
Contents

Lefschetz Pencils, Branched Covers and Symplectic Invariants

<i>Denis Auroux and Ivan Smith</i>	1
1 Introduction and Background	1
1.1 Symplectic Manifolds	1
1.2 Almost-Complex Structures	3
1.3 Pseudo-Holomorphic Curves and Gromov–Witten Invariants	5
1.4 Lagrangian Floer Homology	6
1.5 The Topology of Symplectic Four-Manifolds	9
2 Symplectic Lefschetz Fibrations	10
2.1 Fibrations and Monodromy	10
2.2 Approximately Holomorphic Geometry	17
3 Symplectic Branched Covers of \mathbb{CP}^2	22
3.1 Symplectic Branched Covers	22
3.2 Monodromy Invariants for Branched Covers of \mathbb{CP}^2	26
3.3 Fundamental Groups of Branch Curve Complements	30
3.4 Symplectic Isotopy and Non-Isotopy	33
4 Symplectic Surfaces from Symmetric Products	35
4.1 Symmetric Products	35
4.2 Taubes’ Theorem	39
5 Fukaya Categories and Lefschetz Fibrations	42
5.1 Matching Paths and Lagrangian Spheres	43
5.2 Fukaya Categories of Vanishing Cycles	44
5.3 Applications to Mirror Symmetry	48
References	50

Differentiable and Deformation Type of Algebraic Surfaces, Real and Symplectic Structures

<i>Fabrizio Catanese</i>	55
1 Introduction	55
2 Lecture 1: Projective and Kähler Manifolds, the Enriques Classification, Construction Techniques	57
2.1 Projective Manifolds, Kähler and Symplectic Structures	57

2.2	The Birational Equivalence of Algebraic Varieties	63
2.3	The Enriques Classification: An Outline	65
2.4	Some Constructions of Projective Varieties	66
3	Lecture 2: Surfaces of General Type and Their Canonical Models: Deformation Equivalence and Singularities	70
3.1	Rational Double Points	70
3.2	Canonical Models of Surfaces of General Type	74
3.3	Deformation Equivalence of Surfaces	82
3.4	Isolated Singularities, Simultaneous Resolution	85
4	Lecture 3: Deformation and Diffeomorphism, Canonical Symplectic Structure for Surfaces of General Type	91
4.1	Deformation Implies Diffeomorphism	92
4.2	Symplectic Approximations of Projective Varieties with Isolated Singularities	93
4.3	Canonical Symplectic Structure for Varieties with Ample Canonical Class and Canonical Symplectic Structure for Surfaces of General Type	95
4.4	Degenerations Preserving the Canonical Symplectic Structure	96
5	Lecture 4: Irrational Pencils, Orbifold Fundamental Groups, and Surfaces Isogenous to a Product	98
5.1	Theorem of Castelnuovo–De Franchis, Irrational Pencils and the Orbifold Fundamental Group	99
5.2	Varieties Isogenous to a Product	105
5.3	Complex Conjugation and Real Structures	108
5.4	Beauville Surfaces	114
6	Lecture 5: Lefschetz Pencils, Braid and Mapping Class Groups, and Diffeomorphism of ABC-Surfaces	116
6.1	Surgeries	116
6.2	Braid and Mapping Class Groups	119
6.3	Lefschetz Pencils and Lefschetz Fibrations	125
6.4	Simply Connected Algebraic Surfaces: Topology Versus Differential Topology	130
6.5	ABC Surfaces	134
7	Epilogue: Deformation, Diffeomorphism and Symplectomorphism Type of Surfaces of General Type	140
7.1	Deformations in the Large of ABC Surfaces	141
7.2	Manetti Surfaces	145
7.3	Deformation and Canonical Symplectomorphism	152
7.4	Braid Monodromy and Chisini’ Problem	154
	References	159

