Summary

We consider the Coxeter transformation in the context of the McKay correspondence, representations of quivers, and Poincaré series.

We study in detail the Jordan forms of the Coxeter transformations and prove splitting formulas due to Subbotin and Sumin for the characteristic polynomials of the Coxeter transformations. Using splitting formulas we calculate characteristic polynomials of the Coxeter transformation for the diagrams $T_{2,3,r}, T_{3,3,r}, T_{2,4,r}$, prove J. S. Frame's formulas, and generalize R. Steinberg's theorem on the spectrum of the affine Coxeter transformation for the multiply-laced diagrams. This theorem is the key statement in R. Steinberg's proof of the McKay correspondence. For every extended Dynkin diagram, the spectrum of the Coxeter transformation is easily obtained from R. Steinberg's theorem.

In the study of representations π_n of $SU(2)$, we extend B. Kostant's construction of a vector-valued generating function $P_G(t)$. B. Kostant's construction appears in the context of the McKay correspondence and gives a way to obtain multiplicities of irreducible representations ρ_i of the binary polyhedral group G in the decomposition of $\pi_n|G$. In the case of multiply-laced graphs, instead of irreducible representations ρ_i we use restricted representations and induced representations of G introduced by P. Slodowy. Using B. Kostant's construction we generalize to the case of multiply-laced graphs W. Ebeling's theorem which connects the Poincaré series $[P_G(t)]_0$ and the Coxeter transformations. According to W. Ebeling's theorem

$$[P_G(t)]_0 = \frac{\mathcal{X}(t^2)}{\tilde{\mathcal{X}}(t^2)},$$

where $\mathcal{X}$ is the characteristic polynomial of the Coxeter transformation and $\tilde{\mathcal{X}}$ is the characteristic polynomial of the corresponding affine Coxeter transformation.

Using the Jordan form of the Coxeter transformation we prove a criterion of V. Dlab and C. M. Ringel of the regularity of quiver representations, consider necessary and sufficient conditions of this criterion for extended Dynkin diagrams and for diagrams with indefinite Tits form.

We prove one more observation of McKay concerning the Kostant generating functions $[P_G(t)]_i$:

$$(t + t^{-1})[P_G(t)]_i = \sum_{j \leftarrow i} [P_G(t)]_j,$$

where j runs over all successor vertices to i.

A connection between fixed and anti-fixed points of the powers of the Coxeter transformations and Chebyshev polynomials of the first and second kind is established.

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