Linear Systems Control

Errata

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p. 297.

Equation (5.7) should be

$$J^* = \int_{t_0}^{t_1} F(\mathbf{x}^*(t),\ \dot{\mathbf{x}}^*(t),\ t)dt\ .$$

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p. 320.

Example 5.5. **Steady State LQR for the Double Integrator**

The state feedback control law (last equation on page) should be:

$$u(t) = -\mathbf{K}_\infty \mathbf{x}(t) = -\mathbf{R}_2^{-1}\mathbf{B}^T\mathbf{P}_\infty\mathbf{x}(t) = \frac{1}{r_2}[p_{12} \quad p_{22}]$$

$$= -\left[\sqrt{\frac{r_p}{r_2}} \quad \sqrt{\frac{r_p}{r_2}}\sqrt{2\sqrt{r_p r_2} + r_v}\right]\mathbf{x}(t).$$

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p. 346.

Problem 5.8

The equation at the top of the page should read:

$$J = \frac{1}{2}\mathbf{x}^T(t_1)\mathbf{x}(t_1) + \int_{t_0}^{t_1} (\mathbf{x}^T(t)\mathbf{x}(t) + u^2(t))\ dt\,.$$

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p. 346.

Problem 5.10

The problem text should read:

> a. Find an optimal steady state control for the system. Let the input weight parameter for the LQR index be $\mathbf{R}_2 = [r_2]$.
> b. Sketch the movement of the poles of the closed loop system as a function of the weight parameter r_2. Use the numerical values $\alpha = 7.14$ rad/sec, $\beta = 286.3$ rad/(V sec^2).
> c. If the motor damping parameter is (or is not) zero, i.e. $[A_{22}] = \alpha = 0$ (or $[A_{22}] = \alpha \neq 0$) then what does the root curve look like? Explain by inspection, do not calculate.
> d. Find the value of r_2 such that the closed loop response has a 50 m sec response time. Determine by computation or simulation the response of the closed loop system given the initial conditions: $\theta(0) = 0.3$ rad and $\theta(0) = 0$ rad/sec.

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pages 442-443.

***Example 7.2.* Estimation of the States of a Hydraulic Servo**

In this example an error has been made in the correct evaluation of the noise input to the system. This comes about because of the simplification of the problem to the harmonic oscillator form. The formulation of the state equations of the system in equation (7.33) is incorrect when noise is taken into account. Taking the state noise sources into account, equation (7.33) should be:

$$\dot{v}(t) = -\frac{C_f}{M}v + \frac{A_c}{M}\Delta p + v_{11}(t),$$

$$\dot{\Delta p}(t) = -\frac{2\beta A_c}{V}v + \frac{2\beta}{V}q + v_{12}(t),$$

where $v_{11}(t)$ and $v_{12}(t)$ are the state white noise sources. Now multiplying the second equation throuqy by A_c/M, one finds a new set of state equations:

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$$\dot{v}(t) = -\frac{C_f}{M}v + \frac{A_c}{M}\Delta p + v_{11}(t),$$

$$\frac{A_c}{M}\Delta\dot{p}(t) = -\frac{2\beta A_c^2}{MV}v + \frac{2\beta A_c}{MV}q + \frac{A_c}{M}v_{12}(t).$$

Now making the variable substitutions $u = q$, $x_1 = v$, $x_2 = (A_c/M)\Delta p$, $\omega_0^2 = 2\beta A_c^2/(MV)$ and $\bar{\omega}_0^2 = 2\beta A_c/(MV)$, and ignoring the small damping term, C_c/M, these equations become

$$\dot{x}_1(t) = x_2 + v_{11}(t),$$

$$\dot{x}_2(t) = -\omega_0^2 x_1 + \bar{\omega}_0^2 u + a_{cM}v_{12}(t),$$

where $a_c = A_c/M$. This means that the matrix $\mathbf{B}_v$ is

$$\mathbf{B}_v = \begin{bmatrix} 1 & 0 \\ 0 & a_c \end{bmatrix}.$$

Thus equation (7.36) should be

$$\mathbf{0} = \mathbf{B}_v\mathbf{V}_1\mathbf{B}_v^T - \mathbf{QC}^T\mathbf{V}_2^{-1}\mathbf{CQ} + \mathbf{AQ} + \mathbf{QA}^T$$

$$\mathbf{0} = \begin{bmatrix} 1 & 0 \\ 0 & a_c \end{bmatrix}\begin{bmatrix} V_{111} & 0 \\ 0 & V_{122} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & a_c \end{bmatrix} - \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{V_2}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix}\begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} + \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix}\begin{bmatrix} 0 & -\omega_0^2 \\ 1 & 0 \end{bmatrix}$$

and equation (7.37) should be:

$$0 = V_{111} - \frac{q_{11}^2}{V_2} + q_{12},$$

$$0 = 0 - \frac{q_{11}q_{12}}{V_2} - \omega_0^2 q_{11} + q_{22},$$

$$0 = a_c^2 V_{122} - \frac{q_{12}^2}{V_2} - 2\omega_0^2 q_{12}.$$

The solutions of these equations are:

