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## First Order Considerations

### 2.1 Quantum Plane-Wave Born Approximation

In quantum considerations of atomic collisions all atomic particles, electrons and nuclei, are treated quantum mechanically. The simplest quantum mechanical approach for considering nonrelativistic collisions of two structured atomic particles – first order plane-wave (plane-wave Born) approximation – was formulated long ago by Bates and Griffing [18–20]. Later on this approximation was used in many papers which were devoted to the different aspects of the projectile-electron excitation and loss in collisions with neutral atoms and simplest molecules. Earlier reviews, discussing the applications of the first order approximations to the projectile–target collisions, are presented in [1, 2, 21, 22], where also references to very many original papers, published before the middle of the nineties, can be found.

Although already first order calculations can be quite formidable for practical implementations, the formulation of the plane-wave Born approximation for nonrelativistic collisions is *per se* elementary. Here we sketch very briefly how one can derive first order cross sections using an approach which permits a natural generalization for the case of relativistic collisions.

Let us consider a collision between a projectile-ion and a target-atom. The charges of the nuclei of the colliding particles are  $Z_I$  and  $Z_A$ , respectively, and  $v$  is the collision velocity. For simplicity we will assume for the moment that each of the colliding atomic particles has initially only one electron. The  $S$ -matrix element, describing transitions in the colliding system, can be quite generally written as

$$S_{fi} = -i \int_{-\infty}^{+\infty} dt \int d^3\mathbf{x} \varrho_I(\mathbf{x}, t) \varphi_A(\mathbf{x}, t). \quad (2.1)$$

Here  $\varrho_I(\mathbf{x}, t)$  is the transition charge density, created by the projectile at time  $t$  and space point  $\mathbf{x}$ , and  $\varphi_A(\mathbf{x}, t)$  is the transition scalar potential, generated

by the target atom at the same  $t$  and  $\mathbf{x}$ .<sup>1</sup> Throughout the book the indices A and I stand for the atom and ion, respectively. The scalar potential, created by the target in the collision, is a solution of Poisson's equation

$$\Delta\varphi_A(\mathbf{x}, t) = -4\pi\varrho_A(\mathbf{x}, t), \quad (2.2)$$

where  $\varrho_A(\mathbf{x}, t)$  is the transition charge density of the target.

Assuming that the collision velocity is sufficiently high, such that the electrons belonging to the ion and atom can be treated as distinguishable particles, the charge densities are written according to

$$\begin{aligned} \varrho_I(\mathbf{x}, t) &= \int d^3\mathbf{R}_I d^3\mathbf{r} \Psi_{I,f}^*(\mathbf{R}_I, \mathbf{r}, t) [Z_I \delta(\mathbf{x} - \mathbf{R}_I) - \delta(\mathbf{x} - \mathbf{r})] \Psi_{I,i}(\mathbf{R}_I, \mathbf{r}, t), \\ \varrho_A(\mathbf{x}, t) &= \int d^3\mathbf{R}_A d^3\boldsymbol{\lambda} \Psi_{A,f}^*(\mathbf{R}_A, \boldsymbol{\lambda}, t) [Z_A \delta(\mathbf{x} - \mathbf{R}_A) - \delta(\mathbf{x} - \boldsymbol{\lambda})] \Psi_{A,i}(\mathbf{R}_A, \boldsymbol{\lambda}, t). \end{aligned} \quad (2.3)$$

Within the first-order treatment  $\Psi_{I,i}$ ,  $\Psi_{A,i}$  and  $\Psi_{I,f}$ ,  $\Psi_{A,f}$  are approximated by unperturbed initial and final states, respectively, of the colliding particles. The form of these states is well known: they are a product of a plane-wave, representing the motion of the center of mass of the atomic particle, and a function describing the internal motion of the electron in the particle. Further, in (2.3)  $\mathbf{R}_I$  is the coordinate of the projectile nucleus,  $\mathbf{r}$  is the coordinate of the projectile electron with respect to the projectile nucleus,  $\mathbf{R}_A$  the coordinate of the target nucleus and  $\boldsymbol{\lambda}$  the coordinate of the target electron with respect to the target nucleus.

