2 Classical International Trade Theories

This chapter introduces the basic ideas and conclusions of classical international trade theories in mathematical form. Section 2.1 studies Adam Smith's trade theory with absolute advantage. Although Smith's ideas about absolute advantage were crucial for the early development of classical thought for international trade, he failed to create a convincing economic theory of international trade. Section 2.2 examines the theories of comparative advantage. Ricardo showed that the potential gains from trade are far greater than Smith envisioned in the concept of absolute advantage. Section 2.3 develops a two-good, two-factor model. Different from the common dual approach to examining perfectly competitive two-factor two-sector model in the trade literature, we use profit-maximizing approach to demonstrate the most well-known theorems in the Heckscher-Ohlin trade theory. These theorems include the factor price insensitivity lemma, Samuelson' factor price equalization theorem, Stolper-Samuelson theorem, and Rybczynski's theorem. In Sect. 2.4, we illustrate the dual approach for the same economic problems as defined in Sect. 2.3. Section 2.5 examines the Heckscher-Ohlin theory which emphasizes differences between the factor endowments of different countries and differences between commodities in the intensities with which they use these factors. The basic model deals with a long-term general equilibrium in which the two factors are both mobile between sectors and the cause of trade is that different countries have different relative factor endowments. The theory is different from the Ricardian model which isolates differences in technology between countries as the basis for trade. In the Heckscher-Ohlin theory costs of production are endogenous in the sense that they are different in the trade and autarky situations, even when all countries have access to the same technology for producing each good. Section 2.6 introduces the neoclassical theory which holds that the determinants of trade patterns are to be found simultaneously in the differences between the technologies, the factor endowments, and the tastes of different countries. Section 2.7 develops a general equilibrium model for a two-country two-sector two-factor economy, synthesizing the models in the previous sectors. Section 2.8 introduces public goods to the two-sector and two-factor trade model defined in the previous sections. Section 2.9 concludes the chapter. Appendix 2.1 represents a well-known generalization of the Ricardian model to encompass a continuum of goods.

2.1 Adam Smith and Absolute Advantage

Adam Smith (1776) held that for two nations to trade with each other voluntarily, both nations must gain. If one nation gained nothing or lost, it would refuse it. According to Smith, mutually beneficial trade takes place based on absolute advantage. When one nation is more efficient than (or has an absolute advantage over) the other nation is producing a second commodity, then both nations gain by each specializing in the production of the commodity of its absolute advantage and exchanging part of its output with the other nation for the commodity of its absolute disadvantage. For instance, Japan is efficient in producing cars but inefficient in producing computers; on the other hand, the USA is efficient in producing computers but inefficient in cars. Thus, Japan has an absolute advantage over the USA in producing cars but an absolute disadvantage in producing computers. The opposite is true for the USA. Under these conditions, according to Smith, both nations would benefit if each specified in the production of the commodity of its absolute advantage and then traded with the other nation. Japan would specialize in producing cars and would exchange some of the cars for computers produced in the USA. As a result, both more cars and computers would be produced, and both Japan and the USA gain. Through free trade, resources are mostly efficiently utilized and output of both commodities will rise. Smith thus argued that all nations would gain from free trade and strongly advocated a policy of laissez-faire. Under free trade, world resources would be utilized mostly efficiently and world welfare would be maximized.

To explain the concept of absolute advantage, we assume that the world consists of two countries (for instance, England and Portugal). There are two commodities (cloth and wine) and a single factor (labor) of production. Technologies of the two countries are fixed. Assume that the unit cost of production of each commodity (expressed in terms of labor) is constant. Assume that a labor theory of value is employed, that is, goods exchange for each other at home in proportion to the relative labor time embodied in them. Let us assume that the unit costs of production of cloth and wine in terms of labor are respectively 2 and 8 in England;[1] while they are respectively 4 and 6 in Portugal. Applying the labor theory of value, we see

[1] Units for cloth and wine are, for instance, yard and barrel.

that 1 unit of wine is exchanged for 4 units of cloth in England when England does not have trade with Portugal. The ratio is expressed as $1/4$ units of cloth/per wine. The ratio is the relative quantities of labor required to produce the goods in England and can be considered as opportunity costs. The ratio is referred to as the price ratio in autarky. Similarly, 2 units of wine is exchanged for 3 units of cloth in Portugal ($3/2$ units of cloth/per wine). England has an absolute advantage in the production of cloth and Portugal has an absolute advantage in the production of wine because to produce one unit of cloth needs less amount of labor in England than in Portugal and to produce one unit of wine needs more amount of labor in England than in Portugal. Adam Smith argued that there should be mutual benefits for trade because each country has absolute advantage in producing goods. For instance, if the two countries have free trade and each country specified in producing the good where it has absolute advantage. In this example, England is specified in producing cloth and Portugal in producing wine. Also assume that in the international market, one unit of wine can exchange for 3 units of cloth. In England in open economy one can obtain one unit of wine with 3 units of cloth, while in the autarky system one unit of wine requires 4 units of cloth, we see that trade will benefit England. Similarly, in Portugal in open economy one can obtain one unit of cloth with $1/3$ unit of wine instead of $2/3$ unit of wine as in autarky system, trade also benefits Portugal. In this example, we fixed the barter price in open economies with one unit of wine for 3 units of cloth. It can be seen that mutual gains can occur over a wide range of barter prices.

2.2 The Ricardian Trade Theory

Although Smith's ideas about absolute advantage were crucial for the early development of classical thought for international trade, it is generally agreed that David Ricardo is the creator of the classical theory of international trade, even though many concrete ideas about trade existed before his *Principles* (Ricardo, 1817). Ricardo showed that the potential gains from trade are far greater than Smith envisioned in the concept of absolute advantage.

The theories of comparative advantage and the gains from trade are usually connected with Ricardo. In this theory the crucial variable used to explain international trade patterns is technology. The theory holds that a difference in comparative costs of production is the necessary condition for the existence of international trade. But this difference reflects a difference in techniques of production. According to this theory, technological differences between countries determine international division of labor and consumption and trade patterns. It holds that trade is beneficial to all participating countries. This conclusion is against the viewpoint about trade held by the doctrine of mercantilism. In mercantilism it is argued that the regulation and planning of economic activity are efficient means of fostering the goals of nation.

In order to illustrate the theory of comparative advantage, we consider an example constructed by Ricardo. We assume that the world consists of two countries (for instance, England and Portugal). There are two commodities (cloth and wine) and a single factor (labor) of production. Technologies of the two countries are fixed. Let us assume that the unit cost of production of each commodity (expressed in terms of labor) is constant. We consider a case in which each country is superior to the other one in production of one (and only one) commodity. For instance, England produces cloth in lower unit cost than Portugal and Portugal makes wine in lower unit cost than England. In this situation, international exchanges of commodities will occur under free trade conditions. As argued in Sect. 2.1, trade benefits both England and Portugal if the former is specified in the production of cloth and the latter in wine. This case is easy to understand. The Ricardian theory also shows that even if one country is superior to the other one in the production of two commodities, free international trade may still benefit the two countries. We may consider the following example to illustrate the point.

Let us assume that the unit costs of production of cloth and wine in terms of labor are respectively 4 and 8 in England; while they are 6 and 10 in Portugal. That is, England is superior to Portugal in the production of both commodities. It seems that there is no scope for international trade since England is superior in everything. But the theory predicts a different conclusion. It argues that the condition for international trade to take place is the existence of a difference between the comparative costs. Here, we define comparative costs as the ratio between the unit costs of the two commodities in the same countries. In our example comparative costs are $4/8 = 0.5$ and $6/10 = 0.6$ in England and Portugal respectively. It is straightforward to see that England has a relatively greater advantage in the production of cloth than wine: the ratio of production costs of cloth between England and Portugal is $4/6$; the ratio of production costs of wine is $8/10$. It can also seen that Portugal has a relatively smaller disadvantage in the production of wine. The Ricardian model predicts that if the terms of trade are greater than 0.5 and smaller than 0.6, British cloth will be exchanged for Portuguese wine to the benefit of both countries. For instance, if we fix the trade terms at 0.55, which means that 0.55 units of wine exchanges for one unit of cloth, then in free trade system in England one unit of cloth exchanges for 0.55 units of wine (rather than 0.5 as in isolated system) and in Portugal 0.55 (rather than 0.6) unit of wine exchanges for one unit of cloth. The model thus concludes that international trade is beneficial to both countries. It is straightforward to show that the terms of trade must be strictly located between the two comparative costs (i.e., between 0.5 and 0.6 in our example). It is readily verified that if the terms of trade were equal to either comparative cost, the concerned country would have no economic incentive to trade; if the terms of trade were outside the interval between the comparative costs, then some country will suffer a loss by engaging in international trade.

We now formally describe the Ricardian model.[2] The assumptions of the Ricardian model are as follows: (1) Each country has a fixed endowment of resources, and all units of each particular resource are identical; (2) The economy is characterized of perfect competition; (3) The factors of production are perfectly mobile between sectors within a country but immobile between countries;[3] (4) There is only one factor of production, labor and the relative value of a commodity is based solely on its labor content;[4] (5) Technology is fixed and different countries may have different levels of technology; (6) Unit costs of production are constant; (7) Factors of production are fully employed; (8) There is no trade barrier, such as transportation costs or government-imposed obstacles to economic activity.

First, we consider that the world economy consists of two countries, called Home and Foreign. Only two goods, wine and cloth, are produced. The technology of each economy can be summarized by labor productivity in each country, represented in terms of the unit labor requirement, the number of hours of labor required to produce a unit of wine or a unit of cloth. Let a_W and a_C stand respectively for the unit labor requirements in wine and cloth production, and Q_W and Q_C for levels of production of wine and cloth in Home. For Foreign, we will use a convenient notation throughout this book: when we refer some aspect of Foreign, we will use the same symbol that we use for Home, but with a tilde $\sim$. Correspond-

[2] The Ricardian model presented below can be found in standard textbooks on international economics. This section is referred to Krugman and Obstfeld (2006). A formal analysis is referred to Borkakoti (1998: Chap. 6).

[3] This assumption implies that the prices of factors of production are the same in different sectors within each country and may differ between countries.

[4] The assumption of a single factor of production can be replaced by that any other inputs are measured in terms of the labor embodied in production or the other inputs/labor ratio is the same in all industries

ingly, we define $\tilde{a}_W$, $\tilde{a}_C$, $\tilde{Q}_W$ and $\tilde{Q}_C$ for Foreign. Let two countries' total labor supplies be represented by N and $\tilde{N}$, respectively. The production possibility frontiers of the two sectors in the two countries are given by

$$a_W Q_W + a_C Q_C \leq N,$$
$$\tilde{a}_W \tilde{Q}_W + \tilde{a}_C \tilde{Q}_C \leq \tilde{N}.$$

A production possibility shows the maximum amount of one product that can be produced once the decision on the amount of production of the other product has been made. We rewrite the above two inequalities in the following form

$$\bar{a}_W \bar{Q}_W + \bar{a}_C \bar{Q}_C \leq \bar{N}, \tag{2.2.1}$$

where a variable with macron ¯ stand for both Home and Foreign. Figure 2.2.1 shows Home's production possibility frontier. The absolute value of slope of the line is equal to the opportunity cost of cloth in terms of wine.[5] The slope of the line is equal to a_C / a_W.

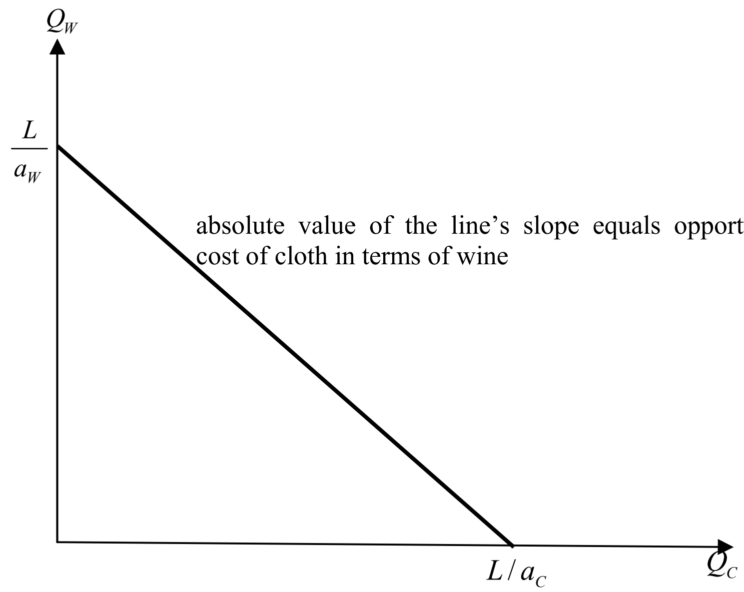

Fig. 2.2.1. Home's production possibility frontier

[5] The opportunity cost is the units of wine the country has to give up in order to produce of an extra unit of cloth.

The production possibility frontiers show the combinations of goods that the economy can produce. To determine what the economy actually produces, we need to know prices. Let P_W and P_C stand respectively for the prices of wine and cloth. As it takes a_W hours to produce a unit of wine, under the assumption of perfect competition the wage per hour is equal to P_W / a_W in the wine sector in Home. Similarly, the wage rate of the cloth sector is P_C / a_C. For a moment, we are concerned with autarky system. As labor is freely mobile between the two sectors, both goods will be produced only when the wage rates equal in the two sectors, that is

$$\frac{P_W}{a_W} = \frac{P_C}{a_C}.$$

Otherwise, if $P_W / a_W > (<) P_C / a_C$, the economy will specialize in the production of wine (cloth). As a_C / a_W is the opportunity cost of cloth in terms of wine, the economy will specialize in the production of cloth (wine) if the relative price of cloth, P_C / P_W, exceeds (is less than) its opportunity cost.

To examine trade, we compare the opportunity costs, a_C / a_W and $\tilde{a}_C / \tilde{a}_W$. The opportunity cost in country may be greater or less than or equal to the opportunity cost in the other country. First, we are concerned with the case when the opportunity cost in Home is lower than it is in Foreign, that is, $a_C / a_W < \tilde{a}_C / \tilde{a}_W$, or equivalently

$$\frac{a_C}{\tilde{a}_C} < \frac{a_W}{\tilde{a}_W}. \tag{2.2.2}$$

The requirement means that the ratio of the labor required to produce a unit of cloth to that required to produce a unit of wine is lower in Home than it is in Foreign. We say that Home's relative productivity in cloth is higher than it is in wine. We also say that Home has a comparative advance in cloth. As Fig. 2.2.1, we can also draw Foreign's production possibility frontier. Since the slope equals the opportunity cost of cloth in terms of wine, Foreign's frontier is steeper than Home's.

It should be noted that the concept of comparative advance involves all the four parameters, $\bar{a}_j$, $j = W, C$. The concept of absolute comparative advantage used in the previous section is different from the concept of comparative advance. When one country can produce a unit of good with less labor than the other country, we say that the former has an absolute advantage in producing that good. In autarky systems relative prices are determined by the relative unit labor requirements. But once the possibility of international trade is allowed, we have to examine how relative prices are determined. We now examine how supply is determined. First, we show that if the relative price of cloth is below a_C / a_W, then there is no supply of cloth. We have already shown that if $P_C / P_W < a_C / a_W$, Home will specialize in the production of wine. Similarly, Foreign will specialize in the production of wine if $P_C / P_W < \tilde{a}_C / \tilde{a}_W$. Under (2.2.2), there is no supply of cloth. When $P_C / P_W = a_C / a_W$, Home will supply any relative amount of the two goods. We draw the relative supply curve (RS) in Fig. 2.2.2, where the horizontal axis is the relative supply, $\left(Q_C + \tilde{Q}_C\right)/\left(Q_W + \tilde{Q}_W\right)$, and the vertical axis is the relative price, P_C / P_W. At $P_C / P_W = a_C / a_W$, the supply curve is flat. If $P_C / P_W > a_C / a_W$, Home specializes in the production of cloth, the output being L / a_C. As long as $P_C / P_W < \tilde{a}_C / \tilde{a}_W$, Foreign will specialize in production of wine, the output being $\tilde{L} / \tilde{a}_W$. We see that if

$$\frac{a_C}{a_W} < \frac{P_C}{P_W} < \frac{\tilde{a}_C}{\tilde{a}_W},$$

the relative output is $\left(L / a_C\right)/\left(\tilde{L} / \tilde{a}_W\right)$. When $P_C / P_W = \tilde{a}_C / \tilde{a}_W$, Foreign is indifferent between producing cloth and wine, resulting in a flat section of the supply curve. Finally, if $P_C / P_W > \tilde{a}_C / \tilde{a}_W$, both Home and Foreign specialize in production of cloth. The relative supply of cloth becomes infinite. In summary, we see that the world relative supply curve consists of steps with flat sections connected by a vertical section.

