

# Preface

This monograph is devoted to construction of novel theoretical approaches of modeling non-homogeneous structural members as well as to development of new and economically efficient (simultaneously keeping the required high engineering accuracy) computational algorithms of nonlinear dynamics (statics) of strongly nonlinear behavior of either purely continuous mechanical objects (beams, plates, shells) or hybrid continuous/lumped interacting mechanical systems.

In general, the results presented in this monograph cannot be found in the existing literature even with the published papers of the authors and their coauthors. We take a challenging and originally developed approach based on the integrated mathematical–numerical treatment of various continuous and lumped/continuous mechanical structural members, putting emphasis on mathematical and physical modeling as well as on the carefully prepared and applied novel numerical algorithms used to solve the derived nonlinear partial differential equations (PDEs) mainly via Bubnov-Galerkin type approaches.

The presented material draws on the fields of bifurcation, chaos, control, and stability of the objects governed by strongly nonlinear PDEs and ordinary differential equations (ODEs), and may have a positive impact on interdisciplinary fields of nonlinear mechanics, physics, and applied mathematics. We show, for the first time in a book, the complexity and fascinating nonlinear behavior of continual mechanical objects, which cannot be found in widely reported bifurcational and chaotic dynamics of lumped mechanical systems, i.e., those governed by nonlinear ODEs. On the other hand, we illustrate also how the strongly nonlinear PDEs modeled by chains of many (infinitely many) coupled nonlinear oscillators may also exhibit nonlinear behavior well known and studied via simple systems, usually in one or two degrees-of-freedom mechanical objects.

In the Introduction an overview of both Ritz and Bubnov-Galerkin methods with emphasis on achievements of Eastern countries is given. A historical origin of both the Bubnov-Galerkin and Galerkin procedures is described and their status versus that of other projection approaches is addressed.

In Chap. 1 a general theory of non-homogeneous shells is introduced. First, fundamental relations and assumptions are given, shells' non-homogeneities are

introduced, and then the governing variational equations and equations of motion are derived. After the boundary and initial conditions are introduced, the equations are cast to non-dimensional form and the so-called variable parameters of a shell stiffness are defined. In addition, a flexural stiffness coefficient of a shell element is formally introduced. The section devoted to generalized functions completes this chapter.

Chapter 2 deals with the static instability problems of rectangular plates. First, fundamental concepts of the theory of elastic stability are illustrated and discussed. Second, two fundamental formulas of the energy-based criterion of bifurcational stability loss of an elastic continuous mechanical system are derived. In addition, advantages and disadvantages of today's stability investigation approaches are critically revisited, placing emphasis on problems not yet satisfactorily solved. In the next section various methods devoted to stability investigations are briefly addressed, exhibiting their strong and weak points regarding applications with particular attention to computational advantages of the Galerkin methods. In Sect. 2.4 the Bubnov-Galerkin method of high-order approximations and the associated numerical algorithm are presented. The rectangular form additions of other materials to a shell are described in Sect. 2.5, whereas the next section deals with static shell stability problems. Finally, the central square element, cross addition element, and perforation-type non-homogeneities are introduced.

In Chap. 3 vibrations of rectangular shells are studied. Linear and weakly nonlinear vibrations are revisited in Sect. 3.1, and then natural vibrations of non-homogeneous shells applying the Bubnov-Galerkin method of higher order approximations are analyzed. Section 3.3 is devoted to investigation of free nonlinear vibrations of homogeneous plates and shells regarding any choice of control parameters. The relatively extensive Sect. 3.4 addresses the spectral analysis of a stress-strain problem of any plate/shell approximated by systems of  $n$  degrees of freedom. A harmonic process convergence and spectral analysis of free nonlinear vibrations are illustrated and discussed in Sects. 3.5 and 3.6, respectively.

In Chap. 4 dynamic stability loss of rectangular shells is addressed. A background containing types of dynamic buckling exhibited by perfect constructions, as well as the concept of finite-time stability, is given in Sects. 4.1–4.3. Mathematical modeling of dynamical systems, problems of synchronization, chaos, and quasi-periodicity are also briefly revisited. Sections 4.6–4.10 refer to both static and dynamic bifurcations and their numerical estimations. Stability loss of homogeneous shells subjected to an action of transversal loads is rigorously studied in Sect. 4.11.

Chapter 5 concerns stability of closed cylindrical shells subjected to an axially non-symmetrical load action. In the beginning (Sect. 5.1), equations of motion are derived, and then the influence of imperfections on shell stability is studied. Both static and dynamic problems of buckling with the use of the Bubnov-Galerkin method of higher approximations are analyzed and many computational results are reported.

Composite shells are studied in Chap. 6. First, equations governing behavior of composite shells are derived, and then both static and dynamic problems of stability loss of composite shells are addressed.

In Chap. 7 the problem of interaction between flexible construction and a moving lumped body is reduced to that of essentially simpler ones, i.e., that of vibrations

subject to moving force  $P_0$  and that of displacement in the domain of the moving masses under the action of the mentioned force. Advantages of the proposed method are illustrated and discussed. It is characteristic for the modeling of interaction between a construction and moving bodies that the impact on the construction is expressed by the weight and inertial forces of the objects moving on the studied construction. This crucial feature of the applied approach constitutes also the essential difficulty in the mathematical analysis of the problem. Let us note that most publications in this field and related to the analysis of the interaction between a shell and a moving mass are based on the application of a geometrically linear model or on the assumption that a movable mass does not tear off the leading construction. Apart from that, calculations were usually conducted with the use of approximated methods with only a few first approximations taken into account. This chapter deals with this problem in a complex and detailed way and it provides the methods leading to the proper and accepted solution in engineering.

