energy;
- viscous dissipation takes place at the smallest length scales of eddies, far removed from the larger scales eddies contain most of the kinetic energy; the smallest scales \gg molecular scales;
- fluctuations cover a wide frequency range and a wide range of eddy sizes or length scales;
- occurring at high Reynolds numbers.

Turbulence level: based on the averaging of the specific kinetic energy

$$\frac{1}{2} \overline{v_i^2} = \frac{1}{2} \overline{(\bar{v}_i + v'_i)(\bar{v}_i + v'_i)} = \frac{1}{2} \bar{v}_i^2 + \frac{1}{2} \overline{v_i'^2} \quad (10)$$

three-dimensional

$$\frac{1}{2}\overline{v_i^2} = \frac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2}) + \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \quad (11)$$

turbulence level:

$$Tu = \sqrt{\frac{\frac{1}{3}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})}{(\overline{u^2} + \overline{v^2} + \overline{w^2})}} \quad (12)$$

in the special case of isotropic turbulence and unidirectional flow $\bar{v}_i = \{\bar{v}_x = U; 0; 0\}$:

$$Tu = \frac{\sqrt{\overline{u'^2}}}{U} = \frac{u'_{rms}}{U} \quad (13)$$

Transition:

the fluid flow change from laminar to turbulent flow.

Boundary layer flow:

- in the mean flow sense (MORFEY):

flow next to a solid surfaces within which the mean flow $\bar{u}(y)$ varies with distance y from the wall, from zero at the wall (at $y = 0$) to 99% of its free-stream value at $y = \delta$, δ is the boundary layer thickness;

- in the acoustic sense (MORFEY):

a thin region produced by a sound field next to a solid boundary, within which the oscillatory velocity parallel to the wall drops to zero as the wall is approached, as a result of viscosity. The acoustic boundary layer thickness is

$$\delta = \sqrt{\frac{2\nu}{\omega}} \ll \lambda \quad (14)$$

Reynolds stress:

- $\rho v_i v_j$ in unsteady fluid flow;

v_i, v_j are fluid velocity components in any of the three orthogonal Cartesian coordinate directions;

$\rho v_i v_j$ represents the transfer rate of j -components fluid momentum, per unit area;

the double divergence of $\rho v_i v_j$ represents a source term in Lighthill's

inhomogeneous wave equation (acoustic analogy for aerodynamic sound generation);

- in turbulent flows:

the time-average Reynolds stress $\overline{\rho v_i' v_j'}$ is a term in the time-averaged momentum equation, as the negative of an effective stress;

$\overline{\rho v_i' v_j'}$ represents the mean momentum flux due to turbulent eddies;

the Reynolds stress tensor is $\tau_{ij} = \overline{\rho v_i' v_j'}$ with normal stress if $i = j$, and shear stress if $i \neq j$ (MORFEY)

N.2 Some Tools in Fluid Mechanics and Aeroacoustics

► See also: Telionis (1981), Johnson (1998), Schlichting (1997), Lauchle (1996), Liu (1988), Hussain (1970), Reynolds (1972)./

N.2.1 Averaging

General quantity: $f(\vec{x}, t)$

Spatial average: $\bar{f}_{\text{spatial}} = \frac{1}{V} \int_V f(\vec{x}, t) dV = \bar{f}^s(t)$ (1)

Time average: $\bar{f}_{\text{time}} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(\vec{x}, t) dt = \bar{f}^t(\vec{x}) = \bar{f}$ (2)

with: abbreviation: \bar{f} .

Root mean square: $f_{\text{rms}} = \sqrt{\bar{f}^2} = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f^2(\vec{x}, t) dt} = \bar{f}^{\text{rms}}(\vec{x})$ (3)

the square root of the mean square value.