The supply curve is plotted in Fig. 2.2.2. The relative demand curve (RD) is plotted as in Fig. 2.2.2. As the relative price of cloth rises, consumers will tend to purchase less cloth and more wine. The equilibrium relative price of cloth is determined at the intersection of the RD and RS. From two different RDs, we have two different equilibrium points, A and B, as illustrated in Fig. 2.2.2. At equilibrium point A, each country specializes in production of the good in which it has a comparative advantage: Home produces cloth and Foreign produces wine. At equilibrium point B, Home produces both cloth and wine. Foreign still specializes in producing wine. Foreign still specializes in producing in the good in which it has a comparative advantage. We also see that except the case that one of the two countries does not completely specialize, the relative price in trade system is somewhere between its autarky levels in the two countries.

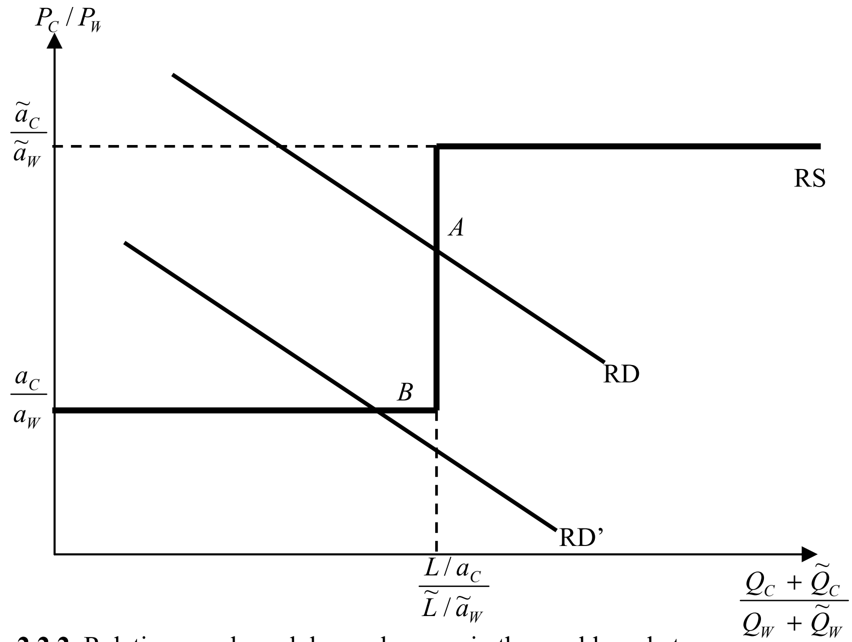

Fig. 2.2.2. Relative supply and demand curves in the world market

We have demonstrated that countries with different technologies will specialize in production of different goods. To see that trade benefits the two countries, consider how Home uses an hour of labor. Home could use the hour either to produce $1/a_W$ units of wine or $1/a_C$ units of cloth. This cloth could be traded for wine, obtaining $(P_C / P_W)/a_C$ units of wine. There will be more wine than the hour could have produced directly as long as $(P_C / P_W)/a_C > 1/a_W$, that is $P_C / P_W / > a_C / a_W$, which holds if both countries specialize in producing one good. This implies that Home uses more effectively its labor in trade than in autarky. This is similarly holds for Foreign. Both countries gain by trade.

One of the attractive features of the Ricardian model is that its modeling structure allows virtually all the results obtained for the simple two-commodity and two-country case to be extended to many countries and many commodities, even though some new features appear in high dimensions.[6] For example, when the global economy consists of many commodities but only two countries, commodities can be ranked by comparative costs in a chain of decreasing relative labor costs:

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[6] For instance, Ethier (1974), Chang (1979), Jones and Neary (1985) and Neary (1985).

$$\frac{a_{21}}{a_{11}} > \frac{a_{22}}{a_{12}} > \cdots > \frac{a_{2j}}{a_{1j}} > \cdots > \frac{a_{2n}}{a_{1n}}, \tag{2.2.3}$$

in which a_{ij} is country i 's labor requirement per unit output in sector j, $i = 1, 2$, $j = 1, 2, ..., n$. Demand conditions determine where the chain is broke. The comparative unit costs ensure that country 1 must export all commodities to the left of the break and import all those to the right, with at most one commodity produced in common.

This theory may be represented in different ways. For instance, we may interpret the theory of comparative costs in terms of optimization. We refer the following example to Gandolfo (1994a). We consider a simple case in which the world economy consists of two countries and produces two commodities. Here, we consider the benefits from international trade in terms of an increase in the quantity (rather than utility) of goods which can be obtained from the given amount of labor. Our optimal problem is to maximize each country's real income under constraints of the fixed labor and technology. We use P_x and P_y to denote the absolute prices of cloth and wine (expressed in terms of some external unit of measurement, for instance, gold). Under the assumptions of free trade, perfect competition and zero-transportation cost, the Home price ratio is equal between the two countries. The exchange ratio of the two goods, P_x / P_y, is taken as given. Let x_j and y_j denote respectively country j' s outputs of cloth and wine and N_j stand for country j' s fixed labor force. Country j' s optimal problem is defined by

$$Max\ Y_j = \left(\frac{P_x}{P_y}\right) x_j + y_j,$$

subject to

$$a_{1j} x_j + a_{2j} y_j \le N_j, \quad x_j, y_j \ge 0, \quad j = 1, 2, \tag{2.2.4}$$

in which Y_j is country j' s real national income measured in terms of good y and a_{1j} and a_{2j} are respectively country j' s unit costs of production of cloth and wine. The optimal problems defined by (2.2.4) can find an easy graphic solution. It can be shown that international trade and international specification occur as the consequence of the maximization of the real national income of each country.

The Ricardian model assumes that production costs are independent of factor prices and the composition of output. The model throws no light on issues related to the internal distribution of income since it assumes either a single mobile factor or multiple mobile factors, which are used in equal proportions in all sectors. From this theory, we can only determine the limits within which the terms of trade must lie. Since it lacks consideration of demand sides, the theory cannot determine how and at what value the terms of trade are determined within the limits. This is a serious limitation of this theory because a trade theory should be able to explain not only the causes and directions of trade but also to determine the terms of trade.[7]

2.3 The $2 \times 2 \times 2$ Trade Model and the Core Theorems in Trade Theory

The Ricardian theory is concerned with technology. The theory has a single factor of production. Nevertheless, economic activities involve many factors. The Heckscher-Ohlin international trade theory is concerned with factors of production. Before introducing the Heckscher-Ohlin theory in the next section, we develop a two-good, two-factor model. Different from the common dual approach to examining perfectly competitive two-factor two-sector model in the trade literature,[8] we use profit-maximizing approach to demonstrate the most well-known theorems in the Heckscher-Ohlin trade theory. In Sect. 2.4, we illustrate the dual approach for the same economic problems.

We are concerned with a single country. Assume that there are two factors of production, labor and capital. Their total supplies, N and K, are fixed. The economy produces two goods with the following Cobb-Douglas production functions[9]

[7] The terms of trade measures the relationship between the price a country receives for its exports versus the price a country pays for its imports. The higher the ratio, the more favorable terms of trade are for the country (Sawyer and Sprinkle, 2003: Chap. 2). In general, we need to introduce demand theory to determine the terms of trade.

[8] The dual approach is referred to, for instance, Woodland (1977, 1982), Mussa (1979), and Dixit and Norman (1980). The geometric approach to the problem is referred to, for instance, Lerner (1952), Findlay and Grubert (1959), and Gandolfo (1994a).

[9] The specified form is mainly for convenience of analysis. It can be shown that if the production functions are neoclassical, then the essential conclusions of this

$$F_j = A_j K_j^{\alpha_j} N_j^{\beta_j}, \quad j = 1, 2, \quad \alpha_j, \beta_j > 0, \quad \alpha_j + \beta_j = 1, \tag{2.3.1}$$

where K_j and N_j are respectively capital and labor inputs of sector j. We assume perfect competition in the product markets and factor markets. We also assume that product prices, denoted by p_1 and p_2, are given exogenously. This assumption is acceptable, for instance, if the country is open and small. Assume labor and capital are freely mobile between the two sectors and are immobile internationally. This implies that the wage and rate of interest are the same in different sectors but may vary between countries. Let w and r stand for wage and rate of interest respectively. Profits of the two sectors, π_j, are given by

$$\pi_j = p_j F_j - w N_j - r K_j.$$

The marginal conditions for maximizing profits are given by

$$r = \frac{\alpha_j p_j F_j}{K_j}, \quad w = \frac{\beta_j p_j F_j}{N_j}. \tag{2.3.2}$$

The amount of factors employed in each sector is constrained by the endowments found in the economy. These resource constraints are given

$$K_1 + K_2 = K, \quad N_1 + N_2 = N. \tag{2.3.3}$$

It is more realistic to use "$\le$" instead of "$=$". We will use equalities for simplicity of discussion.

Equations (2.3.2) and (2.3.3) contain 6 variables, N_j, K_j, w and r, and 6 equations for given p_j, N and K. We now show that the six variables can be solved as functions of p_j, N and K. First, from Eqs. (2.3.2), we have

$$\frac{\alpha_1 p_1 F_1}{K_1} = \frac{\alpha_2 p_2 F_2}{K_2}, \quad \frac{\beta_1 p_1 F_1}{N_1} = \frac{\beta_2 p_2 F_2}{N_2}. \tag{2.3.4}$$

From these relations, we have $N_1 = \alpha k N_2$, where $\alpha \equiv \alpha_2 \beta_1 / \alpha_1 \beta_2$ and $k \equiv K_1 / K_2$. From $N_1 = \alpha k N_2$ and $N_1 + N_2 = N$, we determine the labor distribution as a function of the ratio of the two sectors' capital inputs as follows

section holds. Some of the discussions in this section are based on Leamer (1984) and Feenstra (2004: Chap. 1).

$$N_1 = \frac{\alpha k N}{1 + \alpha k}, \quad N_2 = \frac{N}{1 + \alpha k}. \tag{2.3.5}$$

Substituting $F_j = A_j K_j^{\alpha_j} N_j^{\beta_j}$ into $\beta_1 p_1 F_1 / N_1 = \beta_2 p_2 F_2 / N_2$ yields

$$\beta_1 p_1 A_1 = \beta_2 p_2 A_2 K_2^{\alpha_2 - \alpha_1} N_2^{\beta_2 - \beta_1} (k\alpha)^{\alpha_1}, \tag{2.3.6}$$

where we also use $K_1 = kK_2$ and $N_1 = \alpha k N_2$. Here we require $\alpha_1 \neq \alpha_2$. From $k = K_1 / K_2$ and $K_1 + K_2 = K$, we have $K_2 = K/(1 + k)$. Substituting this equation and N_2 in (2.3.5) into Eq. (2.3.6) yields

$$\frac{1 + \alpha k}{1 + k} = \alpha_0, \tag{2.3.7}$$

where

$$\alpha_0 \equiv \left(\frac{\beta_1 p_1 A_1}{\alpha^{\alpha_1} \beta_2 p_2 A_2} \right)^{1/(\alpha_2 - \alpha_1)} \frac{N}{K}.$$

We solve the above equation in k as follows

$$k = \frac{1 - \alpha_0}{\alpha_0 - \alpha}. \tag{2.3.8}$$

Two goods are produced if $k > 0$. This is guaranteed if (1) $1 > \alpha_0 > \alpha$ or (2) $1 < \alpha_0 < \alpha$. The parameter, α_0, lies between 1 and α. In the case of $\alpha > 1$, that is, $\alpha_2 > \alpha_1$, we should require

$$\left(\frac{\beta_1}{\beta_2} \right)^{\beta_1} \left(\frac{\alpha_1}{\alpha_2} \right)^{\alpha_1} < \frac{p_2 A_2}{p_1 A_1} \left(\frac{N}{K} \right)^{\alpha_1 - \alpha_2} < \left(\frac{\beta_1}{\beta_2} \right)^{\beta_2} \left(\frac{\alpha_1}{\alpha_2} \right)^{\alpha_2}. \tag{2.3.9}$$

It is direct to show that under $\alpha_2 > \alpha_1$, the right-hand side of (2.3.8) is greater than the left-hand side. Hence, under "proper" combinations of technological levels,[10] relative price and factor endowments, we have a unique positive solution $k > 0$. We can similarly discuss Case (1). In the rest of this section, we require $\alpha_2 > \alpha_1$ and (2.3.8) to be held. Once we

[10] It is difficult to explicitly interpret economic implications of these conditions as a whole.

solve k, it is straightforward to solve all the other variables. From $k = K_1 / K_2$ and $K_1 + K_2 = K$, we have

$$K_1 = \frac{kK}{1+k}, \quad K_2 = \frac{K}{1+k}. \tag{2.3.10}$$

The labor distribution is given by Eqs. (2.3.4). As the distributions of the factor endowments are already determined, it is straightforward for us to calculate the output levels and factor prices.

As the production functions are neoclassical, the wage and rate of interest are determined as functions of capital intensities, K_j / N_j. We now find out the expressions for the capital intensities. Insert $F_j = A_j K_j^{\alpha_j} N_j^{\beta_j}$ in Eqs. (2.3.4)

$$\frac{K_1}{N_1} = \left(\frac{\alpha_1 p A_1}{\alpha_2 A_2}\right)^{1/\beta_1} \left(\frac{K_2}{N_2}\right)^{\beta_2/\beta_1}, \quad \frac{K_1}{N_1} = \left(\frac{\beta_2 A_2}{\beta_1 p A_1}\right)^{1/\alpha_1} \left(\frac{K_2}{N_2}\right)^{\alpha_2/\alpha_1}, \tag{2.3.11}$$

where $p \equiv p_1 / p_2$ and we have repeatedly made use of the fact that $\alpha_j + \beta_j = 1$. We solve Eqs. (2.3.11) as

$$\frac{K_1}{N_1} = a_1 p^{\beta_0}, \quad \frac{K_2}{N_2} = a_2 p^{\beta_0}, \tag{2.3.12}$$

in which $\beta_0 \equiv 1/(\beta_1 - \beta_2)$ and

$$a_1 \equiv \left(\frac{A_1}{A_2}\right)^{\beta_0} \left(\frac{\beta_1}{\beta_2}\right)^{\beta_2 \beta_0} \left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2 \beta_0}, \quad a_2 \equiv \left(\frac{A_1}{A_2}\right)^{\beta_0} \left(\frac{\beta_1}{\beta_2}\right)^{\beta_1 \beta_0} \left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1 \beta_0}.$$

It is important to note that the capital intensities are independent of N and K. From marginal conditions (2.3.2) and $F_1 = A_1 K_1^{\alpha_1} N_1^{\beta_1}$, we have

$$r = \frac{\alpha_1 A_1 p_2^{\beta_1 \beta_0}}{a_1^{\beta_1} p_1^{\beta_2 \beta_0}}, \quad w = \frac{\beta_1 A_1 a_1^{\alpha_1} p_1^{\alpha_2 \beta_0}}{p_2^{\alpha_1 \beta_0}}. \tag{2,3,13}$$

We obtain the well-known factor price insensitivity lemma.