The target scalar potential and the integrals in (2.1) are conveniently evaluated by using Fourier transforms for the charge densities  $\varrho_I$ ,  $\varrho_A$  and the scalar potential  $\varphi_A$ , e.g.

$$\varrho_A(\mathbf{x}, t) = \frac{1}{4\pi^2} \int d\omega d^3\mathbf{k} \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t) \xi_A(\mathbf{k}, \omega),$$

where  $\xi_A$  is the Fourier transform of  $\varrho_A$ . Using the standard procedure of obtaining a cross section from a known  $S$ -matrix transition element, one can show that the cross section for a collision, in which the electron of the projectile makes a transition from an initial internal state  $\psi_0$  into a final internal state  $\psi_n$  and the electron of the target makes a transition from its internal initial state  $u_0$  to a final state  $u_m$ , is given by

$$\sigma_{0 \rightarrow n}^{0 \rightarrow m} = \frac{4}{v^2} \int d^2\mathbf{q}_\perp \frac{|F_{0n}^I(\mathbf{q}) F_{0m}^A(-\mathbf{q})|^2}{q^4}. \quad (2.4)$$

Here  $\mathbf{q} = (\mathbf{q}_\perp, q_{\min})$  is the momentum transfer to the projectile where,  $\mathbf{q}_\perp$  is the two-dimensional part of the momentum, which is perpendicular to the

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<sup>1</sup> Of course, one can take the  $S$ -matrix element in a fully equivalent form where the target charge density is coupled with the projectile scalar potential.

collision velocity  $\mathbf{v}$ , and  $q_{\min}$  is the minimum momentum transfer to the projectile given by

$$q_{\min} = \frac{\varepsilon_n - \varepsilon_0 + \epsilon_m - \epsilon_0}{v}. \quad (2.5)$$

In (2.5)  $\varepsilon_{0(n)}$  and  $\epsilon_{0(m)}$  are the initial (final) electron energies in the internal states of the projectile and target, respectively. Further, in (2.4)

$$\begin{aligned} F_{0n}^{\text{I}}(\mathbf{q}) &= Z_{\text{I}}\delta_{n0} - \langle \psi_n | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_0 \rangle, \\ F_{0m}^{\text{A}}(\mathbf{q}) &= Z_{\text{A}}\delta_{m0} - \langle u_m | \exp(i\mathbf{q} \cdot \boldsymbol{\rho}) | u_0 \rangle \end{aligned} \quad (2.6)$$

are the form-factors of the ion and atom.

When we consider collisions of a projectile carrying initially an electron with a target, we will be interested in the study of those collisions, where the projectile electron makes a transition, i.e. when it gets excited or lost and  $n \neq 0$ . In what follows we will not consider collisions where  $n = 0$ , i.e. collisions which are elastic for the projectile. While final states of the projectile are observed in experiment, there is often no experimental information about the final state of the target. Therefore, in order to describe theoretically such a situation, one has to calculate the cross section

$$\sigma_{0 \rightarrow n} = \sum_m \sigma_{0 \rightarrow n}^{0 \rightarrow m}, \quad (2.7)$$

where the summation has to be performed over all possible final states of the target including the continuum. It is convenient to split the first order cross section (2.7) into two parts and discuss them separately.

### 2.1.1 Elastic Target Mode

One part represents the contribution to the cross section (2.7) from collisions in which the target electron remains in the initial state, i.e. from collisions, where this electron can be considered as ‘passive’. This part reads

$$\sigma_{0 \rightarrow n}^{\text{s}} = \frac{4}{v^2} \int d^2\mathbf{q}_{\perp} Z_{\text{A,eff}}^2(\mathbf{q}_0) \frac{|\langle \psi_n | \exp(i\mathbf{q}_0 \cdot \mathbf{r}) | \psi_0 \rangle|^2}{q^4}. \quad (2.8)$$

Here  $\mathbf{q}_0 = (\mathbf{q}_{\perp}, \frac{\varepsilon_n - \varepsilon_0}{v})$  and  $Z_{\text{A,eff}} = Z_{\text{A}} - \langle u_0 | \exp(-i\mathbf{q}_0 \cdot \boldsymbol{\rho}) | u_0 \rangle$  is the effective charge of the target which is ‘seen’ by the electron of the projectile in collisions where the target does not change its internal state. Considering this effective charge as a function of the momentum transfer, one can note the following important points (see also [2, 21]). The value of the effective charge  $Z_{\text{A,eff}}$  varies in the limits  $Z_{\text{A}} - 1 < Z_{\text{A,eff}} < Z_{\text{A}}$ .<sup>2</sup> The charge  $Z_{\text{A,eff}}$  approaches its lower and upper limits in collisions where the momentum transfer  $q_0$  is

<sup>2</sup> If the target contains  $N_{\text{A}}$  electrons then  $Z_{\text{A}} - N_{\text{A}} < Z_{\text{A,eff}} < Z_{\text{A}}$ .

much lower and much larger, respectively, than a typical electron momentum in the initial target state. It is seen that the effect of the target electron(s) in collisions, where the target remains in its initial internal state, is to weaken the field of the target nucleus acting on the projectile electron, i.e. to partially or completely screen the nucleus.

