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Fuzzy Decision Making and Maximization Decision Making

Due to incomplete knowledge and information, it is not enough to use precise mathematics to model a complex system. In order to represent the vagueness in everyday life, Zadeh introduced the concept of fuzzy sets in 1965. Based on this concept, Bellman and Zadeh presented the fuzzy decision theory. They defined decision-making in a fuzzy environment with a decision set which unifies a fuzzy objective and a fuzzy constraint.

Suppose that fuzzy sets are defined on a set of alternatives, X. Let the fuzzy set for the fuzzy objective be identified as G, the fuzzy set for the constraints as C, and let decision set D be a unifying factor between G and C. Then, decision set D can be defined as an intersection set between fuzzy objective G and fuzzy constraint C, $D = G \cap C$. Decision set D is a fuzzy set, named a fuzzy decision. The corresponding membership function of the decision set D is given by:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)), \forall x \in X. \tag{2.1}$$

More generally, if there are m fuzzy goals G_i $(i = 1, \cdots, m)$ and n fuzzy constraints C_j $(j = 1, \cdots, n)$, then the fuzzy decision is defined by the following fuzzy set

$$D = \{G_1 \cap G_2 \cap \cdots \cap G_m\} \cap \{C_1 \cap C_2 \cap \cdots \cap C_n\}. \tag{2.2}$$

Its membership function is characterized as in the following.

$$\mu_D(x) = \min(\mu_{G_1}(x), \cdots, \mu_{G_m}(x), \mu_{C_1}(x), \cdots, \mu_{C_n}(x)), \forall x \in X. \tag{2.3}$$

The above concepts illustrate that there is no difference between the objective and the constraint in a fuzzy environment.

Bellman and Zadeh proposed a maximization decision. The maximization decision is defined by the following non-fuzzy subset.

$$D^* = \{x^* \in X | x^* = \text{argmax}\{\mu_D(x)\} = \text{argmax}\{\min(\mu_G(x), \mu_C(x))\}\}. \tag{2.4}$$

Where m fuzzy objectives and n fuzzy constraints are given, the maximization decision can be denoted as follows:

$$\begin{aligned} D^* = \{x^* \in X | x^* &= \text{argmax}\{\mu_D(x)\} \\ &= \text{argmax}\{\min(\mu_{G_1}(x), \cdots, \mu_{G_m}(x), \mu_{C_1}(x), \cdots, \mu_{C_n}(x))\}\}. \end{aligned}$$

The maximization decision can be considered to be an optimal decision.

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