Lemma 2.3.1 (Factor Price Insensitivity)
So long as two goods are produced, then each price vector (p_1, p_2) corresponds to unique factor prices (w, r).

This lemma also implies that the factor endowments (N, K) do not affect (w, r). "Factor price insensitivity" is referred to the result that in a two-by-two economy, with fixed product prices, it is possible that the labor force or capital has no effects on its factor price. This property does hold even for the one-sector Ricardian model introduced in Sect. 2.2.

Another direct implication of our analytical results is Samuelson's factor price equalization theorem.[11]

Theorem 2.3.1 (Factor Price Equalization Theorem, Samuelson, 1949) Suppose that two countries are engaged in free trade, having identical technologies but different factor endowments. If both countries produce both goods, then the factor prices (w, r) are equalized across the countries.

When trade takes place, then the relative price, p, is the same across the countries. As the two countries have the identical technologies, that is, α_j and A_j are identical across the countries, from Eqs. (2.3.13) we see that Samuelson's theorem holds. This theorem says that trade in goods may equalize factor prices across the countries even when production factors are immobile. One may consider that trade in goods is a perfect substitute for trade in factors. It should be remarked that in the Ricardian model, this result does not hold – equalization of the product price through trade would not equalize wage rates across countries. In the Ricardian economy, the labor-abundant country would be paying a lower wage.

Another well-known question in the trade literature is that when product prices are changed, how the factor prices will be changed. Taking derivatives of Eqs. (2.3.13) with respect to p_1 and p_2 results in

$$\frac{1}{r}\frac{dr}{dp_1} = \frac{\beta_2}{(\alpha_1 - \alpha_2)p_1}, \quad \frac{1}{w}\frac{dw}{dp_1} = -\frac{\alpha_2}{(\alpha_1 - \alpha_2)p_1},$$

$$\frac{1}{w}\frac{dw}{dp_2} = \frac{\alpha_1}{(\alpha_1 - \alpha_2)p_2}, \quad \frac{1}{r}\frac{dr}{dp_2} = -\frac{\beta_1}{(\alpha_1 - \alpha_2)p_2}, \tag{2.3.14}$$

[11] A general treatment of the subject for any finite dimensional case is referred to Nishimura (1991).

where we use $\beta_0 = -1/(\alpha_1 - \alpha_2)$. If $\alpha_2 > \alpha_1$, an increase in goods 1's price will increase the wage rate and reduce the rate of interest, and an increase in goods 2's price will reduce the wage rate and raise the rate of interest.

We now introduce another important concept. For the two industries, we say that industry 1 is capital-intensive if[12]

$$\frac{K_1}{N_1} > \frac{K_2}{N_2}.$$

If the inequality is inverse, then industry 1 is labor-intensive. From Eq. (2.3.12), it is straightforward to show

$$\frac{K_1}{N_1} - \frac{K_2}{N_2} = (\alpha_1 - \alpha_2)\frac{a_2 p^{\beta_0}}{\alpha_2 \beta_1}. \tag{2.3.15}$$

We see that the sign of $K_1/N_1 - K_2/N_2$ is the same as that of $\alpha_1 - \alpha_2$. If $\alpha_2 > \alpha_1$, then industry 1 is labor intensive.

Another important issue is related to changes in the real values, w/p_j and r/p_j, in terms of goods. As we have already explicitly solved the model, it is straightforward to calculate the effects of these changes. From Eqs. (2.3.14), we have

$$\frac{d(r/p_1)}{dp_1} = \frac{\beta_1}{(\alpha_1 - \alpha_2)}\frac{r}{p_1^2} < 0, \quad \frac{d(w/p_1)}{dp_1} = -\frac{\alpha_1 w}{(\alpha_1 - \alpha_2)p_1^2} > 0. \tag{2.3.16}$$

The real rate of interest falls and real wage rises under $\alpha_2 > \alpha_1$. As the condition of $\alpha_2 > \alpha_1$ implies that industry 1 is labor intensive, the price of goods 1 rises the price of the factor that is intensively used and reduces the price of the other factor. This is the Stolper-Samuelson (1941) Theorem.

Theorem 2.3.2 (Stolper-Samuelson Theorem)
An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor.

The implies that when product price changes because of changes in, for instance, export conditions or tariffs, there will be both gainers and losers due to the change. This implies that trade has distributional consequences

[12] Similarly, we say that industry 1 is labor intensive if $N_1/K_1 > K_2/N_2$.

within the country, which make some people worse off and some better off, even though the aggregated result for the national economy is beneficial.

We now examine effects of changes in the endowments. From Eq. (2.3.8), we have

$$\frac{1}{k}\frac{dk}{dN} = \frac{-(1-\alpha)\alpha_0}{(1-\alpha_0)(\alpha_0-\alpha)N} > 0,$$

$$\frac{1}{k}\frac{dk}{dK} = \frac{(1-\alpha)\alpha_0}{(1-\alpha_0)(\alpha_0-\alpha)K} < 0. \tag{2.3.17}$$

Under $\alpha_2 > \alpha_1$ and $1 < \alpha_0 < \alpha$, $dk/dN > 0$ and $dk/dK < 0$. Hence, an increase in either of the factor endowments reduces the ratio of capital stocks employed by industry 1 and industry 2. From Eqs. (2.3.12), we have

$$\frac{1}{K_1}\frac{dK_1}{dN} = \frac{1}{N_1}\frac{dN_1}{dN} = \frac{1}{(1+k)k}\frac{dk}{dN} > 0,$$
$$\frac{1}{K_2}\frac{dK_2}{dN} = \frac{1}{N_2}\frac{dN_2}{dN} = -\frac{1}{(1+k)}\frac{dk}{dN} < 0,$$

$$\frac{N_1}{N_2}\frac{d(N_1/N_2)}{dN} = \frac{1}{k}\frac{dk}{dN} > 0, \quad \frac{N_1}{N_2}\frac{d(N_1/N_2)}{dK} = \frac{1}{k}\frac{dk}{dK} < 0.$$

An increase in either of the factor endowments reduces the ratio of labor force employed by industry 1 and industry 2. From Eqs. (2.3.10) and (2.3.12), we obtain

$$\frac{1}{K_1}\frac{dK_1}{dK} = \frac{1}{N_1}\frac{dN_1}{dK} = \frac{1}{(1-\alpha_0)K} < 0,$$

$$\frac{1}{K_2}\frac{dK_2}{dK} = \frac{1}{N_2}\frac{dN_2}{dK} = -\frac{1}{(1+k)}\frac{dk}{dK} + \frac{1}{K} > 0. \tag{2.3.18}$$

We note that changes in the endowments have no effect on the wage and the rate of interest. From Eqs. (2.3.2), we directly obtain

$$\frac{1}{F_1}\frac{dF_1}{dK} = \frac{1}{K_1}\frac{dK_1}{dK} < 0, \quad \frac{1}{F_2}\frac{dF_2}{dK} = \frac{1}{K_2}\frac{dK_2}{dK} > 0,$$

$$\frac{1}{F_1}\frac{dF_1}{dN} = \frac{1}{K_1}\frac{dK_1}{dN} > 0, \quad \frac{1}{F_2}\frac{dF_2}{dN} = \frac{1}{K_2}\frac{dK_2}{dN} < 0. \tag{2.3.19}$$

We notice that industry 1 is labor intensive and industry 2 is capital intensive. Equations (2.3.19) state another important theorem in the trade theory.

Theorem 2.3.3 (Rybczynski Theorem, 1955)
An increase in a factor endowment will increase the output of the industry using it intensively, and reduce the output of the other industry.

An often cited example for applying this theorem is the so called "Dutch Disease"[13]. It was observed that the discovery of oil off the coast of the Netherlands had led to an increase in industries making use of this resource and a decrease in other traditional export industries. The Rybczynski theorem predicts that for a small open economy, the increase in the resource would encourage the industry which uses the resource intensively and reduce the other industry, with all the other conditions fixed.

Using our alternative approach to the common dual approach, we have demonstrated the main conclusions about the standard two-factor too-goods model for a small economy. As we have explicitly solved the equilibrium problem with the Cobb-Douglas functions, it is straightforward for us to prove the factor price insensitivity lemma, Samuelson' factor price equalization theorem, Stolper-Samuelson theorem, and Rybczynski's theorem. In fact, as shown in the literature,[14] these theorems hold for general (neoclassical) production functions. In our approach, we use the neoclassical production functions: $F_j = F_j(K_j, N_j)$. Marginal conditions for maximizing profits are given by

$$r = p_j f_j'(k_j), \quad w = p_j\left[f_j(k_j) - k_j f_j'(k_j)\right], \tag{2.3.20}$$

where

$$k_j \equiv \frac{K_j}{N_j}, \quad f_j(k_j) \equiv \frac{F_j(K_j, N_j)}{N_j}.$$

From $p_1 f_1'(k_1) = p_2 f_2'(k_2)$, we find k_2 as a function k_1, denoted as $k_2 = \phi(k_1)$. It can be shown $\phi' > 0$. From

[13] See Corden and Neary (1982) and Jones et al. (1987).
[14] For instance, Borkakoti (1998).

$$p_1\left[f_1(k_1) - k_1 f_1^{'}(k_1)\right] = p_1\left[f_2(\phi(k_1)) - \phi(k_1) f_2^{'}(\phi(k_1))\right].$$

This equation contains a single variable. From this equation, we solve k_1. We see that k_2, w and r are uniquely determined as functions of k_1. As k_1 is independent of N and K, k_2, w and r are also independent of the factor endowments. From Eqs. (2.3.3) and the definitions of k_j, we have the following four equations for the four variables

$$N_1 + N_2 = N, \; K_1 + K_2 = K, \; \frac{K_j}{N_j} = k_j,$$

where k_j are already known. We solve the above equations as

$$N_1 = \frac{K - k_2 N}{k_1 - k_2}, \; N_2 = \frac{k_1 N - K}{k_1 - k_2},$$

$$K_1 = \frac{(K - k_2 N) k_1}{k_1 - k_2}, \; K_2 = \frac{(k_1 N - K) k_2}{k_1 - k_2}.$$

We thus solved all the variables. It is not difficult to examine the comparative statics results of the model with the neoclassical production functions.

Another important case of the 2×2 model is that either capital or labor is specific to the sector so that there is no capital or labor movement. A common assumption is that capital is specific to the sector but labor can move freely between the sectors. The rental rates for capital employed by two sectors may vary. The 2×2 model with specific-factors is called the Ricardo-Viner or Jones-Neary model.[15] The production functions are now given by $F_j(K_j^*, N_j)$, where K_j^* are fixed levels of capital. The marginal conditions for capital are given by

$$r_j = p_j f_j^{'}(k_j),$$

where r_j is the rate of interest for capital j. It is straightforward to analyze behavior of the factor-specific model.[16]

[15] See Viner (1931, 1950), Jones (1971) and Neary (1978a, 1978b).

[16] See Wong (1995) and Markusen et al. (1995). The specific-factor model is extended in two directions. First, Mussa (1974), Mayer (1974a) and Grossman (1983) regard sector specificity as a short-run phenomenon and in the long run capital is mobile. Second, the extension is to treat the capital stocks in the two sec-

2.4 The Dual Approach to the Two-Good, Two-Factor Model

We now illustrate the dual approach commonly used in the literature of international economics to examine the equilibrium properties of the two-good two-sector model in Sect. 2.3.[17] This section is based on Feenstra (2004: Chap. 1).[18]

The basic assumptions are similar to the assumptions in Sect. 2.3. When a symbol stands for the same variable, we will not explain it. The neoclassical production functions are $y_j = F_j(K_j, N_j)$, where y_j is the output of good j. The resource constraints are

$$K_1 + K_2 \leq K, \ \ N_1 + N_2 \leq N. \tag{2.4.1}$$

Maximizing the amount of good 2, $y_2 = F_2(K_2, N_2)$, subject to a given amount of good 1, $y_1 = F_1(K_1, N_1)$, and the resource constraints (2.4.1), yields $y_2 = h(y_1, K, N)$. Under the assumptions of perfect competition, the economy will maximize gross domestic product (GDP)

$$G(p_1, p_2, K, N) = \max_{y_j \geq 0} p_1 y_1 + p_2 y_2 \quad \text{s.t.:} \ \ y_2 = h(y_1, K, N).$$

The first-order condition for this problem is

$$p = \frac{p_1}{p_2} = -\frac{\partial h}{\partial y_1} = -\frac{\partial y_2}{\partial y_1}.$$

The economy produces where the relative price is equal to the slope of the production possibility frontier. The function, G, has some "nice properties" for analyzing the equilibrium problem. Taking derivatives of this function with respect to prices yields

tors as two different types of factors. This implies a three-factor, two-sector model (see, for instance, Batra and Casas, 1976; Ruffin, 1981; Thompson, 1986; and Wong, 1990).

[17] The dual approach has been widely applied in static trade theory. Except this example, this study does not follow this approach in deriving the classical results of trade theory.

[18] Explanations in detail and geometric illustrations are referred to Feenstra (2004). We also refer this case to Appleyard and Field (2001).

$$\frac{\partial G}{\partial p_j} = y_j + \left(p_1 \frac{\partial y_1}{\partial p_j} + p_2 \frac{\partial y_2}{\partial p_j} \right) = y_j ,$$

where we use the envelope theorem.[19]

The unit-cost functions which are dual to the production functions, $F_j(K_j, N_j)$, are defined by

$$c_j(r, w) = \min_{K_j, N_j \geq 0} \{rK_j + wN_j \,|\, F_j(K_j, N_j) \geq 1\}. \qquad (2.4.2)$$

Because of the assumption of constant returns to scale, the unit-costs are equal to both marginal cost and average costs. The unit-cost functions are nondecreasing and concave in (r, w). Let us express the optimal solution of problem (2.4.2) as

$$c_j(r, w) = ra_{jK}(r, w) + wa_{jN}(r, w),$$

where a_{jK} and a_{jN} are respectively the optimal choice of K_j and N_j. They are functions of (r, w). According to the envelope theorem, we have

$$\frac{\partial c_j}{\partial r} = a_{jK} , \quad \frac{\partial c_j}{\partial w} = a_{jN} . \qquad (2.4.3)$$

The zero-profit conditions are represented by

$$p_j = c_j(r, w), \quad j = 1, 2. \qquad (2.4.4)$$

The full employment conditions are now represented by

$$a_{1K}y_1 + a_{2K}y_2 = K, \quad a_{1N}y_1 + a_{2N}y_2 = N. \qquad (2.4.5)$$

We now have four equations, (2.4.4) and (2.4.5) and four variables, r, w, y_1 and y_2, with four parameters, p_1, p_2, K and N. We are prepared to prove the factor price insensitivity lemma, Samuelson' factor price

[19] The theorem states that when we differentiate a function that has been maximized with respect to an exogenous variable, then we can ignore the changes in the endogenous variables in this derivative. In fact, by taking partial derivatives of $y_2 = h(y_1, K, N)$ with respect to p_j and using $\partial h / \partial y_1 = - p_1 / p_2$, we obtain

$$p_1 \frac{\partial y_1}{\partial p_j} + p_2 \frac{\partial y_2}{\partial p_j} = 0.$$

equalization theorem, Stolper-Samuelson theorem, and Rybczynski's theorem.