Smoothings of Singularities and Deformation Types of Surfaces

<i>Marco Manetti</i>	169
1 Introduction	169

2	Deformation Equivalence of Surfaces	174
2.1	Rational Double Points	174
2.2	Quotient Singularities	178
2.3	RDP-Deformation Equivalence	181
2.4	Relative Canonical Model	182
2.5	Automorphisms of Canonical Models	183
2.6	The Kodaira–Spencer Map	184
3	Moduli Space for Canonical Surfaces	187
3.1	Gieseker’s Theorem	188
3.2	Constructing Connected Components: Some Strategies	189
3.3	Outline of Proof of Gieseker Theorem	190
4	Smoothings of Normal Surface Singularities	194
4.1	The Link of an Isolated Singularity	194
4.2	The Milnor Fibre	196
4.3	\mathbb{Q} -Gorenstein Singularities and Smoothings	197
4.4	T -Deformation Equivalence of Surfaces	201
4.5	A Non Trivial Example of T -Deformation Equivalence	203
5	Double and Multidouble Covers of Normal Surfaces	204
5.1	Flat Abelian Covers	204
5.2	Flat Double Covers	205
5.3	Automorphisms of Generic Flat Double Covers	207
5.4	Example: Automorphisms of Simple Iterated Double Covers	209
5.5	Flat Multidouble Covers	210
6	Stability Criteria for Flat Double Covers	213
6.1	Restricted Natural Deformations of Double Covers	214
6.2	Openess of $N(a, b, c)$	217
6.3	RDP-Degenerations of Double Covers	218
6.4	RDP-Degenerations of $\mathbb{P}^1 \times \mathbb{P}^1$	221
6.5	Proof of Theorem 6.1	222
6.6	Moduli of Simple Iterated Double Covers	223
	References	225

Lectures on Four-Dimensional Dehn Twists

<i>Paul Seidel</i>	231
1 Introduction	231
2 Definition and First Properties	235
3 Floer and Quantum Homology	249
4 Pseudo-Holomorphic Sections and Curvature	259
References	265

Lectures on Pseudo-Holomorphic Curves and the Symplectic Isotopy Problem

<i>Bernd Siebert and Gang Tian</i>	269
1 Introduction	269
2 Pseudo-Holomorphic Curves	270

XII Contents

2.1	Almost Complex and Symplectic Geometry	270
2.2	Basic Properties of Pseudo-Holomorphic Curves	272
2.3	Moduli Spaces	273
2.4	Applications	276
2.5	Pseudo-Analytic Inequalities	279
3	Unobstructedness I: Smooth and Nodal Curves	281
3.1	Preliminaries on the $\bar{\partial}$ -Equation	281
3.2	The Normal $\bar{\partial}$ -Operator	282
3.3	Immersed Curves	286
3.4	Smoothings of Nodal Curves	287
4	The Theorem of Micallef and White	288
4.1	Statement of Theorem	288
4.2	The Case of Tacnodes	289
4.3	The General Case	291
5	Unobstructedness II: The Integrable Case	292
5.1	Motivation	292
5.2	Special Covers	292
5.3	Description of the Deformation Space	294
5.4	The Holomorphic Normal Sheaf	296
5.5	Computation of the Linearization	299
5.6	A Vanishing Theorem	300
5.7	The Unobstructedness Theorem	301
6	Application to Symplectic Topology in Dimension Four	302
6.1	Monodromy Representations – Hurwitz Equivalence	303
6.2	Hyperelliptic Lefschetz Fibrations	304
6.3	Braid Monodromy and the Structure of Hyperelliptic Lefschetz Fibrations	307
6.4	Symplectic Noether–Horikawa Surfaces	309
7	The \mathcal{C}^0 -Compactness Theorem for Pseudo-Holomorphic Curves	311
7.1	Statement of Theorem and Conventions	311
7.2	The Monotonicity Formula for Pseudo-Holomorphic Maps	312
7.3	A Removable Singularities Theorem	315
7.4	Proof of the Theorem	316
8	Second Variation of the $\bar{\partial}_J$ -Equation and Applications	320
8.1	Comparisons of First and Second Variations	321
8.2	Moduli Spaces of Pseudo-Holomorphic Curves with Prescribed Singularities	323
8.3	The Locus of Constant Deficiency	324
8.4	Second Variation at Ordinary Cusps	328

9 The Isotopy Theorem	332
9.1 Statement of Theorem and Discussion	332
9.2 Pseudo-Holomorphic Techniques for the Isotopy Problem	333
9.3 The Isotopy Lemma	334
9.4 Sketch of Proof	336
References	339
List of Participants	343

Symplectic 4-Manifolds and Algebraic Surfaces

Lectures given at the C.I.M.E. Summer School held in
Cetraro, Italy, September 2-10, 2003

Auroux, D.; Catanese, F.; Manetti, M.; Seidel, P.;

Siebert, B.; Smith, I.; Tian, G. - Catanese, F.; Tian, G.

(Eds.)

2008, XIV, 354 p. 15 illus., Softcover

ISBN: 978-3-540-78278-0