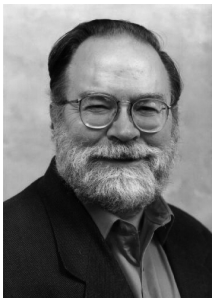
Editorial Introduction

This book is a collection of original papers and state-of-the-art contributions written by leading experts in the areas of differential equations, chaos and variational problems in honour of Arrigo Cellina and James A. Yorke, whose remarkable scientific carrier was a source of inspiration to many mathematicians, on the occasion of their 65th birthday.

Arrigo Cellina and James A. Yorke were born on the same day: August 3, 1941. Both received their Ph.D. degrees from the University of Maryland, where they met first in the late 1960s, at the Institute for Fluid Dynamics and Applied Mathematics. They had offices next to each other and though they were of the same age, Yorke was already Assistant Professor, while Cellina was a Graduate Student. Each one of them had a small daughter, and this contributed to their friendship.

Arrigo Cellina



James A. Yorke

Yorke arrived at the office every day with a provision of cans of Coca Cola, his daily ration, that he put in the air conditioning fan, to keep cool. Cellina says that he was very impressed by Yorke's way of doing mathematics; Yorke could prove very interesting new results using almost elementary mathematical tools, little more than second year Calculus.

From those years, he remembers for example the article *Noncontinuable solutions of differential-delay equations* where Yorke shows, in an elementary way but with a clever use of the extension theorem, that the basic theorem of continuation of solutions to ordinary differential equations cannot be valid for functional

equations (at that time very fashionable). In the article *A continuous differential equation in Hilbert space without existence*, Yorke gave the first example of the nonexistence of solutions to Cauchy problems for an ordinary differential in a Hilbert space. Furthermore, in a joint paper with one of his students, Saperstone, he proved a controllability theorem without using the hypothesis that the origin belongs to the interior of the set of controls. This is just a sample of important problems to which Yorke made nontrivial contributions.

Yorke went around always carrying in his pocket a notebook where he annotated the mathematical problems that seemed important for future investigation. In those years Yorke's collaboration with Andrezj Lasota began, which produced outstanding results in the theory of "chaos". Yorke became famous even in nonmathematical circles for his mathematical model for the spread of gonorrhoea. While traditional models were not in accord with experimental data, he proposed a simple model based on the existence of two groups of people and proved that this model fits well the experimental data. Later, in a 1975 paper entitled *Period three implies chaos* with T.Y. Lee, Yorke introduced a rigorous mathematical definition of the term "chaos" for the study of dynamical systems. From then on, he played a leading role in the further research on chaos, including its control and applications.

Yorke's goals to explore interdisciplinary mathematics were fully realized after he earned his Ph.D. and joined the faculty of the Institute for Physical Science and Technology (IPST), an institute established in 1950 to foster excellence in interdisciplinary research and education at the University of Maryland. He said: *All along the goal of myself and my fellow researchers here at Maryland has been to find the concepts that the applied scientist needs.* His chaos research group introduced many basic concepts with exotic names like *crises*, the *control of chaos, fractal basin boundary, strange non-chaotic attractors*, and the *Kaplan–Yorke dimension.* One remarkable application of Yorke's theory of chaos has been the weather prediction.

In 2003 Yorke shared with Benoit Mandelbrot of Yale University the prize for *Science and Technology of Complexity* of the Science and Technology Foundation of Japan for the *Creation of Universal Concepts in Complex Systems-Chaos and Fractals.* With this prize, Jim Yorke was recognized for his outstanding achievements in nonlinear dynamics that have greatly advanced the frontiers of science and technology.

Yorke's research has been highly influential, with some of his papers receiving hundreds of citations. He is the author of three books on chaos, of a monograph on gonorrhoea epidemiology, and of more than 300 papers in the areas of ordinary differential equations, dynamical systems, delay differential equations, applied and random dynamical systems.

He believes that a Ph.D. in mathematics is a licence to investigate the universe, and he has supervised over 40 Ph.D. dissertations in the departments of mathematics, physics and computer science.

Currently, Jim Yorke is a Distinguished University Professor of Mathematics and Physics, and Chair of the Mathematics Department of the University of Maryland.

Arrigo Cellina received a Ph.D. degree in mathematics in 1968 and went back to Italy, where he was Assistant Professor and then Full Professor at the Universities of Perugia, Florence, and Padua, at the International School for Advanced Studies (SISSA) in Trieste, and at the University of Milan. He was a member of the scientific committee and then Director (1999–2001) of the International Mathematical Summer Centre (CIME) in Florence, Italy, and also a member of the scientific council of CIM (International Centre for Mathematics) seated in Coimbra, Portugal. Presently he is Professor at the University of Milan "Bicocca" and coordinator of the Doctoral Program of this university.