The projectile–target collision mode, in which the target does not change its internal state (while the projectile does), is often called the *elastic mode*, implying that it is elastic only for the target. This mode is also referred to as *screening* because of the screening (or shielding) role of the atomic electrons, which in this mode counteract to the field of the atomic nucleus and reduce the total atomic field acting on the electron of the ion. Below we will use both these expressions.

### 2.1.2 Inelastic Target Mode

The second part of the cross section (2.7) describes collisions in which the target electron makes transitions. It reads

$$\begin{aligned}\sigma_{0 \rightarrow n}^a &= \sum_{m \neq 0} \sigma_{0 \rightarrow n}^{0 \rightarrow m} \\ &= \frac{4}{v^2} \sum_{m \neq 0} \int d^2 \mathbf{q}_\perp \frac{|\langle u_m | \exp(-i\mathbf{q} \cdot \boldsymbol{\rho}) | u_0 \rangle \langle \psi_n | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_0 \rangle|^2}{q^4}.\end{aligned}\tag{2.9}$$

Equation (2.9) deals with the collision mode where not only the electron of the projectile but also that of the target are ‘active’ in the collision. This collision mode is called *doubly inelastic* or simply *inelastic*.

According to the first order approximation, the inelastic mode is not influenced by the interaction between the electron of the projectile and the nucleus of the target and the projectile electron undergoes a transition solely due to the interaction with the electron of the target. The latter is sometimes referred to as the two-center dielectronic interaction (TCDI) [1].

Contributions from collisions, in which the target changes its initial internal state, increase the total cross section (2.7). This action of the target electron is just opposite to that in the elastic mode, where the electron by screening the target nucleus decreases the cross section value compared to that in collisions with the bare atomic nucleus. Therefore, the inelastic collision mode is also often termed as *antiscreening*.

### 2.1.3 Collisions with Large Momentum Transfer. Free Collision Model

Let us consider collisions in which the minimum momentum transfer  $q_{\min}$ , given by (2.5), and, thus, the total momentum transfer  $q$  are much larger than

a typical momentum of the target electron in the initial target state. Such a situation can occur if the atomic number  $Z_I$  of the projectile substantially exceeds that of the target  $Z_A$  and the collision velocity is not too high.

*Elastic Mode.* For the elastic mode the effective charge  $Z_{A,\text{eff}} = Z_A - \langle u_0 | \exp(-i\mathbf{q}_0 \cdot \boldsymbol{\rho}) | u_0 \rangle$ , because of the rapid oscillations of the integrand due to the factor  $\exp(-i\mathbf{q}_0 \cdot \boldsymbol{\rho})$ , becomes approximately equal to the charge  $Z_A$  of the bare target nucleus. Therefore, in collisions with a large momentum transfer the shielding effect of the target electron is very weak and the transition of the electron of the projectile is almost solely caused by its interaction with the nucleus of the atomic target.

*Inelastic Mode.* In collisions with large momentum transfers the rapid oscillations of the term  $\exp(-i\mathbf{q} \cdot \boldsymbol{\rho})$  in the integrands of the transition matrix elements  $\langle u_m | \exp(-i\mathbf{q} \cdot \boldsymbol{\rho}) | u_0 \rangle$  can make them negligible. These oscillations, however, can be compensated in the case when final states of the target electron are continuum states, where the electron momentum  $\mathbf{k}$  with respect to the target nucleus is close to  $-\mathbf{q}$ , i.e. where  $\mathbf{k} \approx -\mathbf{q}$  or, by separating the transverse and longitudinal parts,  $\mathbf{k}_\perp \approx -\mathbf{q}_\perp$  and  $k_z \approx -q_{\min}$ .

The condition  $\mathbf{k}_\perp \approx -\mathbf{q}_\perp$  simply implies that nearly the whole transverse momentum transfer to the target has to be taken by the target electron alone.