As $c_j(r, w)$ does not contain K and N, from Eqs. (2.4.4) we can solve the factor prices as unique functions of the product prices under certain conditions.[20] That is, Lemma 2.3.1 holds according to the dual approach. It is straightforward to see that Samuelson's factor price equalization theorem also holds. To prove the Stolper-Samuelson theorem, we take total differentiation of Eqs. (2.4.4)

$$dp_j = a_{jK} dr + a_{jN} dw, \ j = 1, 2,$$

where we use Eqs. (2.4.3). We may rewrite the above equations as

$$\frac{dp_j}{p_j} = \frac{ra_{jK}}{c_j}\frac{dr}{r} + \frac{wa_{jw}}{c_j}\frac{dw}{w}, \ j = 1, 2.$$

Let $\theta_{jK} \equiv ra_{jK} / c_j$ and $\theta_{jN} \equiv wa_{jN} / c_j$ respectively denote the cost shares of capital and labor. Then, the above equations can be expressed as

$$\hat{p}_j = \theta_{jK}\hat{r} + \theta_{jN}\hat{w}, \ j = 1, 2,$$

in which a variable with circumflex ^ represents the percentage change of the variable, for instance, $\hat{p}_j = dp_j / p_j = d\ln p_j$.[21] We solve the two linear equations in $\hat{r}$ and $\hat{w}$ as

$$\hat{r} = \frac{(\theta_{1N} - \theta_{2N})\hat{p}_2 - (\hat{p}_1 - \hat{p}_2)\theta_{2N}}{\theta_{1N} - \theta_{2N}},$$

$$\hat{w} = \frac{(\theta_{1K} - \theta_{2K})\hat{p}_1 + (\hat{p}_1 - \hat{p}_2)\theta_{1K}}{\theta_{1N} - \theta_{2N}},$$

where we use $\theta_{jK} + \theta_{jN} = 1$. For convenience of discussion, assume henceforth that industry 1 is labor intensive, that is, $L_1 / K_1 > L_2 / K_2$. We have the following relations

[20] These conditions are that both goods are produced and factor intensity reversals do not occur. The latter means that the two zero-profit conditions intersect only once.

[21] Expressing the equation using the cost shares and percentage changes follow Jones (1965) and is referred to as "Jones' algebra".

$$\frac{L_1}{K_1} > \frac{L_2}{K_2} \Leftrightarrow \theta_{1N} > \theta_{2N} \Leftrightarrow \theta_{1K} < \theta_{2K}.$$

Moreover, suppose that the relative price of good 1 increases, so that $\hat{p} = \hat{p}_1 - \hat{p}_2 > 0$. With these assumptions, we have

$$\hat{r} < \hat{p}_2, \quad \hat{w} > \hat{p}_1 > \hat{p}_2.$$

The above inequalities are the contents of the Stolper-Samuelson theorem.

To confirm the Rybczynski theorem, totally differentiate Eqs. (2.4.5)

$$a_{1K} dy_1 + a_{2K} dy_2 = dK,$$
$$a_{1N} dy_1 + a_{2N} dy_2 = dN,$$

where we use the fact that the wage and rate of interest are independent of the resource endowments. Rewrite the above equations as

$$\lambda_{1K} \hat{y}_1 + \lambda_{2K} \hat{y}_2 = \hat{K},$$
$$\lambda_{1N} \hat{y}_1 + \lambda_{2N} \hat{y}_2 = \hat{N}, \tag{2.4.6}$$

where $\lambda_{jK} \equiv y_j a_{jK} / K = K_i / K$ and $\lambda_{jN} \equiv y_j a_{jN} / N = N_j / N$ respectively denote the fraction of capital and the labor force employed in industry j. We also have $\lambda_{jK} + \lambda_{jN} = 1$. As industry 1 is labor intensive, we have $\lambda_{1N} - \lambda_{2N} > 0$. First, we examine the case of $\hat{N} > 0$ and $\hat{K} = 0$. We solve Eqs. (2.4.6) as

$$\hat{y}_1 = \frac{\lambda_{2K} \hat{N}}{\lambda_{2K} - \lambda_{2N}} > \hat{N}, \quad \hat{y}_2 = \frac{-\lambda_{1K} \hat{N}}{\lambda_{2K} - \lambda_{2N}} < 0. \tag{2.4.7}$$

We can similarly examine the case of $\hat{N} = 0$ and $\hat{K} > 0$. The Rybczynski theorem is thus proved.

2.5 The Heckscher-Ohlin Theory

The classical distinction introduced by Ricardo and maintained by most of his followers has factors of production trapped within national boundaries. Only final commodities can be traded. The Heckscher-Ohlin theory shows that international trade in commodities could alleviate the discrepancy between countries in relative factor endowments. This takes places indirectly when countries export those commodities that use intensively the factors in relative abundance. In 1933, Ohlin, a Swedish economist, published his renowned *Interregional and International Trade*. The book built an economic theory of international trade from earlier work by Heckscher (another Swedish economist, Ohlin's teacher) and his own doctoral thesis.[22] The theory is now known as the Heckscher-Ohlin model, one of the standard models in the literature of international economics. Ohlin used the model to derive the so-called Heckscher-Ohlin theorem, predicting that nations would specialize in industries most able to utilize their mix of national resources efficiently. Importing commodities that would use domestic scarce factors if they were produced at home can relieve the relative scarcity of these factors. Hence, free trade in commodities could serve to equalize factor prices between countries with the same technology, even though the production inputs do not have an international market.

The Ricardian model and Heckscher-Ohlin model are two basic models of trade and production. They provide the pillars upon which much of pure theory of international trade rests. The so-called Heckscher-Ohlin model has been one of the dominant models of comparative advantage in modern economics. The Heckscher-Ohlin theory emphasizes the differences between the factor endowments of different countries and differences between commodities in the intensities with which they use these factors. The basic model deals with a long-term general equilibrium in which the two factors are both mobile between sectors and the cause of trade is different countries having different relative factor endowments. This theory deals with the impact of trade on factor use and factor rewards. The theory is different from the Ricardian model which isolates differences in technology between countries as the basis for trade. In the Heckscher-Ohlin theory costs of production are endogenous in the sense that they are different in the trade and autarky situations, even when all countries have access to the same technology for producing each good. This model has been a main stream of international trade theory. According to Ethier (1974), this theory has four "core proportions". In the simple case of two-commodity and two-country world economy, we have these four propositions as follows: (1) the factor-price equalization theorem by Lerner (1952) and Samuelson (1948, 1949), stating that free trade in final goods alone brings about complete international equalization of factor prices; (2) the Stolper-Samuelson theory by Stolper and Samuelson (1941), saying that an in-

[22] The original 1919 article by Heckscher and the 1924 dissertation by Ohlin have been translated from Swedish and edited by Flam and Flanders (Heckscher and Ohlin, 1991).

crease in the relative price of one commodity raises the real return of the factor used intensively in producing that commodity and lowers the real return of the other factor; (3) the Rybczynski theorem by Rybczynski (1955), stating that if commodity prices are held fixed, an increase in the endowment of one factor causes a more than proportionate increase in the output of the commodity which uses that factor relatively intensively and an absolute decline in the output of the other commodity; and (4) the Heckscher-Ohlin theorem by Heckscher (1919) and Ohlin (1933), stating that a country tends to have a bias towards producing and exporting the commodity which uses intensively the factor with which it is relatively well-endowed.

The previous section has already confirmed the factor price insensitivity lemma, Samuelson' factor price equalization theorem, Stolper-Samuelson theorem, and Rybczynski's theorem. We now confirm the Heckscher-Ohlin theorem. The original Heckscher-Ohlin model considers that the only difference between countries is the relative abundances of capital and labor. It has two commodities. Since there are two factors of production, the model is sometimes called the "$2\times 2\times 2$ model." The Heckscher-Ohlin theorem holds under, except the assumptions for the two-product two-factor model developed in Sect. 2.3, the following assumptions: (1) capital and labor are not available in the same proportion in both countries; (2) the two goods produced either require relatively more capital or relatively more labor; (3) transportation costs are neglected; (4) consumers in the world have the identical and homothetic taste. Like in Sect. 2.2, we call the two countries as Foreign and Home. We will use the same symbol as in Sect. 2.3 and the variables for Foreign with a tilde ~. We assume that Home is labor abundant, that is, $N/K > \tilde{N}/\tilde{K}$. The two countries have identical technologies. We also assume that good 1 is labor intensive. Trade is balanced, that is, value of exports being equal to value of imports. Under these assumptions, the following Heckscher-Ohlin theorem holds.

Theorem 2.5.1 (Heckscher-Ohlin Theorem)
Each country will export the good that uses its abundant factor intensively.

The theorem implies that Home exports good 1 and Foreign exports 2. In order to determine trade directions, we need mechanisms to determine prices of goods. The analytical results in Sect. 2.3 and or the dual theory in Sect. 2.4 cannot yet determine prices. To determine trade directions, we further develop the economic model in Sect. 2.3. We now introduce a utility function to determine prices in autarky. After we determine the prices in autarky, we can then determine the directions of trade flows. The consumer's utility-maximizing problem is described as

$$Max\ C_1^{\xi_1} C_2^{\xi_2},\ \ s.t.: p_1 C_1 + p_2 C_2 = Y,$$

where C_j is the consumption level of good j, ξ_1 and ξ_2 are positive parameters, and Y is the total income given by

$$Y = rK + wL.$$

For simplicity, we require $\xi_1 + \xi_2 = 1$. The optimal solution is given by

$$p_j C_j = \xi_j Y.$$

As $C_j = F_j$, we have

$$\frac{p_1 F_1}{p_2 F_2} = \frac{\xi_1}{\xi_2}.$$

Substituting $r = \alpha_j p_j F_j / K_j$ into the above equation yields $k = \xi$, where we use $k = K_1 / K_2$ and $\xi = \alpha_1 \xi_1 / \alpha_2 \xi_2$. Substituting Eq. (2.3.8) into the above equation yields

$$np^{1/(\alpha_2 - \alpha_1)} = \frac{1 + \alpha\xi}{(\xi + 1)\alpha^*}, \tag{2.5.1}$$

where

$$n \equiv \frac{N}{K},\ \ \alpha^* \equiv \left(\frac{\beta_1 A_1}{\alpha^{\alpha_1} \beta_2 A_2}\right)^{1/(\alpha_2 - \alpha_1)}.$$

Equation (2.5.1) determines the relative price in Home. According to the assumptions that the two countries have the identical technology and preference, the values of the parameters for Foreign corresponding to α, ξ and α^* are equal to the values of α, ξ and α^*. We thus have

$$np^{1/(\alpha_2 - \alpha_1)} = \tilde{n}\tilde{p}^{1/(\alpha_2 - \alpha_1)}. \tag{2.5.2}$$

The assumption of $N / K > \tilde{N} / \tilde{K}$ implies $n > \tilde{n}$. For $n > \tilde{n}$ and Eq. (2.5.2) to hold, we should have

$$p^{1/(\alpha_2 - \alpha_1)} < \tilde{p}^{1/(\alpha_2 - \alpha_1)}.$$

The assumption that good 1 is labor intensive implies $\alpha_2 > \alpha_1$. As $\alpha_2 - \alpha_1 > 0$, we have $p < \tilde{p}$, that is

$$\frac{p_1}{p_2} < \frac{\tilde{p}_1}{\tilde{p}_2}.$$

Hence, when the two countries are in autarky, the relative price in Home is lower than the relative price in Foreign. This implies that when the two countries start to conduct trade, good 1 is exported to Foreign and good 2 is imported from Foreign. We have thus confirmed the Heckscher-Ohlin theorem. It should be noted that we do not determine trade volumes. We will not further examine this model in this section as we will study a more general trade equilibrium model later on.

The Heckscher-Ohlin model was a break-through because it showed how comparative advantage might be related to general features of a country's capital and labor. Although the theory cannot describe how these features vary over time, it can be used to provide insights into some simple dynamic trade issues. In the light of modern analysis, Ohlin's original work was not sophisticated. The original model has been generalized and extended since the 1930s. Mundell (1957) first developed a geometric exploration of the model with substitute relationship between factor movements and commodity trade in a Heckscher-Ohlin setting. Here, by trade in commodities being a substitute for international mobility of factors we mean that the volume of trade in commodities is diminished if factors are allowed to see their highest return in global markets. Mundell analyzed a two-country economy in which the two countries share the same technologies for producing the same two commodities with different factor endowments. Free trade leads to a trade pattern that the relatively capital-abundant country exports its relatively capital-intensive commodity, and the return to capital equalized between countries. Notable contributions were made by Paul Samuelson, Ronald Jones, and Jaroslav Vanek.[23] In the modern literature, these syntheses are sometimes called the Heckscher-Ohlin-Samuelson (HOS) model and the Heckscher-Ohlin-Vanek (HOV) model. We now mention a few basic results from the HOV model.[24]

The economy of the HOV model consists of C countries (indexed by $i = 1, ..., C$), J industries (indexed by $j = 1, ..., J$), and M factors (indexed by $\kappa, \ell = 1, ..., M$). Assume that technologies and tastes are identical across countries and factor price equalization prevails under free trade.

[23] These developments introduce many real-world considerations into the basic analytical framework, even though the fundamental role of variable factor proportions in driving international trade remains.

[24] The rest of this section is based on Feenstra (2004: Chaps. 2 and 3). Wong (1997) represents a comprehensive treatment of the subject.

Let $a_{j\kappa}$ stand for the amount of factor κ needed for one unit of production in industry j, and $A \equiv [a_{j\kappa}]^T_{M\times J}$. The matrix is valid for any country. Let Y^i and D^i represent respectively the $(J\times 1)$ vectors of outputs in each industry and demands of each good in country i. Country $i's$ net exports vector is

$$T^i = Y^i - D^i.$$

The factor content of trade is defined as $F^i \equiv AT^i$, which is a $(M\times 1)$ vector. Let F^i_κ and F^i_ℓ represent respectively the individual positive and negative components of F^i. The HOV model reveals the relation between the factor content of trade and the endowments of the country. We note that AY^i represents the demand for factors in the country. Let $V^i = AY^i$. Since product prices are equalized across countries, the consumption vectors of all countries must be proportional to each other. Hence, we can express $D^i = s^i D^w$, where D^w is the world consumption vector and s^i is the share of country i in the world consumption. As trade is balanced, s^i also represents the country's share in the world GDP. Since world consumption equals world production, we have

$$AD^i = s^i AD^w = s^i AY^w = s^i V^w.$$

We thus have the following relations

$$F^i \equiv AT^i = V^i - s^i V^w. \tag{2.5.3}$$

This equation represents the content of the HOV theorem. If country $i's$ endowment of factor κ relative to the world endowment exceeds its share of world GDP, that is, $V^i_\kappa / V^w_\kappa > s^i$, that is, $F^i > 0$, we say that country i is abundant in that factor. When we have two factors, capital and labor, then the following theorem holds.

Theorem 2.5.2 (Leamer, 1980)

Let there be only two production factors, capital and labor. If capital is abundant relative to labor in country i, then the HOV theorem implied by Eqs. (2.5.3) means that the capital/labor ratio embodied in production for country i exceeds the capital/labor ratio embodied in consumption

$$\frac{K^i}{L^i} > \frac{K^i - F^i_K}{L^i - F^i_\ell}, \tag{2.5.4}$$

where K^i and L^i are respectively the capital and labor endowments for country i.

