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## Preface

In the past decades now a famous class of evolution equations has been discovered and intensively studied, a class including the nowadays celebrated *Korteweg-de Vries* equation, *sine-Gordon* equation, *nonlinear Schrödinger* equation, etc. The equations from this class are known also as *the soliton equations* or equations solvable by the so-called *Inverse Scattering Transform Method*. They possess a number of interesting properties, probably the most interesting from the geometric point of view of being that most of them are *Liouville integrable* Hamiltonian systems. Because of the importance of the soliton equations, a dozen monographs have been devoted to them. However, the great variety of approaches to the soliton equations has led to the paradoxical situation that specialists in the same field sometimes understand each other with difficulties. We discovered it ourselves several years ago during a number of discussions the three of us had. Even though by friendship binds us, we could not collaborate as well as we wanted to, since our individual approach to the field of integrable systems (finite and infinite dimensional) is quite different. We have become aware that things natural in one approach are difficult to understand for people using other approaches, though the objects are the same, in our case – *the Recursion (generating) Operators* and their applications to finite and infinite dimensional (not necessarily integrable) Hamiltonian systems. Since even between us, in order to overcome our differences, we needed some serious efforts, we decided that it was time to bring together the analytic and geometric aspects, if not of the theory of the soliton equations (this would be too ambitious) but at least the analytic and the geometric aspects of the so-called Recursion Operators, which are among the powerful tools for the study of soliton equations. We had to do it in such a way, that a specialist in one of the approaches can read and understand the value of the other approach. However, the material we started to collect soon began growing rapidly, and we realized that a book should be written on this topic. The realization of the book project took longer than we expected – more than six years. But now we are happy that we are able to present a text which in our opinion reflects our original ideas.

The book has two parts, the first is dedicated to the analytic approach to the Recursion operators, the second, to the geometric nature of these operators, that is, to their interpretation as mixed tensor fields with special geometric properties over the manifold of potentials.

As we mentioned, we expect that the book will be useful to specialists in the Recursion Operator approach to the soliton equations. However, with an intent to target a larger audience, we have included some other important topics, such as the construction of the soliton solutions, for example. We have tried to develop the material in such a way that the book proves useful for graduate students who want to enter this interesting field of research.

The present book is based on some material that has become already classical, as well as on some of our works. The last few have been written in collaboration with many other friends and colleagues, namely:

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