In Italy, the International School for Advanced Studies (SISSA) was established in 1978, in Trieste, as a dedicated and autonomous scientific institute to develop top-level research in mathematics, physics, astrophysics, biology and neuroscience, and to provide qualified graduate training to Italian and foreign laureates, to train them for research and academic teaching.

SISSA was the first Italian school to set up post-laurea courses aimed at a Ph. D. degree (Doctor Philosophiae). Cellina was one of the professors, founders and, for several years, the Coordinator of the Sector of Functional Analysis and Applications at SISSA, from 1978 until 1996.

I was lucky to have been initiated to mathematical research on Aubin–Cellina's book *Differential inclusions* in a research seminar at the University of Bucharest. Three years later I began my Ph.D. studies on differential inclusions at SISSA, under the supervision of Arrigo Cellina. I arrived at SISSA coming from Florence where I spent a very rewarding and training period of one year as a Research Fellow of GNAFA under the supervision of Roberto Conti, and I remember that Arrigo welcomed me with a kindness equal to his erudition.

Always available to discuss and to help his students to overcome difficulties, not only of mathematical orders, Arrigo taught me a lot more than differential inclusions. I remember with great pleasure his beautiful lessons, the long hours of reflection in front of the blackboard in his office, as well as the walks along the sea or in the park of Miramare.

I remember SISSA of those days as a very exciting environment. A community of researchers worked there, while several others were visiting SISSA and gave short courses or seminars concerning their new results. The Sector of Functional Analysis and Applications was located in a beautiful place, close to the Castle of Miramare, and near the International Centre for Theoretical Physics (ICTP), with an excellent library where we could spend much of our time. Without a doubt, this has been a very fruitful and rewarding period of my life, both as a scientific and as a life experience. Cellina's contribution has been significant.

Cellina's scientific work has always been highly original, introducing entirely new techniques to attack the difficult problems he considered. He introduced the notion of *graph approximate selection* for upper semicontinuous multifunctions,

thus establishing a basic connection between ordinary differential inclusions and differential inclusions. He also introduced the *fixed-point approach* to prove the existence of differential inclusions based on *continuous selections from multifunctions with decomposable values*.

The Baire category method, for the analysis of differential inclusions without convexity assumptions, has been developed starting from Cellina's seminal paper *On the differential inclusion* $x' \in [-1, 1]$, published in 1980 by the *Rendiconti dell'Academia dei Lincei*. Eventually this method recently found applications to problems of the Calculus of Variations, without convexity or quasi-convexity assumptions, as well as to implicit differential equations. This year, it found even more striking new applications to the construction of deep counterexamples in the theory of multidimensional fluid flow.

More recently, Cellina's research activity was devoted to the area of the Calculus of Variations, where he obtained important results on the validity of the Euler-Lagrange equation, on the regularity of minimizers, on necessary and sufficient conditions for the existence of minima, and on uniqueness and comparison of minima without strict convexity.

The book *Differential Inclusions*, co-authored by Cellina with J. P. Aubin and published by Springer, as well as several of his eighty papers published in first-class journals, are now classic references to their subject. Cellina also edited several volumes with lectures and seminars of CIME sessions, published by Springer in the subseries *Fondazione C.I.M.E.* of the *Lecture Notes in Mathematics* series.

Cellina mentored ten Ph.D. students: seven of them while at SISSA and three others at the university of Milan. Among his former students, many are now Professors in prestigious universities in Italy, Portugal, Chile or other countries. Several more mathematicians continue to be inspired by his ground-breaking ideas.

Cellina and Yorke during the conference in Aveiro

In June 2006, I had the privilege to organize in Aveiro (Portugal) with my colleagues from the *Functional Analysis and Applications* research group, the conference *Views on ODEs*, in celebration of the 65th birthday of Arrigo Cellina and James A. Yorke. Several friends, former students and collaborators, presently leading experts in differential equations, chaos and variational problems, gathered in Aveiro on this occasion to discuss their new results. The present volume collects thirty-two original papers and state-of-the-art contributions of participants to this conference and brings the reader to the frontier of research in these modern fields of research.

I wish to thank Professor Haim Brezis for accepting to publish this book as a volume of the series *Progress in Nonlinear Differential Equations and Their Applications*. I also thank Thomas Hempfling for the professional and pleasant collaboration during the preparation of this volume. Finally, I gratefully acknowledge partial financial support from the Portuguese Foundation for Science and Technology (FCT) under the Project POCI/MAT/55524/2004 and from the *Mathematics and Applications* research unit of the University of Aveiro.

Aveiro, October 2007 Vasile Staicu

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