This theorem holds for only two product factors. It has become clear that the results for the $2 \times 2 \times 2$ model are not valid for many countries with many factors and many products.[25] The Heckscher-Ohlin model has been extended and generalized in many other ways. For instance, Purvis (1972) proposed a trade model, showing that trade in commodities and mobility of factors might be complements. By complements, it means that opening up factor mobility could cause the previous level of international trade in commodities to rise. In Purvis' framework, the pattern of trade might reflect different technologies between countries that happen to be endowed with the same factor endowment proportions. If the home country has an absolute technological advantage in producing the labor-intensive commodity which will be exported in the free trade system, its wage rate will be higher. Free migration attracts the foreign labor because of the higher wage. Consequently, free trade expands the volume of exports. In this case, trade in commodities and factor mobility is complements. Markusen (1983) synthesized the ideas in the two approaches, concluding that if trade is a refection of endowment differences, commodities and factors are substitutes, while if trade is prompted by other differences, they can be compliments. A further examination of these ideas is referred to Jones (2000). Leontief tried to empirically test the theory, concluding that the theory is empirically not valid.[26] Leontief observed that the United States had a lot of capital. According to the Heckscher-Ohlin theory, the United States should export capital-intensive products and import labor-intensive products. But he found that that it exported products that used more labor than the products it imported. This observation is known as the Leontief paradox. From the assumptions made in the Heckscher-Ohlin theory, it is evident that the assumptions are strict. An early attempt to solve the paradox was made by Linder in 1961. The Linder hypothesis emphasizes demand aspects of international trade in contrast to the usual

[25] Reviews about the literature on equilibrium trade models with many goods and many factors is referred to, for instance, Wong (1997) and Feenstra (2004).

[26] Many other researches are conducted to test the theory, for instance, Leamer (1980), Bowen et al. (1987), Trefler (1993, 1995), and Davis and Weinstein (2001).

supply-oriented theories involving factor endowments. Linder predicted that nations with similar demands would develop similar industries. These nations would then trade with each other in similar but differentiated goods. Batra and Beladi (1990) propose a two-good and two-factor trade model with unemployment. It is observed that in spite of the presence of trade, the capital-rich countries have higher wages but lower capital rents than the labor surplus and/or land-rich countries. This conflicts with the factor-price equalization theorems of the Heckscher-Ohlin international trade theory. Also, labor-rich countries usually export either labor-intensive or land-using commodities. Assuming that wages are institutionally fixed, Battra and Beladi demonstrate some phenomena which are not compatible with what the Heckscher-Ohlin theory predicts.

2.6 The Neoclassical Trade Theory

The Ricardian theory failed to determine the terms of trade, even though it can be used to determine the limits in which the terms of trade must lie. The Heckscher-Ohlin theory provides simple and intuitive insights into the relationships between commodity prices and factor prices, factor supplies and factor rewards, and factor endowments and the pattern of production and trade. Although the Heckscher-Ohlin model was the dominant framework for analyzing trade in the 1960s, it had neither succeeded in supplanting the Ricardian model nor had been replaced by the specific-factor trade models. Each theory has been refined within its own 'scope'. Each theory is limited to a range of questions. It is argued that as far as general ideas are concerned, the Heckscher-Olin theory may be considered as a special case of the neoclassical theory introduced in this section as it accepts all the logical promises of neoclassical methodology.[27] The Heckscher-Olin theory may be seen as a special case of the neoclassical trade theory in which production technology and preferences are internationally identical.

It was recognized long ago that in order to determine the terms of trade, it is necessary to build trade theory which not only takes account of the productive side but also the demand side.[28] The neoclassical theory holds that the determinants of trade patterns are to be found simultaneously in the differences between the technologies, the factor endowments, and the tastes of different countries.[29] Preference accounts for the existence of international trade even if technologies and factor endowments were com-

[27] For instance, Gandolfo (1994a).
[28] For instance, Negishi (1972), Dixit and Norman (1980), and Jones (1979).
[29] See Mill (1848) and Marshall (1890).

pletely identical between countries. As an illustration of the neoclassical trade theory, we show how Mill solved the trade equilibrium problem and how this problem can be solved with help of modern analytical tool. Mill introduced the equation of international demand, according to which the terms of trade are determined so as to equate the value of exports and the value of imports. Mill argued: "the exports and imports between the two countries (or, if we suppose more than two, between each country and the world) must in the aggregate pay for each other, and must therefore be exchanged for one another at such values as will be compatible with the equation of the international demand.[30]" He initiated the theory of reciprocal demand which is one of the earliest examples of general equilibrium analysis in trade theory. In Chap. 18, book 3 of his *Principles*, he showed the existence of trade equilibrium, using a simplified model and explicitly solving equations in the model numerically. He assumed that there exists only one factor of production and production is subjected to constant returns to scale and requires on the demand side as follows: "Let us therefore assume, that the influence of cheapness on demand conforms to some simple law, common to both countries and to both commodities. As the simplest and most convenient, let us suppose that in both countries any given increase of cheapness produces an exactly proportional increase of consumption; or, in other words, that the value expended in the commodity, the cost incurred for the sake of obtaining it, is always the same, whether that cost affords a greater or a smaller quantity of the commodity.[31]" As a numerical example, consider that the world economy consists of Germany and England and the economic system has two goods, cloth and linen. Let us assume that in Germany 10 yards of cloth was exchanged for 20 yards of linen and that England wants to sell $1{,}000{,}000$ yards of cloth to Germany. If Germany wants $800{,}000$ yards of cloth, this is equal to $1{,}600{,}000$ yards of linen at German exchange ratio. Since German expended value in cloth is constant, England will receive $1{,}600{,}000$ yards of linen in exchange of $1{,}000{,}000$ yards of cloth, replacing Germany supply of cloth entirely. Under the assumption mentioned above and some additional requirements, Mill explicitly solved the international exchange ratio of two commodities in terms of coefficients of production in two countries and by so doing showed the existence of trade equilibrium. Chipman pointed out that the case analyzed by Mill can be treated as a problem of non-linear

[30] Mill (1848: 596).
[31] Mill (1848: 598).

programming and the existence of trade equilibrium can be proved by the existence theorem of a solution of non-linear programming.[32]

We now use analytical methods to prove the existence of trade equilibrium as shown by Mill.[33] This example also illustrates the difference between the Ricardian theory and the neoclassical theory. Let subscript indexes 1 and 2 represent respectively Germany and England. We denote the amount of cloth and linen produced by country j respectively y_{jc} and y_{jl} which are non-negative. If we denote the total amount of cloth (linen) produced in country j when the country is completely specified in producing cloth (linen) by a_{jc} (a_{jl}), the possible sets of y_{jc} and y_{jl} are given by

$$\frac{y_{jc}}{a_{jc}} + \frac{y_{jl}}{a_{jl}} \le 1, \quad y_{jc}, y_{jl} \ge 0, \quad j = 1, 2. \tag{2.6.1}$$

The above two equations mean that the demand for labor does not exceed the supply in each country. We denote respectively the prices of cloth and linen by p_c and p_l. At equilibrium country j should choose (y_{jc}, y_{jl}) such that the following GDP is maximized

$$\frac{p_c}{p_l} y_{jc} + y_{jl}.$$

Multiplying (2.6.1) by a_{jc} ($j = 1, 2$) and adding the two equations, we get

$$y_c + \frac{a_{1c}}{a_{1l}} y_{1l} + \frac{a_{2c}}{a_{2l}} \le a_c,$$

where

$$y_c \equiv y_{1c} + y_{2c}, \quad a_c \equiv a_{1c} + a_{2c}.$$

If we assume that Germany has the comparative advantage in linen, i.e., $a_{1c} / a_{1l} < a_{2c} / a_{2l}$, from the above inequality we get

$$\frac{y_c}{a_{1c}} + \frac{y_l}{a_{1l}} \le \frac{a_c}{a_{1c}}, \tag{2.6.2}$$

[32] See Chipman (1965a, 1965b) and Negishi (1972).
[33] See Negishi (1972).

where $y_l \equiv y_{1l} + y_{2l}$. Similarly multiplying (2.6.1) by a_{jl}, we get

$$\frac{y_c}{a_{2c}} + \frac{y_l}{a_{2l}} \le \frac{a_l}{a_{2l}}, \tag{2.6.3}$$

where $a_l \equiv a_{1l} + a_{2l}$. In order to describe the demand, let $x_c\ (\ge 0)$, $x_l\ (\ge 0)$ and $R\ (\ge 0)$ respectively stand for the demand for cloth, demand for linen, and income measured in terms of the factor of production. Maximizing the following utility

$$U = x_c x_l,$$

subject to the budget constraint $p_c x_c + p_l x_l = R$ yields the demand functions

$$x_c = \frac{R}{2p_c}, \quad x_l = \frac{R}{2p_l},$$

which satisfy Mill's assumption. Since the two countries have an identical preference structure but different incomes, we have that country $j's$ demand for cloth and linen, X_{jc} and X_{jl}, are given by

$$X_{jc} = \frac{R_j}{2p_c}, \quad X_{jl} = \frac{R_j}{2p_l}, \quad j = 1, 2, \tag{2.6.4}$$

where R_j is country $j's$ income. Since demands for commodities cannot exceed supplies at the equilibrium of free international trade, we have

$$X_c \le y_{1c} + y_{2c}, \quad X_l \le y_{1l} + y_{2l}, \tag{2.6.5}$$

where

$$X_c \equiv X_{1c} + X_{2c}, \quad X_l \equiv X_{1l} + X_{2l}.$$

Introduce the world utility function as

$$U = \log X_c + \log X_l.$$

We maximize this U subject to (2.6.1) and (2.6.5). The Lagrangean is given by

$$\log X_c + \log X_l + p_c(y_{1c} + y_{2c} - X_c) + p_l(y_{1l} + y_{2l} - X_l)$$
$$+ \sum_{j=1}^{2} w_j \left(\frac{y_{jc}}{a_{jc}} + \frac{y_{jl}}{a_{jl}} - 1 \right).$$

It is shown that the Lagrangean has a strictly positive saddle point at which (2.6.1) and (2.6.5) are satisfied with equality at the saddle point. In fact, this saddle point is an equilibrium of free international trade, with p_c / p_l, w_1 / p_l and w_2 / p_l respectively satisfying the price of cloth, the price of factor of production in Germany and in England. Since the world total income is equal to

$$p_c X_c + p_l X_1 = w_1 + w_2,$$

we have $R_j = w_j$. By (2.6.4) we get X_{jc} and X_{jl} which is an optimal solution of the problem that country j maximizes its utility subject to its budget constraint with the given world prices.

2.7 A General Two-Country Two-Good Two-Factor Trade Model

Section 2.3 examined a two-good two-factor model with fixed prices. Section 2.5 determined prices for an autarky economy by studying households' utility-maximizing behavior. Section 2.6 showed how the neoclassical economic trade theory determines trade pattern for a two-country world with a single factor. This section develops a general equilibrium model for a two-country two-sector two-factor economy, synthesizing the models in the previous sectors.[34]

2.7.1 The General Equilibrium Model

The two countries are called Home and Foreign. Assume that there are two factors of production, labor and capital. For Foreign, we will use the same symbol that we use for Home, but with a tilde ~. Home's and Foreign's total supplies of capital and labor are fixed and are denoted respectively by,

[34] This section will not analyze pattern of specializations in detail, as we will examine similar issues in Chap. 7 when dealing with economic structures with capital accumulation.

N and K, $\tilde{N}$ and $\tilde{K}$. Each economy may produce two goods with the following Cobb-Douglas production functions

$$\bar{F}_j = \bar{A}_j \bar{K}_j^{\bar{\alpha}_j} \bar{N}_j^{\bar{\beta}_j}, \quad j = 1, 2, \ \bar{\alpha}_j, \bar{\beta}_j > 0, \ \bar{\alpha}_j + \bar{\beta}_j = 1, \tag{2.7.1}$$

where $\bar{K}_j$ and $\bar{N}_j$ are respectively capital and labor inputs of sector j in Home and Foreign. A variable with macron ¯ stands for both Home and Foreign. We assume perfect competition in the product markets and factor markets. Let p_j stand for the price of good j. Assume labor and capital are freely mobile between the two sectors and are immobile internationally. This implies that the wage and rate of interest are the same in different sectors but may vary between countries. Let $\bar{w}$ and $\bar{r}$ stand for, respectively, wage and rate of interest in Home and Foreign. Marginal conditions for maximizing profits are given by

$$\bar{r} = \frac{\bar{\alpha}_j p_j \bar{F}_j}{\bar{K}_j}, \quad \bar{w} = \frac{\bar{\beta}_j p_j \bar{F}_j}{\bar{N}_j}. \tag{2.7.2}$$

The amount of factors employed in each sector is constrained by the endowments found in the economy. These resource constraints are given

$$\bar{K}_1 + \bar{K}_2 = \bar{K}, \quad \bar{N}_1 + \bar{N}_2 = \bar{N}. \tag{2.7.3}$$

Each country's income is given by

$$\bar{Y} = \bar{r}\bar{K} + \bar{w}\bar{N}. \tag{2.7.4}$$

The consumer's utility-maximizing problems are described as

$$Max\ \bar{C}_1^{\bar{\xi}_{01}} \bar{C}_2^{\bar{\xi}_{02}}, \quad s.t.: p_1\bar{C}_1 + p_2\bar{C}_2 = \bar{Y}, \ \bar{\xi}_{01}, \bar{\xi}_{02} > 0,$$

where $\bar{C}_j$ is the consumption level of good j in Home and Foreign. The optimal solution is given by

$$p_j \bar{C}_j = \bar{\xi}_j \bar{Y}_j, \quad j = 1, 2, \tag{2.7.5}$$

where

$$\bar{\xi}_j \equiv \frac{\bar{\xi}_{0j}}{\bar{\xi}_{01} + \bar{\xi}_{02}} > 0, \quad j = 1, 2.$$

We now describe trade balances. The total output of world production of any good is equal its total consumption. That is

$$C_j + \tilde{C}_j = F_j + \tilde{F}_j . \tag{2.7.6}$$

Let X_j and $\tilde{X}_j$ stand for respectively the amount of (net) imports of good j by Home and Foreign. When the variable is negative (positive), then the country exports (imports) that good. A country's consumption plus its exports is equal to its total product. That is

$$\bar{C}_j = \bar{F}_j + \bar{X}_j , \quad j = 1, 2. \tag{2.7.7}$$

The sum of the net exports for any good in the world is equal to zero, that is

$$X_j + \tilde{X}_j = 0, \quad j = 1, 2. \tag{2.7.8}$$

From Eqs. (2.7.7) and (2.7.8), we directly obtain Eqs. (2.7.6). Hence, two equations in (2.7.6)-(2.7.8) are redundant.

In terms of value, any country is in trade balance, that is

$$p_1 \bar{X}_1 + p_2 \bar{X}_2 = 0.$$

From these conditions and Eqs. (2.7.8), we have[35]

$$X_1 = p\tilde{X}_2 , \quad pX_2 = \tilde{X}_1 ,$$

where $p \equiv p_1 / p_2$.

We now solve the model. We have 26 variables, p_1, p_2, $\bar{E}_j$, $\bar{X}_j$, $\bar{F}_j$, $\bar{N}_j$, $\bar{K}_j$, $\bar{w}$ and $\bar{r}$, to determine. First, from Eqs. (2.7.2), we have

$$\frac{\bar{\alpha}_1 p_1 \bar{F}_1}{\bar{K}_1} = \frac{\bar{\alpha}_2 p_2 \bar{F}_2}{\bar{K}_2}, \quad \frac{\bar{\beta}_1 p_1 \bar{F}_1}{\bar{N}_1} = \frac{\bar{\beta}_2 p_2 \bar{F}_2}{\bar{N}_2}. \tag{2.7.9}$$

From these relations, we have $\bar{N}_1 = \bar{\alpha}\bar{k}\bar{N}_2$, where $\bar{\alpha} \equiv \bar{\alpha}_2 \bar{\beta}_1 / \bar{\alpha}_1 \bar{\beta}_2$ and $\bar{k} \equiv \bar{K}_1 / \bar{K}_2$. From $\bar{N}_1 = \bar{\alpha}\bar{k}\bar{N}_2$ and $\bar{N}_1 + \bar{N}_2 = \bar{N}$, we determine the labor distribution as a function of the ratio of the two sectors' capital inputs as follows

$$\bar{N}_1 = \frac{\bar{\alpha}\bar{k}\bar{N}}{1 + \bar{\alpha}\bar{k}}, \quad \bar{N}_2 = \frac{\bar{N}}{1 + \bar{\alpha}\bar{k}}. \tag{2.7.10}$$

[35] This is also obtainable from Walras's law.