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$$
\begin{aligned}
q_{12} &= -\omega_0^2 V_2 \pm \sqrt{\omega_0^4 V_2^2 + a_c^2 V_{122} V_2},\\
q_{11} &= \sqrt{V_2(V_{111} + 2q_{12})},\\
q_{22} &= \omega_0^2 q_{11} + \frac{q_{11}q_{12}}{V_2},
\end{aligned}
$$

where the positive sign has to be selected in the first equation in order for **Q** to be positive definite. The Kalman gain is then as given in equation (7.39) in the testbook.

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p. 470.

Equation (7.170) should read:

$$
\begin{aligned}
\dot{\mathbf{Q}}_{11} &= [\mathbf{A}(t) - \mathbf{L}(t)\mathbf{C}(t)]\mathbf{Q}_{11} + \mathbf{Q}_{11}[\mathbf{A}(t) - \mathbf{L}(t)\mathbf{C}(t)]^T\\
&\quad + \mathbf{B}_v(t)\mathbf{V}_1(t)\mathbf{B}_v^T(t) + \mathbf{L}(t)\mathbf{V}_2(t)\mathbf{L}^T(t).
\end{aligned}
$$

Equation (7.173) should then be:

$$
\begin{aligned}
\dot{\mathbf{Q}}_{11} &= \mathbf{A}(t)\mathbf{Q}_{11}(t) + \mathbf{Q}_{11}(t)\mathbf{A}^T(t) - \mathbf{L}(t)\mathbf{C}(t)\mathbf{Q}_{11}(t) - \mathbf{Q}_{11}(t)\mathbf{C}^T(t)\mathbf{L}^T(t)\\
&\quad + \mathbf{B}_v(t)\mathbf{V}_1(t)\mathbf{B}_v^T(t) + \mathbf{L}(t)\mathbf{V}_2(t)\mathbf{L}^T(t).
\end{aligned}
$$

Then equation (7.174) should be changed to:

$$
\mathbf{B}_v(t)\mathbf{V}_1(t)\mathbf{B}_v^T(t) - \mathbf{Q}_{11}(t)\mathbf{C}^T(t)\mathbf{V}_2^{-1}(t)\mathbf{C}(t)\mathbf{Q}_{11}(t).
$$

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p. 473.

Example 7.10. **LQG Controller for the Hydraulic Servo**

There is an error in the second last paragraph on the page. This paragraph should read:

To design a Kalman filter the state and measurement noise intensities must be specified as well as the noise scaling matrix, $\mathbf{B}_v$. Here it is given that $\mathbf{B}_v = diag[1 \quad (A_c/M)] = diag[1 \quad 300]$ (see example 7.2, p. 442, above) and the noise intensities selected were $\mathbf{V}_1 = diag[1 \cdot 10^5 \quad 500]$ and $V_2 = diag[1]$. This gives the Kalman gain,

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$\mathbf{L} = 10^2 \cdot [3.17 \quad 1.07]^T$. The eigenfrequencies of the Kalman filter can be determined to be $\lambda_0 = (-1.583 \pm j4.302) \cdot 10^2$. These results were found using the MATLAB routine, **`lqe:`** **`[L,P,Este] = lqe(A,G,C,V1,V2,N), G = Bv*V1*Bv', N = 0`**. This routine also yielded the following results for the variances of the estimation error and state estimates:

$$\mathbf{Q}_{11} = \begin{bmatrix} 3.17 \cdot 10^{-5} & 10.7 \cdot 10^{-5} \\ 10.7 \cdot 10^{-5} & 6.65 \end{bmatrix} \cdot 10^7,$$

$$\mathbf{Q}_{22} = \begin{bmatrix} 9.91 \cdot 10^{-6} & -5.01 \cdot 10^{-3} \\ -5.01 \cdot 10^{-3} & 3.32 \end{bmatrix} \cdot 10^7.$$

Simulation proved less reliable, probably due to the sample time which was used for practical reasons (1 msec) and the fact that only one noise realization was used. In principle the results of 10 noise realizations (10 different seeds) should have been averaged to find the final result. The results using simulink were incorrect with a factor of two. The difficulty is the large spread in the numbers and the difficulty involved in integrating numerically the white noise in the dynamic system using Matlab/Simulink. For an excellent simulation study of the a similar system one should see the book by Gran, (2007), p. 184.

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