More insight into the collision physics can be obtained by considering the condition  $k_z \approx -q_{\min}$ . Taking into account the explicit form of  $q_{\min}$ , this condition can be rewritten as a quadratic equation for  $k_z$  with the solutions

$$k_z^\pm \approx -v \pm \sqrt{v^2 - k_\perp^2 - 2(\varepsilon_n - \varepsilon_0 - \epsilon_0)}. \quad (2.10)$$

If  $\frac{v^2}{2} < (\varepsilon_n - \varepsilon_0 - \epsilon_0) \approx (\varepsilon_n - \varepsilon_0)$ , then both roots in (2.10) are complex. Physically it means that in such a case, due to the restrictions imposed by the energy-momentum conservation in the collision, there are no target states where the rapidly oscillating factor  $\exp(-i\mathbf{q} \cdot \boldsymbol{\rho})$  can be compensated by a similar term arising from the final motion of the target electron. As a result, the inelastic contribution (2.9) to the cross section (2.7) is negligible in this case.

The roots  $k_z^\pm$ , given by (2.10), become real if  $\frac{v^2}{2} > (\varepsilon_n - \varepsilon_0 + k_\perp^2/2 - \epsilon_0)$ . If, in addition, we assume that  $\frac{v^2}{2} \gg (\varepsilon_n - \varepsilon_0 + k_\perp^2/2 - \epsilon_0)$ , then these roots are given by  $k_z^+ \approx -(0.5k_\perp^2 + \varepsilon_n - \varepsilon_0 - \epsilon_0)/v$  and  $k_z^- \approx -2v$ . In the rest frame of the projectile these roots correspond to an electron having the  $z$ -component of the momentum approximately equal to  $v$  and  $-v$ , respectively, where  $v > 0$  is the velocity of the incident target. Analysis shows that the contribution of the electrons with  $k_z \approx k_z^-$  to the inelastic cross section is much smaller than that of the electrons with  $k_z \approx k_z^+$  and can be neglected. A rough estimate for the contribution to the inelastic cross section (2.9) from collisions in which  $k_z \approx -(0.5k_\perp^2 + \varepsilon_n - \varepsilon_0)/v$  can be easily obtained if one neglects the dependence of  $q_{\min}$  on the final energy of the target electron. In such a case the integration over the final continuum states of the target electron in (2.9)

is elementary performed by assuming that, because of large  $k$ , these states can be approximated by plane waves. The result is

$$\sigma_{0 \rightarrow n}^a \simeq \frac{4}{v^2} \int d^2 \mathbf{q}_\perp \frac{|\langle \psi_n | \exp(i \mathbf{q}_0 \cdot \mathbf{r}) | \psi_0 \rangle|^2}{q_0^4}, \quad (2.11)$$

where  $\mathbf{q}_0 = (\mathbf{q}_\perp, \varepsilon_n - \varepsilon_0/v)$ . This cross section can be interpreted as describing transitions of the electron in the projectile under the action of a fast free electron which has initially velocity  $v$  with respect to the projectile.

Combining (2.11) and (2.8) and taking into account that  $Z_{A,\text{eff}} \approx Z_A$ , we see that the cross section (2.7) in collisions with large momentum transfers can be approximated by

$$\sigma_{0 \rightarrow n} \approx (Z_A^2 + 1) \sigma_{0 \rightarrow n}^{\text{pr}}. \quad (2.12)$$

In the above expression  $\sigma_{0 \rightarrow n}^{\text{pr}}$  is the cross section for collisions in which the projectile electron makes a transition  $0 \rightarrow n$  due to the interaction with a point-like unit charge moving with velocity  $v$  in the projectile frame. According to (2.12) the target nucleus and the target electron act incoherently in the collision. If the atom has initially  $Z_A$  electrons the factor  $Z_A^2 + 1$  in (2.12) should be replaced by  $Z_A^2 + Z_A$ . Equation (2.12) is the essence of the free collision model introduced long ago by Bohr [23]. This model, in particular, suggests that the relative importance of the elastic mode in the projectile–target collisions should rapidly increase with increasing atomic number of the neutral target.

The free collision model is quite simple and physically appealing but not very accurate. Better results for the cross sections can be obtained by applying the so called impulse approximation which is closely related to the free collision model. The application of the impulse approximation to the projectile electron excitation and loss was discussed in a review article [22] where also references to original articles were given. The impulse approximation takes into account the inner motion of the electrons in the target atom by averaging the projectile cross sections over the momentum distribution of these electrons in their initial bound state. An insightful discussion of the relationship between the plane-wave Born and impulse approximations was presented in [24].