Substituting $\bar{F}_j = \bar{A}_j \bar{K}_j^{\bar{\alpha}_j} \bar{N}_j^{\bar{\beta}_j}$ into $\bar{\beta}_1 p_1 \bar{F}_1 / \bar{N}_1 = \bar{\beta}_2 p_2 \bar{F}_2 / \bar{N}_2$ yields

$$\bar{\beta}_1 p_1 \bar{A}_1 = \bar{\beta}_2 p_2 \bar{A}_2 \bar{K}_2^{\bar{\alpha}_2 - \bar{\alpha}_1} \bar{N}_2^{\bar{\beta}_2 - \bar{\beta}_1} \bar{\alpha}^{\bar{\alpha}_1}, \tag{2.7.11}$$

where we also use $\bar{N}_1 = \bar{\alpha} \bar{k} \bar{N}_2$. As in Sect. 2.3, we require $\bar{\alpha}_1 \neq \bar{\alpha}_2$. From $\bar{k} = \bar{K}_1 / \bar{K}_2$ and $\bar{K}_1 + \bar{K}_2 = \bar{K}$, we have $\bar{K}_2 = \bar{K} / \left(1 + \bar{k}\right)$. Substituting $\bar{K}_2 = \bar{K} / \left(1 + \bar{k}\right)$ and $\bar{N}_2$ in (2.7.10) into Eq. (2.7.11) yields

$$\frac{1 + \bar{\alpha} \bar{k}}{1 + \bar{k}} = \bar{\alpha}_0 p^{\bar{\upsilon}}, \tag{2.7.12}$$

where

$$p \equiv \frac{p_1}{p_2}, \ \ \bar{\alpha}_0 \equiv \left(\frac{\bar{\beta}_1 \bar{A}_1}{\bar{\alpha}^{\bar{\alpha}_1} \bar{\beta}_2 \bar{A}_2} \right)^{\bar{\upsilon}} \frac{\bar{N}}{\bar{K}}, \ \ \bar{\upsilon} \equiv \frac{1}{\bar{\alpha}_2 - \bar{\alpha}_1}.$$

We solve the above equation in k as follows

$$\bar{k} = \frac{1 - \bar{\alpha}_0 p^{\bar{\upsilon}}}{\bar{\alpha}_0 p^{\bar{\upsilon}} - \bar{\alpha}}. \tag{2.7.13}$$

The two goods are produced in Home if $k > 0$ and in Foreign if $\tilde{k} > 0$. We have $\bar{k} > 0$ if (1) $1 > \bar{\alpha}_0 p^{\bar{\upsilon}} > \bar{\alpha}$ or (2) $1 < \bar{\alpha}_0 p^{\bar{\upsilon}} < \bar{\alpha}$. The variables, $\bar{\alpha}_0 p^{\bar{\upsilon}}$, lies between 1 and $\bar{\alpha}$. In the case of $\bar{\alpha} > 1$, that is, $\bar{\alpha}_2 > \bar{\alpha}_1$, we should require[36]

$$\left(\frac{\bar{\beta}_1}{\bar{\beta}_2} \right)^{\bar{\beta}_1} \left(\frac{\bar{\alpha}_1}{\bar{\alpha}_2} \right)^{\bar{\alpha}_1} < \frac{\bar{A}_2}{p \bar{A}_1} \left(\frac{\bar{N}}{\bar{K}} \right)^{\bar{\alpha}_1 - \bar{\alpha}_2} < \left(\frac{\bar{\beta}_1}{\bar{\beta}_2} \right)^{\bar{\beta}_2} \left(\frac{\bar{\alpha}_1}{\bar{\alpha}_2} \right)^{\bar{\alpha}_2}. \tag{2.7.14}$$

It is direct to show that under $\bar{\alpha}_2 > \bar{\alpha}_1$, the right-hand side of (2.3.8) is greater than the left-hand side. Hence, under proper combinations of technological levels, relative price and factor endowments, we have a unique positive solution $k > 0$. In the rest of this section, for simplicity we require $\bar{\alpha}_2 > \bar{\alpha}_1$ in Home and Foreign. We omit the other possibilities of $\tilde{\alpha}_2 \leq \tilde{\alpha}_1$ and $\alpha_2 \geq \alpha_1$ or $\tilde{\alpha}_2 \geq \tilde{\alpha}_1$ and $\alpha_2 \leq \alpha_1$.

[36] The conditions guarantee that both countries produce two goods. If these conditions are not satisfied, then one or two countries may specialize in producing a single good.

Once we solve $\bar{k}$, it is straightforward to solve all the other variables. From $\bar{k} = \bar{K}_1 / \bar{K}_2$ and $\bar{K}_1 + \bar{K}_2 = \bar{K}$, we have

$$\bar{K}_1 = \frac{\bar{k}\bar{K}}{1+\bar{k}}, \quad \bar{K}_2 = \frac{\bar{K}}{1+\bar{k}}. \tag{2.7.15}$$

The labor distribution is given by Eqs. (2.7.10). As the distributions of the factor endowments are determined as unique functions of the relative price, we can calculate the output levels and factor prices.

As the production functions are neoclassical, the wage and rate of interest are determined as functions of capital intensities, $\bar{K}_j / \bar{N}_j$. We now find the expressions for the capital intensities. Insert $\bar{F}_j = \bar{A}_j \bar{K}_j^{\bar{\alpha}_j} \bar{N}_j^{\bar{\beta}_j}$ in Eqs. (2.7.9)

$$\frac{\bar{K}_1}{\bar{N}_1} = \left(\frac{\bar{\alpha}_1 p \bar{A}_1}{\bar{\alpha}_2 \bar{A}_2}\right)^{1/\bar{\beta}_1} \left(\frac{\bar{K}_2}{\bar{N}_2}\right)^{\bar{\beta}_2/\bar{\beta}_1}, \quad \frac{\bar{K}_1}{\bar{N}_1} = \left(\frac{\bar{\beta}_2 \bar{A}_2}{\bar{\beta}_1 p \bar{A}_1}\right)^{1/\bar{\alpha}_1} \left(\frac{\bar{K}_2}{\bar{N}_2}\right)^{\bar{\alpha}_2/\bar{\alpha}_1}. \tag{2.7.16}$$

We solve Eqs. (2.3.11) as

$$\frac{\bar{K}_1}{\bar{N}_1} = \bar{a}_1 p^{\bar{\upsilon}}, \quad \frac{\bar{K}_2}{\bar{N}_2} = \bar{a}_2 p^{\bar{\upsilon}}, \tag{2.7.17}$$

in which

$$\bar{a}_1 \equiv \left(\frac{\bar{A}_1}{\bar{A}_2}\right)^{\bar{\upsilon}} \left(\frac{\bar{\beta}_1}{\bar{\beta}_2}\right)^{\bar{\beta}_2 \bar{\upsilon}} \left(\frac{\bar{\alpha}_1}{\bar{\alpha}_2}\right)^{\bar{\alpha}_2 \bar{\upsilon}}, \quad \bar{a}_2 \equiv \left(\frac{\bar{A}_1}{\bar{A}_2}\right)^{\bar{\upsilon}} \left(\frac{\bar{\beta}_1}{\bar{\beta}_2}\right)^{\bar{\beta}_1 \bar{\upsilon}} \left(\frac{\bar{\alpha}_1}{\bar{\alpha}_2}\right)^{\bar{\alpha}_1 \bar{\upsilon}}.$$

We note that the capital intensities are independent of $\bar{N}$ and $\bar{K}$. From marginal conditions (2.7.2) and $\bar{F}_1 = \bar{A}_1 \bar{K}_1^{\bar{\alpha}_1} \bar{N}_1^{\bar{\beta}_1}$, we have

$$\bar{r} = \frac{\bar{\alpha}_1 \bar{A}_1 p_2^{\bar{\beta}_1 \bar{\upsilon}}}{\bar{a}_1^{\bar{\beta}_1} p_1^{\bar{\beta}_2 \bar{\upsilon}}}, \quad \bar{w} = \frac{\bar{\beta}_1 \bar{A}_1 \bar{a}_1^{\bar{\alpha}_1} p_1^{\bar{\alpha}_2 \bar{\upsilon}}}{p_2^{\bar{\alpha}_1 \bar{\upsilon}}}. \tag{2.7.18}$$

From the definitions of $\bar{Y}$ and the marginal conditions, it is straightforward to show $\bar{Y} = \bar{F}_1 + \bar{F}_2$. From this equation, $\bar{Y} = \bar{r}\bar{K} + \bar{w}\bar{N}$ and $\bar{F}_j = \bar{A}_j \bar{K}_j^{\bar{\alpha}_j} \bar{N}_j^{\bar{\beta}_j}$, we have

$$\bar{r}\bar{K} + \bar{w}\bar{N} = \bar{A}_1 \bar{K}_1^{\bar{\alpha}_1} \bar{N}_1^{\bar{\beta}_1} + \bar{A}_2 \bar{K}_2^{\bar{\alpha}_2} \bar{N}_2^{\bar{\beta}_2}.$$

Substituting Eqs. (2.7.18) and (2.7.17) into the above equation yields

$$\left(\frac{\overline{\alpha}_1 \overline{A}_1 \overline{K}}{\overline{a}_1^{\beta_1}} + \overline{\beta}_1 \overline{A}_1 \overline{a}_1^{\overline{\alpha}_1} p^{\overline{\upsilon}} \overline{N}\right) p_2 = \overline{A}_1 \overline{N}_1 \overline{a}_1^{\overline{\alpha}_1} p^{(\overline{\alpha}_1 + \overline{\beta}_2)\overline{\upsilon}} + \overline{A}_2 \overline{N}_2 \overline{a}_2^{\overline{\alpha}_2} p^{\overline{\upsilon}} .$$

Insert Eqs. (2.7.10) and (2.7.13) in the above equation

$$\left(\overline{n} + \overline{b} p^{\overline{\upsilon}}\right) p_1 = \overline{A}_1 \overline{\alpha} \overline{a}_1^{\overline{\alpha}_1} \left(1 - \overline{\alpha}_0 p^{\overline{\upsilon}}\right) + \left(\overline{\alpha}_0 p^{\overline{\upsilon}} - \overline{\alpha}\right) \overline{A}_2 \overline{a}_2^{\overline{\alpha}_2} p, \tag{2.7.19}$$

where

$$\overline{n} \equiv \frac{(1 - \overline{\alpha}) \overline{\alpha}_0 \overline{\alpha}_1 \overline{A}_1 \overline{K}}{\overline{a}_1^{\beta_1} \overline{N}}, \quad \overline{b} \equiv (1 - \overline{\alpha}) \overline{\alpha}_0 \overline{\beta}_1 \overline{A}_1 \overline{a}_1^{\overline{\alpha}_1} .$$

Dividing the two equations in (2.7.19) yields

$$\Omega(p) \equiv \frac{n + bp^{\upsilon}}{\tilde{n} + \tilde{b} p^{\tilde{\upsilon}}} - \frac{\left(1 - \alpha_0 p^{\upsilon}\right) A_1 \alpha a_1^{\alpha_1} + \left(\alpha_0 p^{\upsilon} - \alpha\right) A_2 a_2^{\alpha_2} p}{\left(1 - \tilde{\alpha}_0 p^{\tilde{\upsilon}}\right) \tilde{A}_1 \tilde{\alpha} \tilde{a}_1^{\tilde{\alpha}_1} + \left(\tilde{\alpha}_0 p^{\tilde{\upsilon}} - \tilde{\alpha}\right) \tilde{A}_2 \tilde{a}_2^{\tilde{\alpha}_2} p} = 0. \tag{2.7.20}$$

The equation, $\Omega(p) = 0$, contains a single variable, p. Once we determine a meaningful solution of the equation, all the other variables in the system are uniquely determined as functions of the solution.

Lemma 2.7.1
Assume that $\alpha_2 > \alpha_1$ and $\tilde{\alpha}_2 > \tilde{\alpha}_1$. If the equation, $\Omega(p) = 0$, has a positive solution satisfying (2.7.14), then each country produces two goods. The world trade equilibrium is determined by the following procedure: p by (2.7.20) $\rightarrow$ p_1 by (2.7.19) $\rightarrow$ $p_2 = p_1 / p$ $\rightarrow$ $\overline{r}$ and $\overline{w}$ by (2.7.18) $\rightarrow$ $\overline{Y} = \overline{r}\overline{K} + \overline{w}\overline{L}$ $\rightarrow$ $\overline{C}_j$, $j = 1, 2$, by (2.7.5) $\rightarrow$ $\overline{k}$ by (3.7.13) $\rightarrow$ $\overline{K}_j$ by (3.7.15) $\rightarrow$ $\overline{N}_j$ by (3.7.10) $\rightarrow$ $\overline{F}_j$ by (3.7.1) $\rightarrow$ $E_j = -\tilde{E}_j = F_j - C_j$.

It is difficult to interpret the conditions for $\Omega(p) = 0$ to have meaningful solutions. As the problem is difficult to analyze, we are concerned with a special case. First, we examine the Heckscher-Ohlin model, in which all aspects, except the factor endowments, of the two economies are identical. From the definitions of the parameters and Eq. (2.7.20), the relative price is determined by

$$\frac{n + bp^{\upsilon}}{n + \tilde{b} p^{\upsilon}} - \frac{\left(1 - \alpha_0 p^{\upsilon}\right) A_1 \alpha a_1^{\alpha_1} + \left(\alpha_0 p^{\upsilon} - \alpha\right) A_2 a_2^{\alpha_2} p}{\left(1 - \tilde{\alpha}_0 p^{\upsilon}\right) A_1 \alpha a_1^{\alpha_1} + \left(\tilde{\alpha}_0 p^{\upsilon} - \alpha\right) A_2 a_2^{\alpha_2} p} = 0.$$

in which we use $n = \tilde{n}$. From the above equation, we have

$$p = \left(\frac{a_2\alpha_1 + \beta_1 a_1}{a_2\alpha_1 + \alpha\beta_1 a_1} \right) \frac{\alpha a_1^{\alpha_1} A_1}{a_2^{\alpha_2} A_2}.$$

Further calculating yields

$$p = \frac{\alpha_1\beta_1}{\alpha_2\beta_2} > 0. \tag{2.7.21}$$

where we use

$$\frac{a_2}{a_1} = \left(\frac{\beta_1}{\beta_2} \right)\left(\frac{\alpha_2}{\alpha_1} \right), \quad \frac{a_1^{\alpha_1} A_1}{a_2^{\alpha_2} A_2} = \frac{\beta_2}{\beta_1}.$$

If p satisfies (2.7.14), that is

$$\left[\frac{A_1}{A_2}\left(\frac{\beta_1}{\beta_2} \right)^{1+\beta_1} \left(\frac{\alpha_1}{\alpha_2} \right)^{1+\alpha_1} \right]^{1/\upsilon} < \frac{\overline{K}}{\overline{N}} < \left[\frac{A_1}{A_2}\left(\frac{\beta_1}{\beta_2} \right)^{1+\beta_2} \left(\frac{\alpha_1}{\alpha_2} \right)^{1+\alpha_2} \right]^{1/\upsilon},$$

then the problem has a unique equilibrium point and each country produces two goods. The relative price is not dependent on any production factor. From Eq. (2.7.19), we get

$$p_1 = \frac{\left(1 - \alpha_0 p^{\upsilon}\right) A_1 \alpha a_1^{\alpha_1} + \left(\alpha_0 p^{\upsilon} - \alpha\right) A_2 a_2^{\alpha_2} p}{n + bp^{\upsilon}}, \tag{2.7.22}$$

where we use $\alpha_0 = a_2 N / K$. The prices are related to factor endowments. Following Lemma 2.7.1, we can determine all the other variables. Home's net export of good 1 is given by

$$F_1 - C_1 = F_1 - \frac{\xi_1 Y}{p_1}. \tag{2.7.23}$$

From $Y = rK + wN$ and Eqs. (2.7.2), we have

$$\frac{Y}{p_1} = \frac{\alpha_1 F_1 K}{K_1} + \frac{\beta_j F_1 N}{N_1}.$$

Insert this equation into (2.7.23)

$$X_1 = \left(\frac{\alpha_1 \xi_1 K}{K_1} + \frac{\beta_1 \xi_1 N}{N_1} - 1 \right) F_1.$$