Chapter 8 deals with a novel approach to study chaotic vibrations of deterministic mechanical systems represented by shallow sector-type spherical shells. Scales of vibration character of such shells being transversally and harmonically excited vs. control parameters are constructed. Scenarios to chaos are illustrated and discussed. Control of chaotic state applying synchronous action of harmonic loading torque is proposed. Investigations are carried out using the qualitative theory of differential equations and nonlinear dynamics.

In Chap. 9 scenarios of transition from harmonic to chaotic motions are illustrated and discussed. First, a historical background of the problem is described. The Landau-Hopf scenario; the Ruelle, Takens, and Newhouse scenario; the Feigenbaum scenario; and the Pomeau-Manneville scenario are addressed, among others.

In Chap. 10 complex vibrations of closed cylindrical shells of infinite length and circular cross section subjected to transversal local load in the frame of the classical nonlinear theory are studied. A transition from PDEs to ODEs is carried out using the higher order Bubnov-Galerkin approach and Fourier representation. On the other hand, the Cauchy problem is solved using the fourth-order Runge-Kutta method. In the first part of this work static problems of the theory of closed cylindrical shells are studied. Reliability of the obtained results is verified by comparing them with the results taken from the literature. The second part is devoted to the analysis of stability, bifurcation, and chaos of closed cylindrical shells. In particular, an influence of sign-changeable external pressure and the control parameters such as magnitude of pressure measured by  $\varphi_0$ , relative linear shell dimension  $\lambda = \frac{l}{R}$ , frequency  $\omega_p$ , and amplitude  $q_0$  of external transversal load on the shell's nonlinear dynamics are studied.

Chapter 11 is devoted to control of temporal-spatial chaos exhibited by cylindrical shells. Process of controlling chaos is understood as the transformation of chaotic dynamics into regular, or the other chaotic ones, but of different characteristics, with the use of small external periodic input functions and by the influence of transverse load applied in anti-phase.

In Chap. 12 chaotic vibrations of flexible rectangular shells forced by transversal harmonic load are analyzed via application of the qualitative theory of differential equations and nonlinear dynamics. An infinitely dimensional problem is reduced to a finitely dimensional one with the application of the Bubnov-Galerkin method

with higher approximations and the method of finite differences with approximation  $O(\Delta^2)$ . An initial problem is solved with the use of Runge-Kutta method of the fourth order. It is shown that within the range of harmonic vibrations, the results obtained from both methods are fully convergent, whereas in the range of chaos such convergence can be obtained only in relation to the character of vibrations, i.e., in relation to the frequency spectra. The increase in the number of element partitions in the method of finite differences and the number of approximations in the Bubnov-Galerkin method leads to better results, but there is some threshold value beyond which further calculations are impossible.

Chapter 13 deals with both regular and chaotic vibrations, various bifurcations, and scenarios exhibited by three-layered nonlinear uncoupled beams with constraints. The finite difference approximation is applied and the reliability of the numerical results is first rigorously discussed. New scenarios of transition to chaos and synchronization phenomena are reported, and the essential influence of four boundary conditions on various nonlinear behaviors is outlined.

In Chap. 14 dynamics of physically dissipative nonlinear multi-layer sandwich of three beams is analyzed. The boundary conditions are arbitrary. The transversal load can be applied either simultaneously to all beams or separately to each of the beams. The finite difference method is used to solve the governing equations. Different types of beam material are considered: ideally elastic-plastic, elastic-plastic with linear straightness, and pure aluminum. Some new bifurcation and chaotic phenomena of the system are reported and discussed.

Chapter 15 deals with complex vibrations of a flexible Euler-Bernoulli type beam driven by a dynamic load, and the various type of inputs on its edge are studied. The governing equations include damping terms with damping coefficients  $\varepsilon_1$ ,  $\varepsilon_2$  associated with deflection  $w$  and displacement  $u$ , respectively. Damping coefficients  $\varepsilon_1$ ,  $\varepsilon_2$  and the transversal load coefficients  $(q_0, \omega_p)$  serve as control parameters. The formulated infinite dimensional problem is reduced to that of finite dimension applying the finite difference method with approximation  $O(h^2)$  with regard to spatial coordinates and it is solved via the fourth-order Runge-Kutta method. This approach enabled identification of damping coefficients  $\varepsilon_1$  and  $\varepsilon_2$ , as well as investigation of elastic waves generated by an impact introduced through a mass (lumped body) moving at constant velocity. The introduced analysis is supported by applied achievements of the qualitative theory of differential equations and nonlinear dynamics.

Finally, this book is accessible to readers with a fundamental knowledge of applied mathematics, differential equations, and modern theory of nonlinear dynamical systems. One of the co-authors (J. Awrejcewicz) acknowledges the financial support by the Polish Ministry of Science and Higher Education for years 2005–2008 (grant No. 4 07A 03128) regarding the book part devoted to impact phenomena.

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