## 2.2 Semi-Classical Approach

In the theory of fast ion–atom collisions quite often only electrons are treated quantum mechanically whereas the nuclei of the colliding partners are regarded as classical particles and their relative motion is described in terms of a classical trajectory. Such an approach is called *semi-classical*. Although the impact parameter, according to quantum mechanics, in general does not represent a measurable quantity, the semi-classical approach has important

merits. First, by considering transition probabilities as a function of the impact parameter, one can get an additional insight into the collision physics. Second, the impact parameter consideration is usually more convenient for developing treatments which go beyond the first order approximation in the projectile–target interaction. Third, formulating a theory in terms of impact parameter allows one to apply the independent electron approximation for evaluating cross sections of multielectron transitions.

Let us now consider the projectile–target collision using the semi-classical approach<sup>3</sup>. We shall assume that the target nucleus, having a charge  $Z_A$ , is at rest and taken as the origin of our reference frame. In this frame the nucleus of the projectile-ion with a charge  $Z_I$  ( $Z_I \gg 1$ ) moves along a straight-line classical trajectory  $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$ , where  $\mathbf{b}$  is the impact parameter and  $\mathbf{v}$  the projectile velocity. A straight-line trajectory becomes a good approximation starting with collision energies of a few thousand electron volts and is certainly an excellent approximation for the collision energies of interest for this book. The projectile initially carries an electron bound in the ground state.

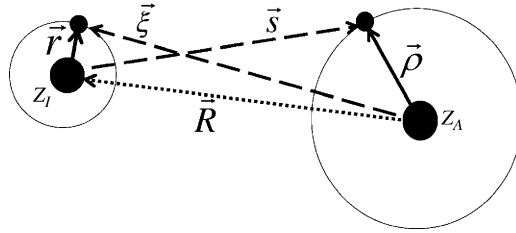
For simplicity we shall consider that the target also has only one electron. We denote the coordinates of the electron of the target and that of the projectile, given with respect to the target nucleus, by  $\boldsymbol{\rho}$  and  $\boldsymbol{\xi}$ , respectively (see Fig. 2.1). Further,  $\mathbf{s}$  and  $\mathbf{r}$  are coordinates of the target and projectile electrons with respect to the projectile nucleus.

The electronic system of the colliding particles is described by the time-dependent Schrödinger equation

$$\left( i \frac{\partial}{\partial t} - H_I^{\text{el}} - H_A^{\text{el}} - V \right) \Psi(\mathbf{r}, \boldsymbol{\rho}, t) = 0, \quad (2.13)$$

where  $\Psi(\mathbf{r}, \boldsymbol{\rho}, t)$  is the time-dependent wave function describing the electronic degrees of freedom. In (2.13)  $H_I^{\text{el}}$  and  $H_A^{\text{el}}$  are the electronic Hamiltonians of the ion and atom, respectively, and

$$V = \frac{Z_I Z_A}{R(t)} - \frac{Z_I}{s} - \frac{Z_A}{\xi} + \frac{1}{|\boldsymbol{\xi} - \boldsymbol{\rho}|} \quad (2.14)$$



**Fig. 2.1.** Schematic representation of space coordinates characterizing the projectile–target collision.

<sup>3</sup> To our knowledge, for the projectile-electron excitation and loss the semi-classical approach was for the first time applied in [25].

is the interaction between the ion and the atom. The term  $Z_I Z_A/R(t)$ , representing the inter-nuclear interaction, is independent of the electron coordinates. In fast collisions this interaction does not influence cross sections for the electron transitions (integrated over the impact parameter) and below will be ignored.

Within the first order approximation, and assuming again that the electrons of the ion and atom are distinguishable, the initial and final states of the electrons of the colliding particles are approximated by the product of the undistorted initial and final states of the colliding particles

$$\begin{aligned}\chi_i(t) &= u_0(\boldsymbol{\rho}) \exp(-i\epsilon_0 t) \psi_0(\boldsymbol{\xi} - \mathbf{R}(t)) \exp(-i\varepsilon_0 t) \exp(i\mathbf{v} \cdot \boldsymbol{\xi} - iv^2 t/2), \\ \chi_f(t) &= u_m(\boldsymbol{\rho}) \exp(-i\epsilon_m t) \psi_n(\boldsymbol{\xi} - \mathbf{R}(t)) \exp(-i\varepsilon_n t) \exp(i\mathbf{v} \cdot \boldsymbol{\xi} - iv^2 t/2).\end{aligned}\quad (2.15)$$

In (2.15)  $u_0$  and  $u_m$  are the initial and final internal states of the target, respectively, given in the target frame. Further,  $\psi_0$  and  $\psi_n$  have similar meanings but are for the projectile and given in the projectile rest frame. The term  $\exp(i\mathbf{v} \cdot \boldsymbol{\xi}) \exp(-iv^2 t/2)$  is the so called translational factor<sup>4</sup> (see e.g. [6, 16]).