Substituting $\overline{K}_1 = \bar{k}\overline{K}/\left(1+\bar{k}\right)$ in (2.7.15) and $\overline{N}_1 = \overline{\alpha}\bar{k}\overline{N}/\left(1+\overline{\alpha}\bar{k}\right)$ in (2.7.10) into the above equation yields

$$X_1 = \left(\frac{\left(\alpha_0 p^{\upsilon} - \alpha\right)}{1-\alpha_0 p^{\upsilon}}\frac{\alpha_1}{\alpha_2} - \frac{\xi_2}{\xi_1}\right)\xi_1 F_1 .$$

It is straightforward to solve all the other variables in the system. We see that different trade patterns may occur in this equilibrium model with heterogeneous tastes and technologies. For instance, country 1 may specialize in production of good 1 and country 2 produce goods 1 and 2. Although this is a simple neoclassical trade model with the Cobb-Douglass production functions and utility functions, it is difficult to get explicit conclusions about trade.[37]

2.8 Public Goods and International Trade

The Ricardian theory is concerned with technology. The Heckscher-Ohlin international trade theory is mainly concerned with factors of production. We have used two-sector and two-factor trade models to show the core trade theorems. This section introduces important determinant, public goods, of international trade to the two-sector and two-factor trade model defined in the previous sections. Public goods are incorporated trade theories in different ways.[38] This section is influenced by Abe (1990).[39]

2.8.1 The Two-Sector Two-Factor Model with Public Input

The world consists of Home and Foreign. As Foreign is similar to Home, first we are concerned with Home. The economy produces two goods,

[37] By examining all possible cases in this simple model, one can obtain many of the important insights that the traditional international trade theories provide. Adding tariffs and transport costs to the model is conceptually easy and can provide more insights into reality. In Sect. 2.8, we will introduce public good into the model, showing how public goods may affect trade pattern.

[38] With regard to public economics, see, Auerbach and Feldetein (1990, 1991) and Jha (1998, 2003). For trade with public sectors, see, for instance, Manning and McMillan (1979), Tawada and Abe (1984), Okamoto (1985), and Ishikawa (1988).

[39] Abe (1990) applies the cost-minimization approach, while this section uses profit-maximization approach with the Cobb-Douglas functions.

called good 1 and good 2. There are two primary factors, labor and capital, and one pure public intermediate good. The total supplies of capital and labor, K and N, are fixed. Let G stand for the amount of the public intermediate good. For firms G is given. The economy produces two goods with the following Cobb-Douglas production functions

$$F_j = A_j G^{v_j} K_j^{\alpha_j} N_j^{\beta_j}, \ j = 1, 2, \ v_j \geq 0, \ \alpha_j, \beta_j > 0, \ \alpha_j + \beta_j = 1, \tag{2.8.1}$$

where K_j and N_j are respectively capital and labor inputs of sector j. We assume perfect competition in the product markets and factor markets. We also assume that product prices, denoted by p_1 and p_2, are given exogenously. Marginal conditions for maximizing profits are given by

$$r = \frac{\alpha_j p_j F_j}{K_j}, \ w = \frac{\beta_j p_j F_j}{N_j}. \tag{2.8.2}$$

In the rest of this section, we choose $p_2 = 1$ and express $p = p_1$. Public good is also produced by combining capital and labor. The production function of the public sector is specify as

$$G = A_p K_p^{\alpha_p} N_p^{\beta_p}, \ \alpha_p, \beta_p > 0, \ \alpha_p + \beta_p = 0, \tag{2.8.3}$$

where K_p and N_p are respectively capital and labor inputs of the public sector and A_p is the productivity. Assume that the amount of public good is fixed by the government and the public good production is financed by the income tax.[40] The total cost of the public sector is $rK_p + wN_p$. Minimizing the total cost subject to the constraint (2.8.3), we obtain the following marginal conditions

$$\omega = \frac{\beta_p K_p}{\alpha_p N_p},$$

where $\omega \equiv w/r$. From this equation and Eq. (2.8.3), we can express the optimal levels of K_p and N_p as functions of r, w and G as follows

[40] This assumption follows Abe (1990). Indeed, there are different ways of financing public good sector (see Jha, 1998). In a growth model with public good proposed by Zhang (2005a), tax rates on producers are fixed by the government. The common approach to determining levels of public goods is to assume that the government makes decision on tax and/or public goods by maximizing some social welfare function.

$$N_p = \frac{A_\alpha G}{\omega^{\alpha_p}}, \;\; K_p = A_\beta G \omega^{\beta_p}, \tag{2.8.4}$$

where

$$A_\alpha = \left(\frac{\beta_p}{\alpha_p}\right)^{\alpha_p} \frac{1}{A_p}, \;\; A_\beta = \left(\frac{\alpha_p}{\beta_p}\right)^{\beta_p} \frac{1}{A_p}.$$

Let τ stand for the tax rate on the total income, $Y = rK + wN$. Then we have

$$rK_p + wN_p = \tau(rK + wN).$$

From this equation and (2.8.4), we can determine the tax rate as a function of r, w and G

$$\tau = \frac{\tau_0 G \omega^{\beta_p}}{1 + n\omega}, \tag{2.8.5}$$

where

$$\tau_0 \equiv \left[\left(\frac{\alpha_p}{\beta_p}\right)^{\beta_p} + \left(\frac{\beta_p}{\alpha_p}\right)^{\alpha_p}\right]\frac{1}{KA_p}, \;\; n \equiv \frac{N}{K}.$$

We determine the tax rate as a function of the public good and the wage-rental ratio. The amount of factors employed in each sector is constrained by the endowments found in the economy. These resource constraints are given

$$K_1 + K_2 + K_p = K, \;\; N_1 + N_2 + N_p = N. \tag{2,8,6}$$

The consumer's utility-maximizing problem is described as

$$Max\ C_1^{\xi_1} C_2^{\xi_2}, \;\; s.t.: p_1 C_1 + p_2 C_2 = (1 - \tau)Y,$$

where C_j is the consumption level of good j, ξ_1 and ξ_2 are positive parameters. For simplicity, we require $\xi_1 + \xi_2 = 1$. The optimal solution is given by

$$p_j C_j = (1 - \tau)\xi_j Y. \tag{2.8.7}$$

For an isolated economy, we also have $C_j = F_j$.

We have thus described the model for Home without trade. We can solve equilibrium problem of Foreign's economy in the same way. We now examine how trade direction is determined.

2.8.2 Equilibrium for an Isolated Economy

First, we will determine equilibrium of an economy in autarky. As $C_j = F_j$, from Eqs. (2.8.7) we have

$$\frac{pF_1}{F_2} = \frac{\xi_1}{\xi_2}. \tag{2.8.8}$$

Substituting $r = \alpha_j p_j F_j / K_j$ into the above equation yields $k = \xi$, where we use $k = K_1 / K_2$ and $\xi \equiv \alpha_1 \xi_1 / \alpha_2 \xi_2$. From Eqs. (2.8.2), we have

$$\frac{\alpha_1 p_1 F_1}{K_1} = \frac{\alpha_2 p_2 F_2}{K_2}, \quad \frac{\beta_1 p_1 F_1}{N_1} = \frac{\beta_2 p_2 F_2}{N_2}.$$

From these relations, we have

$$N_1 = \alpha N_2, \tag{2.8.9}$$

where we use $k = \xi$ and $\alpha \equiv \beta_1 \xi_1 / \beta_2 \xi_2$. From Eqs. (2.8.2), we also obtain

$$\omega = \frac{\beta_1 K_1}{\alpha_1 N_1}. \tag{2.8.10}$$

Insert (2.8.4) in (2.8.6)

$$K_1 \left(1 + \frac{1}{\xi}\right) = K - A_\beta G \omega^{\beta_p}, \quad N_1 \left(1 + \frac{1}{\alpha}\right) = N - \frac{A_\alpha G}{\omega^{\alpha_p}}. \tag{2.8.11}$$

We are interested in the case that the both goods are produced, that is, we should have $0 < K_1 < K$ and $0 < N_1 < N$. From (2.8.11), we see that for $\omega > 0$, the conditions are satisfied if

$$\frac{K}{A_\beta G} > \omega^{\beta_p} > \frac{A_\alpha G}{N}. \tag{2.8.12}$$

This implies that the amount of public good should not be too large for given K and N; otherwise the problem has no solution or the economy may specialize in producing a single good.

From Eqs. (2.8.11) and (2.8.10), we obtain

$$\Omega(\omega) \equiv \omega + \left(A_\beta - \alpha_0 A_\alpha\right)\frac{G}{\alpha_0 N}\omega^{\beta_p} - \frac{K}{\alpha_0 N} = 0, \tag{2.8.13}$$

where

$$\alpha_0 \equiv \frac{\alpha_1(1 + 1/\xi)}{\beta_1(1 + 1/\alpha)} = \frac{\alpha_1\xi_1 + \alpha_2\xi_2}{\beta_1\xi_1 + \beta_2\xi_2}.$$

The equation contains a single variable, ω. In the case of $A_\beta - \alpha_0 A_\alpha = 0$, we solve $\omega = K/\alpha_0 N$. We note that by the definitions of the parameters we have

$$A_\beta - \alpha_0 A_\alpha = \frac{\left[\left(\alpha_p - \alpha_1\right)\xi_1 + \left(\alpha_p - \alpha_2\right)\xi_2\right]A_\alpha}{\beta_p\left(\beta_1\xi_1 + \beta_2\xi_2\right)}. \tag{2.8.14}$$

We see that the term $A_\beta - \alpha_0 A_\alpha$ may be either positive or negative. As it is difficult to explicitly interpret conclusions, we just assume that $\Omega(\omega) = 0$ has at least one positive solution which satisfies (2.8.12). As

$$\Omega' = 1 + \left(A_\beta - \alpha_0 A_\alpha\right)\frac{\beta_p G}{\alpha_0 N}\omega^{-\alpha_p},$$

we see that if $A_\beta - \alpha_0 A_\alpha > 0$, then the solution is unique. Once we determine ω, then we determine all the variables by the following procedure: K_1 and N_1 by (2.8.11) $\rightarrow$ N_2 by (2.8.9) $\rightarrow$ K_2 by (2.8.8) $\rightarrow$ N_p and K_p by (2.8.4) $\rightarrow$ τ by (2.8.5) $\rightarrow$ F_j by (2.8.1) $\rightarrow$ C_j by (2.8.8) $\rightarrow$ $p = \alpha\beta_2 F_2/\beta_1 F_1$ [41] $\rightarrow$ r and w by (2.8.2).

2.8.3 Trade Patterns and Public Good Supplies

Section 2.8.2 solves the equilibrium problem when there is no trade between the two economies. We cannot solve the problem explicitly without

[41] This relation is obtained by Eqs. (2.8.2).

further specifying parameter values.[42] For explaining the role of public goods, we are interested in the situation when the two countries are identical in all aspects, except that the two countries have different levels of public goods.
To determine directions of trade, we first determine the relative prices before trade liberalization. Taking derivatives of Eq. (2.8.13), we have

$$\left[1+\left(A_\beta-\alpha_0 A_\alpha\right)\frac{\beta_p G}{\alpha_0 N}\omega^{-\alpha_p}\right]\frac{d\omega}{dG}=-\left(A_\beta-\alpha_0 A_\alpha\right)\frac{\omega^{\beta_p}}{\alpha_0 N}. \tag{2.8.15}$$

In the case of $A_\beta-\alpha_0 A_\alpha=0$, $d\omega/dG=0$. In the case of $A_\beta-\alpha_0 A_\alpha>0$, we have $d\omega/dG<0$. In the case of $A_\beta-\alpha_0 A_\alpha<0$, from Eq. (2.8.3) we have

$$1+\left(A_\beta-\alpha_0 A_\alpha\right)\frac{\beta_p G}{\alpha_0 N}\omega^{-\alpha_p}=\frac{K-\left(A_\beta-\alpha_0 A_\alpha\right)G\alpha_p\omega^{\beta_p}}{\alpha_0\omega N}>0.$$

Hence, we have $d\omega/dG>0$. We conclude that the sign of $d\omega/dG$ is the opposite to that of $A_\beta-\alpha_0 A_\alpha$. From (2.8.14), we see that the sign of $d\omega/dG$ is the same as the sign of the following term

$$\xi\equiv\left(\alpha_1-\alpha_p\right)\xi_1+\left(\alpha_2-\alpha_p\right)\xi_2. \tag{2.8.16}$$

The above discussions are valid for Foreign as well. As the two countries are identical (except in $\overline{G}$[43]), we see that in the case of $\xi>0$, if $G>(<)\,\tilde{G}$, then $\omega>(<)\,\tilde{\omega}$; and in the case of $\xi<0$, if $G>(<)\,\tilde{G}$, then $\omega<(>)\,\tilde{\omega}$, when the two countries are in isolation.

We now compare p and $\tilde{p}$. From Eqs. (2.8.1) and (2.8.2), we have

$$p=\frac{\alpha\beta_2 A_2 G^{v_2}K_2^{\alpha_2}N_2^{\beta_2}}{\beta_1 A_1 G^{v_1}K_1^{\alpha_1}N_1^{\beta_1}}.$$

Substituting $N_1=\alpha N_2$, and $k=\xi$ into the above equation yields

$$p=AG^{v_2-v_1}\omega^{\alpha_2-\alpha_1}, \tag{2.8.17}$$

where we also use Eq. (2.8.10) and

[42] As the procedure of determining all the variables are explicitly given, it is straightforward to simulate various possibilities with computer.
[43] The macron is defined as before.

$$A \equiv \frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_2}{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_1}.$$

Taking derivatives of Eq. (2.8.17) with respect to $\overline{G}$ yields

$$\frac{1}{p}\frac{dp}{dG} = \frac{\nu_2 - \nu_1}{G} + \frac{\alpha_2 - \alpha_1}{\omega}\frac{d\omega}{dG}.$$

Insert Eq. (2.8.15) in the above equation

$$\frac{1}{p}\frac{dp}{dG} = \frac{\nu_2 - \nu_1}{G} - \frac{\left(A_\beta - \alpha_0 A_\alpha\right)\left(\alpha_2 - \alpha_1\right)}{\left[1 + \left(A_\beta - \alpha_0 A_\alpha\right)\beta_p G \omega^{-\alpha_p} / \alpha_0 N\right]\omega^{\alpha_p} \alpha_0 N}.$$

This result is important for determining trade patterns.

