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## Preface

*Complex Nonlinearity: Chaos, Phase Transitions, Topology Change and Path Integrals* is a graduate-level monographic textbook. This is a book about *prediction & control* of general *nonlinear dynamics* of high-dimensional *complex systems* of various physical and non-physical nature and their underpinning geometro-topological change.

The book starts with a textbook-like expose on *nonlinear and chaotic dynamics* (Chapter 1). After an introduction into attractors and chaos, a brief history of chaos theory is given. Then, temporal chaotic dynamics is developed, both in its continuous form (nonlinear differential equations) and in its discrete form (nonlinear iteration maps). Spatio-temporal chaotic dynamics of nonlinear partial differential equations follows with some physiological examples. The Chapter ends with modern techniques of chaos-control, both temporal and spatio-temporal.

The *dynamical edge of chaos* physically corresponds to the *phase transitions*. Therefore, Chapter 2 continues exposé on complex nonlinearity, from the point of view of phase transitions and the related field of *synergetics*. After the introduction and classification of equilibrium phase transitions, a brief on Landau's theory is given (providing a background for order-parameters and synergetics). The concept is subsequently generalized into non-equilibrium phase transitions, together with important examples of oscillatory, fractal and noise-induced transitions. This core Chapter of the book also introduces the concept of partition function, together with its general, path-integral description. After that the basic elements of Haken's synergetics are presented, and subsequently developed into synergetics of *attractor neural networks*.

While the natural stage for linear dynamics comprises of flat, Euclidean geometry (with the corresponding calculation tools from linear algebra and analysis), the natural stage for nonlinear dynamics is curved, *Riemannian geometry* (with the corresponding tools from tensor algebra and analysis). In both cases, the system's (kinetic) energy is defined by the metric form, either Euclidean or Riemannian. The extreme nonlinearity – chaos – corresponds to the *topology change* of this curved geometrical stage, usually called configu-

ration manifold. Chapter 3 elaborates on geometry and topology change in relation with complex nonlinearity and chaos.

Chapter 4 develops general nonlinear dynamics, both continuous and discrete, deterministic and stochastic, in the unique form of *path integrals* and their *action–amplitude formalism*. This most natural framework for representing both phase transitions and topology change starts with *Feynman’s sum over histories*, to be quickly generalized into the *sum over geometries and topologies*. This Chapter also gives a brief on general dynamics of fields and strings, as well as a path–integral based introduction on the chaos field theory. The Chapter concludes with a number of non–physical examples of complex nonlinear systems defined by path integrals.

The last Chapter puts all the previously developed techniques together and presents the *unified form of complex nonlinearity*. Here we have chaos, phase transitions, geometrical dynamics and topology change, all working together in the form of path integrals. The concluding section is devoted to discussion of hard vs. soft complexity, using the synergetic example of human bio-mechanics.

The objective of the present monograph is to provide a serious reader with a serious scientific tool that will enable them to actually *perform* a competitive research in modern complex nonlinearity. The monograph includes a comprehensive bibliography on the subject and a detailed index. For all mathematical questions, the reader is referred to our book *Applied Differential Geometry: A Modern Introduction*. World Scientific, Singapore, 2007.

Target readership for this monograph includes all researchers and students of complex nonlinear systems (in physics, mathematics, engineering, chemistry, biology, psychology, sociology, economics, medicine, etc.), working both in industry (i.e., clinics) and academia.

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