The semi-classical first order amplitude reads

$$a_{fi}^{(1)}(\mathbf{b}) = -i \int_{-\infty}^{+\infty} dt \langle \chi_f(t) | V(t) | \chi_i(t) \rangle. \quad (2.16)$$

Inserting the states (2.15) into expression (2.16) and keeping in mind that we consider only collisions, in which the internal state of the projectile changes ( $n \neq 0$ ), it is not difficult to obtain that

$$\begin{aligned}a_{fi}^{(1)}(\mathbf{b}) &= i \int_{-\infty}^{+\infty} dt \exp(i(\varepsilon_n + \epsilon_m - \varepsilon_0 - \epsilon_0)t) \\ &\times \left\langle \psi_n u_m \left| \frac{Z_A}{|\mathbf{R}(t) + \mathbf{r}|} - \frac{1}{|\mathbf{R}(t) + \mathbf{r} - \boldsymbol{\rho}|} \right| \psi_0 u_0 \right\rangle.\end{aligned}\quad (2.17)$$

The straightforward generalization of expression (2.17) to the case, when the atom has  $Z_A$  electrons, yields

$$\begin{aligned}a_{fi}^{(1)}(\mathbf{b}) &= i \int_{-\infty}^{+\infty} dt \exp(i(\varepsilon_n + \epsilon_m - \varepsilon_0 - \epsilon_0)t) \\ &\times \left\langle \psi_n u_m \left| \frac{Z_A}{|\mathbf{R}(t) + \mathbf{r}|} - \sum_{j=1}^{Z_A} \frac{1}{|\mathbf{R}(t) + \mathbf{r} - \boldsymbol{\rho}_j|} \right| \psi_0 u_0 \right\rangle.\end{aligned}\quad (2.18)$$

<sup>4</sup> This factor appears because it is the wave function  $\psi_n(\boldsymbol{\xi} - \mathbf{R}(t)) \exp(i\mathbf{v} \cdot \boldsymbol{\xi}) \exp(-iv^2 t/2 - i\varepsilon_n t)$  (and not merely  $\psi_n(\boldsymbol{\xi} - \mathbf{R}(t)) \exp(-i\varepsilon_n t)$ ), which represents an exact solution of the Schrödinger equation for an undistorted atomic system moving with a constant velocity  $\mathbf{v}$  in an inertial reference frame.



By applying the integral representation

$$\frac{1}{|\mathbf{x}|} = \frac{1}{2\pi^2} \int d^3\mathbf{k} \frac{\exp(i\mathbf{k} \cdot \mathbf{x})}{k^2} \quad (2.19)$$

to the Coulomb potentials  $1/|\mathbf{R}(t) + \mathbf{r}|$  and  $1/|\mathbf{R}(t) + \mathbf{r} - \boldsymbol{\rho}_j|$ , the amplitude (2.18) is transformed into

$$\begin{aligned} a_{fi}^{(1)}(\mathbf{b}) &= \frac{i}{\pi v} \int d^2\mathbf{q}_\perp \exp(-i\mathbf{q}_\perp \cdot \mathbf{b}) \left\langle u_m \left| Z_A - \sum_j^{Z_A} \exp(-i\mathbf{q} \cdot \boldsymbol{\rho}_j) \right| u_0 \right\rangle \\ &\quad \times \frac{\langle \psi_n | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_0 \rangle}{q^2}, \end{aligned} \quad (2.20)$$

where  $\mathbf{q} = (\mathbf{q}_\perp, q_{\min})$  with  $q_{\min}$  given by (2.5). It is easy to show [25] that the semi-classical first-order cross section

$$\sigma_{0 \rightarrow n}^{0 \rightarrow m} = \int d^2\mathbf{b} \left| a_{fi}^{(1)}(\mathbf{b}) \right|^2 \quad (2.21)$$

coincides with that following from the plane-wave Born approximation.

Relativistic Collisions of Structured Atomic Particles

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2008, XII, 286 p. 63 illus., Hardcover

ISBN: 978-3-540-78420-3