The magnitude of υ_j represents the degree of spillover of public input into sector j. If the public good has no effect on the production of sector j, then $\nu_j = 0$. If the public input is effective in increasing the productivity of sector j, the parameter value is high. To determine factor intensities, from $k = \xi$ and $N_1 = \alpha N_2$, we obtain

$$\frac{K_1}{N_1} = \frac{\alpha_1 \beta_2}{\alpha_2 \beta_1}\frac{K_2}{N_2}, \tag{2.8.18}$$

where we use the definitions of ξ and α. We say that sector 1 is relatively capital (labor) intensive if $K_1 / N_1 > (<)\, K_2 / N_2$. From $\alpha_j + \beta_j = 1$, we see that sector 1 is relatively capital (labor) intensive if $\alpha_1 > (<)\, \alpha_2$. We also define that the public sector is capital (labor) intensive relative to the private sectors if

$$\frac{K_p}{N_p} > (<)\, \frac{K_1 + K_2}{N_1 + N_2} = \frac{K - K_p}{N - N_p}.$$

We see that the public sector is relatively capital (labor) intensive if

$$\frac{K_p}{N_p} > (<)\, \frac{K}{N}.$$

From Eqs. (2.8.4), the above inequality is equivalent to

$$\omega > (<) \frac{\beta_p K}{\alpha_p N}.$$

This states that if the wage-rental ratio is higher (lower) than the ratio $\beta_p K / \alpha_p N$, then the public sector is relatively capital (labor) intensive.

We now examine trade pattern. First, we are concerned with the situation when the spillover effects of the public good are the same between the two sectors, i.e., $\nu_1 = \nu_2$. Then, by Eq. (2.8.17), we have

$$\frac{1}{p}\frac{dp}{dG} = \frac{(A_\beta - \alpha_0 A_\alpha)(\alpha_1 - \alpha_2)}{\left[1 + (A_\beta - \alpha_0 A_\alpha)\beta_p G \omega^{-\alpha_p} / \alpha_0 N\right]\omega^{\alpha_p}\alpha_0 N}.$$

We know that denominator is always positive. Hence, the sign of dp / dG is the same as that of

$$\Delta \equiv \left[(\alpha_p - \alpha_1)\xi_1 + (\alpha_p - \alpha_2)\xi_2\right](\alpha_1 - \alpha_2).$$

In the case of $\Delta > 0$, if $G > \widetilde{G}$, then we have $p > \widetilde{p}$. Home imports good 1 and exports 2. According to the above discussions, we have the following lemma.

Lemma 2.8.1
Assume that the two countries have identical preferences, technology, and factor endowments, and the spillover effects of the public good are the same between the two sectors. Then, if $\Delta > (<)\ 0$ and Home supplies more public goods than Foreign, then Home exports (imports) good 2 and imports (exports) good 1.

The case of $\Delta > 0$ occurs, for instance, if $\alpha_p > \alpha_1 > \alpha_2$. It can been seen that with different combinations of α_p, ξ_j and α_j, we have different patterns of trade. Another extreme case is when $\alpha_1 = \alpha_2$. We have

$$\frac{1}{p}\frac{dp}{dG} = \frac{\nu_2 - \nu_1}{G}. \tag{2.8.19}$$

Lemma 2.8.2
Assume that the two countries have identical preferences, technology, and factor endowments, and the two (private) sectors have the same factor in-

tensities. Then, if Home supplies more public goods than Foreign and sector 1's spillover effect is stronger (weaker) than sector 2's, then Home exports (imports) good 1 and imports good 2.

From Eqs. (2.8.19) and (2.8.14), we can explicitly judge the sign of dp/dG in the cases when $\nu_2 - \nu_1$ and Δ have the same sign. If $\nu_2 - \nu_1$ and Δ are positive (negative), then dp/dG is positive (negative). Hence, we have the following lemma.

Lemma 2.8.3
Assume that the two countries have identical preferences, technology, and factor endowments and Home supplies more public goods than Foreign. If $\nu_2 - \nu_1$ and Δ are positive (negative), then Home exports (imports) good 2 and imports (exports) good 1.

If $(\nu_2 - \nu_1)\Delta < 0$, we need further information for judging trade pattern. Like in Abe (1990),[44] We have discussed only the case when the two countries have identical preferences, technology, and factor endowments. It is important to examine what will happen when the two countries have different preferences, technology, factor endowments and public policy.[45]

2.9 Concluding Remarks

Ricardo's initial discussion of the concept of comparative advantage is limited to the case when factors of production are immobile internationally. His arguments about gains from trade between England and Portugal are valid only if English labor and/or Portuguese technology (or climate) are prevented from moving across national boundaries. The Heckscher-Ohlin theory is similarly limited to the study of how movements of commodities can substitute for international movements of productive factors. It is obvious that if technologies are everywhere identical and if production

[44] Abe applies the dual approach. Although the functional forms in Abe's analysis are more general than in this section, as we have explicitly solved the model with different factor endowments, technology and preferences, we can easily discuss more issues which may not be easily discussed by the dual approach.

[45] We don't discuss issues related to validity of the core theorems in trade theory. The problems are examined by Altenburg (1992) in a similar framework as Abe's.

is sufficiently diversified, factor prices become equalized between countries. But if production functions differ between countries, no presumption as to factor equalization remains. Most of early contributions to trade theory deal with goods trade only and ignore international mobility of factors of production. For a long period of time since Ricardo, the classical mobility assumption had been well accepted. This assumption states that all final goods are tradable between countries whereas primary inputs are non-tradable, though they are fully mobile between different sectors of the Home economy. In reality, this classical assumption is invalid in many circumstances. For instance, many kinds of final 'goods', services, are not traded and capitals are fully mobile between countries as well as within Home economies. A great deal of works on trade theory has been concerned with examining consequences of departures from these assumptions. There is an extensive literature on various aspects of international factor mobility.[46] It is also important to introduce transport costs into the models in this section.[47]

To end this chapter, we introduce how to analyze effects of, for instance, a tariff on trade.[48] As we have already solved the model without any trade barriers. We can determine trade direction. For instance, we assume that Home imports good 2 and Foreign imports good 1. We assume that there is no other trade barrier. Let use assume that Home introduces a tariff at *ad valorem* rate, τ. Prices of good 2 differ in Home and Foreign. In Home, the equilibrium price equals $(1+\tau)p_2$, where p_2 is the price of good 2 in Foreign. In the tariff income is given by $\tau p_2(F_2 - C_2)$. This income may be distributed in different ways. We may generally assume that the government distributes $\varphi \tau p_2(F_2 - C_2)$ to the households in Home and the rest to the government expenditure, where the parameter, φ, satisfies $0 \le \varphi \le 1$. With these notations, we can correspondingly determine the equilibrium values of all the variables. After determining the equilibrium values with the given tariff rate, we can then analyze effects of tariff on the two economies. As we can explicitly solve the equilibrium problem, it is not difficult to calculate the effects. Under certain conditions,[49] the tariff

[46] See Jones and Kenen (1984), Ethier and Svensson (1986), Bhagwati (1991), and Wong (1995).

[47] See Steininger (2001: Chap. 2).

[48] A graphical illustration of this case is referred to Bhagwati et al. (1998: Chap. 12).

[49] The condition is presumed stability. See, Jones (1961) and Amano (1968) for the definition of stability.

tends to worsen the terms of trade in Foreign (that is, p_2 / p_1 falls) and encourage the terms of trade in Home (that is, $p_1 / (p_2 + p_2\tau)$ rises).[50]

Appendix

A.2.1 A Ricardian Model with a Continuum of Goods

The single-input version of the Rcardian model has been generalized in different directions. It is straightforward to extend the model to a two-factor model with fixed input-output coefficients. We now represent a well-known generalization of the Ricardian model to encompass a continuum of goods.[51] First, we assume that there is no transaction cost.

We index commodities on an interval $[0, 1]$, in accordance with diminishing home country comparative advantage.[52] A commodity z is associated with each point on the interval. For each commodity there are unit labor requirements, $a(z)$ and $\tilde{a}(z)$ in Home and Foreign. The requirement of diminishing home country comparative advantage on the interval is represented by

$$A(z) \equiv \frac{\tilde{a}(z)}{a(z)}, \quad A'(z) < 0.$$

The relative unit labor requirement function, $A(z)$, is also assumed to be continuous. Let $\overline{w}$ be wages measured in any common unit. Home will produce all those commodities for which domestic unit costs are less than or equal to foreign unit costs. This means that any commodity z will be produced in Home if $a(z)w \le \tilde{a}(z)\tilde{w}$, that is, $\omega \le A(z)$, where $\omega \equiv w / \tilde{w}$. For given ω, from equation $\omega = A(z)$, we uniquely determine

50 The well-known Metzler (1949: 7-8) paradox states that a tariff may actually lower the relative domestic price of the importable.

51 The model below due to Dornbusch et al. (1977). The model is also represented in Rivera-Batiz and Oliva (2003: Sect. 1.2). See also Wilson (1980), Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000, 2007). It should be noted that Dornbusch et al. (1980) propose a model with continuum of goods to examine Heckscher-Ohlin trade theory.

52 An alternative description is to take an interval $[0, \infty]$. See Elliot (1950).

$$z^* = \phi(\omega). \tag{A.2.1.1}$$

Hence, for a given relative wage ω, Home and Foreign will respectively efficiently produce the rages of commodities as follows

$$0 \le z \le \phi(\omega), \ \ \phi(\omega) \le z \le 1.$$

The relative price of a commodity z in terms of any other commodity z', when both goods are produced in Home, is equal to the ratio of home unit labor cost

$$\frac{p(z)}{p(z')} = \frac{a(z)}{a(z')}, \ \ 0 \le z \le \phi(\omega). \tag{A.2.1.2}$$

The relative price of a commodity z produced in Home in terms of any other commodity z'' produced in Foreign is given by

$$\frac{p(z)}{p(z'')} = \frac{\omega a(z)}{\tilde{a}(z'')}, \ \ 0 \le z \le \phi(\omega), \ \ \phi(\omega) \le z \le 1. \tag{A.2.1.3}$$

Assume identical tastes in Home and Foreign and Cobb-Douglas demand functions that associate with commodity z a constant expenditure, $\bar{b}(z)$. We should have

$$b(z) = \tilde{b}(z), \ \ 0 \le z \le 1, \ \ \int_0^1 b(z)dz = 1.$$

Let Y stand for total income and $c(z)$ for demand for commodity z. Then, we have

$$b(z) = \frac{p(z)c(z)}{Y}. \tag{A.2.1.4}$$

We define the fraction of income spent on those goods in which Home has a comparative advantage

$$\Lambda(\phi) \equiv \int_0^{\phi} b(z)dz > 0, \ \ \frac{d\Lambda}{d\phi} = b(\phi) > 0, \ \ 1 > \Lambda(\phi) \ge 0 .$$

The fraction of income spent on commodities produced by Foreign is

$$\tilde{\Lambda}(\phi) \equiv 1 - \int_{\phi}^{1} b(z)dz > 0, \ \ 1 > \tilde{\Lambda}(\phi) \ge 0.$$

Domestic labor income, wN, should equal the total expenditures of the two countries on commodities produced by Home, that is, $wN = \Lambda(\phi)\left(wN + \tilde{w}\tilde{N}\right)$. Hence, $(1 - \Lambda)wN = \Lambda\tilde{w}\tilde{N}$, which states that imports are equal in value to exports. From this equation, we have

$$\omega = \frac{\Lambda\left(z^*\right)}{1 - \Lambda\left(z^*\right)} \frac{\tilde{N}}{N}. \tag{A.2.1.5}$$

This function describes behavior of the demand side, while Eq. (A.2.1.1) shows behavior of the supply side. Equation (A.2.1.5) is illustrated in Fig. A.2.1.1. The curve starts at zero and rises in z^* (to infinity as z^* approaches unity). This equation implies that a proper level of the relative wage ratio is required to equate the demand for domestic labor to the existing supply. Equations (A.2.1.1) and (A.2.1.5) contain two variables, ω and z^*. As shown in Fig. A.2.1.1, there is a unique solution to the equations.

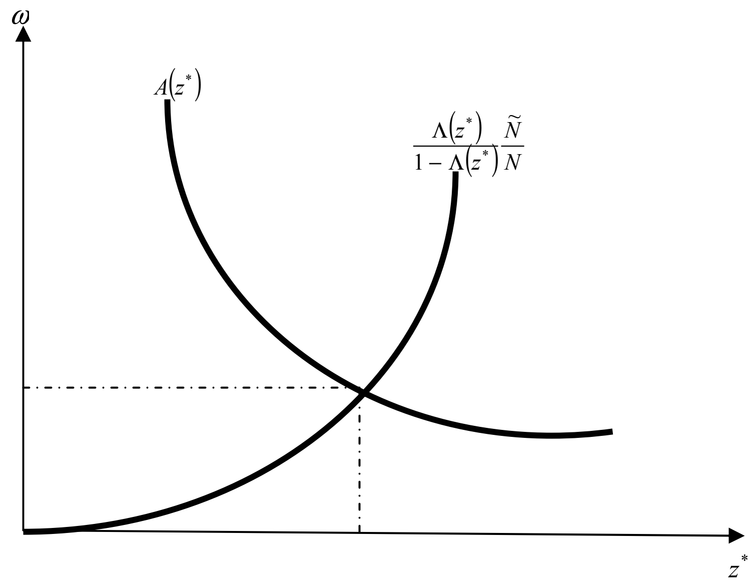

Fig. A.2.1.1. Determination of equilibrium

Once we determine the equilibrium value of z^* (which is the equilibrium borderline of comparative advantage between commodities produced and exported by Home and Foreign. We determine the ranges of production of Home and Foreign as follows $0 \le z \le z^*$ and $z^* \le z \le 1$. The relative price structure is given by Eqs. (A.2.1.2) and (A.2.1.3). The equilibrium levels of production. From $Y = wN + \tilde{w}\tilde{N}$ and Eq. (A.2.1.4), we determine $c(z)$. Let $\overline{N}(z)$ stand for the labor force employed for producing commodity z. Then, the output level of commodity z is equal to $\overline{a}(z)\overline{N}(z)$. From $c(z) = \overline{a}(z)\overline{N}(z)$, we determine $\overline{N}(z)$.

We have thus determined the equilibrium of the Ricardian economy. We now examine effects of changes in some parameters. First, we increase the relative size of labor endowments. An increase in $\tilde{N}/N$ shifts the trade balance equilibrium curve given by (A.2.1.5) upward in proportion to the change in the relative size. From Fig. A.2.1.2, we see that the equilibrium ratio of the relative wages rise and reduces the range of commodities produced in Home. When the labor force is increased, there will initially be a labor excess in Foreign and an excess demand for labor in Home. The resulting increase in Home's wages serves to eliminate the trade surplus and at the same time raise relative unit labor costs in Home. This implies a loss of comparative advantage of Home. We may similarly examine effects of technological change (for instance, through a uniform proportional reduction in $\tilde{a}(z)$).

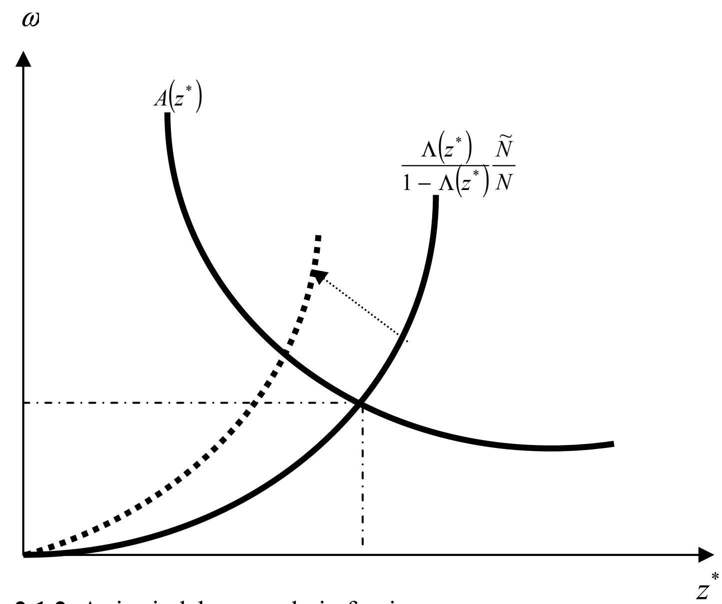

Fig. A.2.1.2. A rise in labor supply in foreign

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