Ensemble average: over N repeated experiments

$$\bar{f}_{\text{ensemble}} = \langle f(\vec{x}, t) \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{\alpha=1}^N f^{(\alpha)}(\vec{x}, t) = \bar{f}^{\text{en}}(\vec{x}, t) \quad (4)$$

Phase average: for periodic flows

$$\bar{f}_{\text{phase}} = \langle f(\vec{x}, t) \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^N f(\vec{x}, t + n\tau) = \bar{f}^{\text{ph}}(\vec{x}, t) \quad (5)$$

with: τ period of an externally imposed fluctuation;

$$\bar{f}_{\text{phase}} = \langle f(\vec{x}, t) \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^N f(\vec{x}, \varphi_0 + n\omega\tau) = \bar{f}^{\text{ph}}(\vec{x}, \varphi_0) \quad (6)$$

with: $0 < \varphi_0 < 2\pi$; $\varphi = \varphi_0 + n\omega\tau$ phase of the periodic flow.

Reynolds averaging: decomposition of a general quantity in the flow in the following form:

$$f = \bar{f} + f'$$

$$\text{with: } \bar{f} = f_0 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f dt \quad \text{mean quantity;} \quad (7)$$

$$\bar{f}' = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f' dt = 0 \quad \text{fluctuating quantity.} \quad (8)$$

Mass-weighted or Favre averaging:

decomposition of a general quantity in the flow in the following form:

$$f = \tilde{f}^f + f''^f$$

$$\text{with: } \tilde{f}^f = \frac{\overline{\rho f}}{\bar{\rho}} \quad \text{filtered part of } f;$$

$$f''^f \quad \text{unresolved or subgrid part of } f$$

$$\tilde{f}''^f = 0.$$

N.2.2 Decomposition (In General)

Decomposition a general flow quantity in three (or four) parts:

$$f(\vec{x}, t) = \bar{f}(\vec{x}) + \tilde{f}(\vec{x}, t) + f'(\vec{x}, t) \quad (9)$$

with:

$\bar{f}(\vec{x})$ time-averaged or mean component, obtained by Reynolds averaging:

$$\bar{f}(\vec{x}) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(\vec{x}, t) dt;$$

$$\bar{\tilde{f}} = 0$$

$$\overline{f'} = 0;$$

$\tilde{f}(\vec{x}, t)$ organized fluctuation: periodicities are in time, periodic mean component of low can be split into the odd modes \tilde{f}^{odd} and in the even modes \tilde{f}^{even} (see L1U):

$$\tilde{f}(\vec{x}, t) = \tilde{f}^{\text{odd}}(\vec{x}, t) + \tilde{f}^{\text{even}}(\vec{x}, t); \quad (10)$$

$f'(\vec{x}, t)$ random fluctuations, incoherent fluctuating flow quantities, e.g. small-scale stochastic fluctuations of the fine-grained turbulence;

$$\tilde{f}_{\text{phase}} = \langle f(\vec{x}, t) \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^N f(\vec{x}, t + n\tau) = \bar{f} + \tilde{f} \quad (11)$$

$$\text{with: } \langle f'(\vec{x}, t) \rangle = 0.$$

$$\tilde{f} = \langle f(\vec{x}, t) \rangle - \bar{f} \quad (12)$$

Phase averaging with period 2τ is denoted by $\langle \langle \rangle \rangle$ so that $\langle \langle \tilde{f}^{\text{odd}} \rangle \rangle = 0$.

Therefore the even modes are obtained from

$$\langle \langle \tilde{f}^{\text{odd}} + \tilde{f}^{\text{even}} \rangle \rangle = \tilde{f}^{\text{even}} \quad (13)$$

and the odd modes from subtracting

$$\tilde{f}^{\text{odd}} = \tilde{f} - \langle \langle \tilde{f}^{\text{odd}} + \tilde{f}^{\text{even}} \rangle \rangle. \quad (14)$$

N.2.3 Decomposition of the Physical Quantities in the Basic Equations

Decomposition:

$$\begin{aligned}\rho &= \bar{\rho} + \rho' \\ p &= \bar{p} + p' \\ v_i &= \bar{v}_i + v'_i.\end{aligned}$$

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = 0 \quad (15)$$

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_i} [\bar{\rho} \bar{v}_i + \bar{\rho} v'_i + \bar{v}_i \rho' + v'_i \rho'] = 0 \quad (16)$$

with assumptions: $\rho', v'_i \ll \bar{\rho}, \bar{v}_i$ and $\bar{\rho} \neq f(\vec{x})$

Continuity equation in the case of mean flow: $\frac{\partial \bar{v}_i}{\partial x_i} = 0$ (17)

and in the case of fluctuating flow $\frac{\partial \rho'}{\partial t} + \bar{\rho} \frac{\partial v'_i}{\partial x_i} + \bar{v}_i \frac{\partial \rho'}{\partial x_i} = 0$ (18)

and with the equation of state $\rho' = \frac{p'}{c_0^2}$:

$$\frac{\partial p'}{\partial t} + \bar{v}_i \frac{\partial p'}{\partial x_i} + \bar{\rho} c_0^2 \frac{\partial v'_i}{\partial x_i} = 0 \quad (19)$$

$$\frac{\bar{D} p'}{Dt} + \bar{\rho} c_0^2 \frac{\partial v'_i}{\partial x_i} = 0 \quad (20)$$

with: $\frac{\bar{D}}{Dt} = \frac{\partial}{\partial t} + \bar{v}_i \frac{\partial}{\partial x_i}$ (21)

Momentum equation (without viscosity)

$$\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} (\rho v_i v_j + p \delta_{ij}) = 0 \quad (22)$$

mean flow $\bar{\rho} \bar{v}_j \frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{p}}{\partial x_i} = 0$ (23)

fluctuating flow (with linearisation) $\bar{\rho} \frac{\partial v'_i}{\partial t} + \bar{\rho} \bar{v}_j \frac{\partial v'_i}{\partial x_j} + \rho' \bar{v}_j \frac{\partial \bar{v}_i}{\partial x_j} + \bar{\rho} \bar{v}_j \frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial p'}{\partial x_i} = 0$ (24)

with constant mean flow

(assumption: \bar{v}_i is uniform) $\bar{\rho} \frac{\partial v'_i}{\partial t} + \bar{\rho} \bar{v}_j \frac{\partial v'_i}{\partial x_j} + \frac{\partial p'}{\partial x_i} = 0$ (25)

Wave equation

following from the continuity equation $\frac{\partial p'}{\partial t} + \bar{v}_i \frac{\partial p'}{\partial x_i} + \bar{\rho} c_0^2 \frac{\partial v'_i}{\partial x_i} = 0$ (26)

and the momentum equations $\bar{\rho} \frac{\partial v'_i}{\partial t} + \bar{\rho} \bar{v}_j \frac{\partial v'_i}{\partial x_j} + \frac{\partial p'}{\partial x_i} = 0$ (27)

Result: convective wave equation:
$$\frac{\partial^2 p'}{\partial x_i^2} - \frac{1}{c_0^2} \left[\frac{\partial}{\partial t} + \bar{v}_i \frac{\partial}{\partial x_i} \right]^2 p' = 0 \quad (28)$$

with:
$$\left[\frac{\partial}{\partial t} + \bar{v}_i \frac{\partial}{\partial x_i} \right]^2 = \left(\frac{\partial^2}{\partial t^2} + 2\bar{v}_i \frac{\partial^2}{\partial x_i \partial t} + \bar{v}_i \bar{v}_j \frac{\partial^2}{\partial x_i \partial x_j} \right) \quad (29)$$

Navier-Stokes equation:

Double decomposition of quantities and time averaging: Reynolds averaging.

Assumptions: incompressible flow

for the mean flow:
$$\bar{v}_j \frac{\partial \bar{v}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j^2} - \frac{\partial (\overline{v'_i v'_j})}{\partial x_j} \quad \text{Reynolds equation} \quad (30)$$

with stress tensor of fluctuating flow
$$\begin{pmatrix} \sigma'_x & \tau'_{xy} & \tau'_{xz} \\ \tau'_{xy} & \sigma'_y & \tau'_{yz} \\ \tau'_{xz} & \tau'_{yz} & \sigma'_z \end{pmatrix} = - \begin{pmatrix} \rho \overline{u'^2} & \rho \overline{u'v'} & \rho \overline{u'w'} \\ \rho \overline{u'v'} & \rho \overline{v'^2} & \rho \overline{v'w'} \\ \rho \overline{u'w'} & \rho \overline{v'w'} & \rho \overline{w'^2} \end{pmatrix} \quad (31)$$

Triple decomposition of quantities and time averaging (TELIONIS):

$$f(\vec{x}, t) = \bar{f}(\vec{x}) + \tilde{f}(\vec{x}, t) + f'(\vec{x}, t)$$

for the mean flow:
$$\bar{v}_j \frac{\partial \bar{v}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j^2} - \frac{\partial (\overline{v'_i v'_j})}{\partial x_j} - \frac{\partial (\overline{\tilde{v}_i \tilde{v}_j})}{\partial x_j} \quad (32)$$

with two Reynolds stress terms on the right-hand side of the equation: non-linear contributions due to the random fluctuations and due to organized fluctuations.

for the organized fluctuations:

$$\frac{\partial \tilde{v}_i}{\partial t} + \bar{v}_j \frac{\partial \tilde{v}_i}{\partial x_j} + \tilde{v}_j \frac{\partial \bar{v}_i}{\partial x_j} + \tilde{v}_j \frac{\partial \tilde{v}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{v}_i}{\partial x_j^2} + \frac{\partial (\overline{\tilde{v}_i \tilde{v}_j})}{\partial x_j} + \frac{\partial (\overline{v'_i v'_j})}{\partial x_j} - \frac{\partial (\overline{\langle v'_i v'_j \rangle})}{\partial x_j} \quad (33)$$

N.2.4 Correlations

$$R_p(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T p(t)p(t+\tau)dt \quad \text{autocorrelation function} \quad (34)$$

$$R_p(0) = \overline{p^2(t)} \quad (35)$$

$$R_p(\xi) = \frac{1}{L} \int_0^L p(x)p(x+\xi)dx \quad (36)$$

$$R = \frac{\overline{v'_1 v'_2}}{\sqrt{\overline{v'^2_1} \overline{v'^2_2}}} \quad \text{correlation function, fluctuations of velocity} \quad (37)$$

N.2.5 Scales

$$\tau_c = \frac{1}{R_p(0)} \int_0^{\infty} R_p(\tau) d\tau \quad \text{integral time scale} \quad (38)$$

$$\Lambda = \frac{1}{R_p(0)} \int_0^{\infty} R_p(\xi) d\xi \quad \text{integral length scale} \quad (39)$$

$$\Lambda_x = \frac{E_{11}(k_1)_{k_1 \rightarrow 0}}{4v_{rms}^2} \quad \begin{array}{l} \text{integral length scale,} \\ \text{limiting value of the power spectrum} \\ \text{as } k_1 \text{ approaches zero.} \end{array} \quad (40)$$

$$E(k) = \alpha \frac{\overline{v_{rms}^2}}{k_e} \frac{(k/k_e)^4}{(1 + (k/k_e)^2)^{17/6}} e^{-2(k/k_v)^2} \quad \begin{array}{l} \text{power spectral density for isotropic} \\ \text{turbulence (von Kármán spectrum)} \end{array} \quad (41)$$

with (LONGATTE):

$$\alpha \approx 1.453$$

$$k_e \approx 0.747/\Lambda$$

$$\Lambda = (\overline{v^2})^{3/2}/\epsilon$$

ϵ = dissipation rate of turbulent kinetic energy

$$k_v = (\epsilon/\overline{v^3})^{1/4}$$

$$l_t^2 = \frac{\overline{p^2(t)}}{\left(\frac{\partial p}{\partial t}\right)^2} \quad \text{differential time scale} \quad (42)$$

$$l_t^2 = - \frac{R_p(0)}{\frac{\partial^2 R_p(0)}{\partial \tau^2}} \quad (43)$$

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